Answers & Solutions

JEE (Main)-28/01/2025, Evening Phase-1

[Computer Based Test (CBT) mode]

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.
(2) This test paper consists of 75 questions. Each subject (MPC) has 25 questions. The maximum marks are 300.
(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.
(4) Section - A : Attempt all questions.
(5) Section - B : Attempt all questions.
(6) Section - A (01-20) contains 20 multiple choice questions which have only one correct answer. Each question carries + 4 marks for correct answer and – 1 mark for wrong answer.
(7) Section - B (21-25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries + 4 marks for correct answer and -1 mark for wrong answer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

1. If $\sum_{r=1}^{13}\left\{\frac{1}{\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}\right\}=a\sqrt{3}+b, a, b\in\mathbb{Z}$, then a^2+b^2 is equal to:

(1) 2
(2) 8
(3) 10
(4) 4

Answer (2)

Sol. $S=\sum_{r=1}^{13}\frac{\sin\left(\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)-\left(\frac{\pi}{4}+\left(\frac{r-1}{4}\right)\frac{\pi}{6}\right)\right)}{\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)\sin\left(\frac{\pi}{4}+\left(\frac{(r-1)\pi}{6}\right)\right)}\times\frac{1}{\sin\left(\frac{\pi}{6}\right)}$

$$\Rightarrow S=2\sum_{r=1}^{13}\frac{\cos\left(\frac{\pi}{4}+\frac{(r-1)\pi}{6}\right)}{\sin\left(\frac{\pi}{4}+\frac{(r-1)\pi}{6}\right)}-\frac{\cos\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}{\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}$$

$$\Rightarrow S=2\sum_{r=1}^{13}\cot\left(\frac{\pi}{4}+\frac{(r-1)\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)$$

$$=2\left(\cot\left(\frac{\pi}{4}+\frac{0\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right)+\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{2\pi}{6}\right)\ldots$$

.

$$+\cot\left(\frac{\pi}{4}+\frac{12\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)$$

$$=2\left(\cot\left(\frac{\pi}{4}\right)-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right)=2\sqrt{3}-2$$

2. Bag B_1 contains 6 white and 4 blue balls, Bag B_2 contains 4 white and 6 blue balls, and Bag B_3 contains 5 white and 5 blue balls. One of the bags is selected at random and a ball is drawn from it. If the ball is white, then the probability, that the ball is drawn from Bag B_2, is

(1) $\frac{2}{5}$
(2) $\frac{1}{3}$
(3) $\frac{2}{3}$
(4) $\frac{4}{15}$

Answer (4)

Sol. $P\left(\frac{B_2}{w}\right)=\frac{P(B_2)\cdot P\left(\frac{w}{B_2}\right)}{P(w)}$

$$P(Bi) = \frac{1}{3}, i = 1,2,3$$

$$P\left(\frac{w}{B_1}\right) = \frac{6}{10}, P\left(\frac{w}{B_2}\right) = \frac{4}{10}, P\left(\frac{w}{B_3} = \frac{5}{10}\right)$$

$$P(w) = \sum_{i=1}^{3} P(B_i) P\left(\frac{w}{B_i}\right) = \frac{1}{3}\left(\frac{6}{10} + \frac{4}{10} + \frac{5}{10}\right)$$

$$= \frac{5}{10} = \frac{1}{2}$$

$$P\left(\frac{B_2}{w}\right) = \frac{\frac{1}{3} \cdot \frac{4}{10}}{\frac{1}{2}} = \frac{8}{30} = \frac{4}{15}$$

3. Let S be the set of all the words that can be formed by arranging all the letters of the word GARDEN. From the set S, one word is selected at random. The probability that the selected word will NOT have vowels in alphabetical order is:
(1) $\frac{1}{2}$
(2) $\frac{1}{3}$
(3) $\frac{1}{4}$
(4) $\frac{2}{3}$

Answer (1)

Sol. V : vowel in alphabet order

$$P(v) = 1 - P(\overline{v})$$

$$= 1 - \frac{\left(\frac{6!}{2!}\right)}{6!} = 1 - \frac{1}{2} = \frac{1}{2}$$

4. Let the coefficients of three consecutive terms T_r, T_{r+1} and T_{r+2} in the binomial expansion of $(a+b)^{12}$ be in a G.P. and let p be the number of all possible values of r. Let q be the sum of all rational terms in the binomial expansion $(\sqrt[4]{3} + \sqrt[3]{4})^{12}$. Then $p + q$ is equal to:
(1) 287
(2) 295
(3) 299
(4) 283

Answer (4)

Sol. Coefficient of

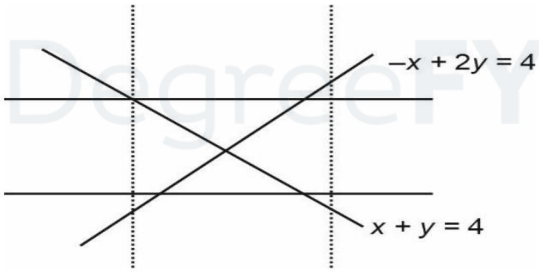
$$T_r, T_{r+1}, T_{r+2} \rightarrow GP$$
$$\Rightarrow \left({}^{12}C_r\right)^2 = {}^{12}C_{r-1} \cdot {}^{12}C_{r+1}$$

but no three consecutive binomial coefficient are in GP
$\Rightarrow P = 0$
Now for $\left(3^{1/4} + 4^{1/3}\right)^{12}, T_{r+1} = {}^{12}C_r (4)^{K/3} (3)^{\frac{12-K}{4}}$ for rational terms K = 0,12
sum of rational terms
$= {}^{12}C_0 4^0 \cdot 3^3 + {}^{12}C_{12} \cdot 4^4 \cdot 3^0$
$= 27 + 256 = 283 = q$
$\therefore p + q = 283$

5. Two equal sides of an isosceles triangle are along $-x + 2y = 4$ and $x + y = 4$. If m is the slope of its third side, then the sum, of all possible distinct values of m, is:
(1) $-2\sqrt{10}$
(2) 6
(3) 12
(4) -6

Answer (2)

Sol.



Slope of the third side = slope of the perpendicular bisector of given lines

$$h: \frac{-x + 2y - 4}{\sqrt{5}} = \pm \frac{x + y - 4}{\sqrt{2}}$$
$$h_1: \sqrt{2}(-x + 2y - 4) = \sqrt{5}(x + y - 4)$$
$$h_2: \sqrt{2}(-x + 2y - 4) = -\sqrt{5}(x + y - 4)$$
$$M_{L_1}: -\left[\frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - 2\sqrt{2}}\right]$$
$$M_{L_2}: -\left[\frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} + 2\sqrt{2}}\right]$$
$$M_{L_1} + M_{L_2} = -\left[\frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - 2\sqrt{2}} + \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} + 2\sqrt{2}}\right]$$
$$= -\left[\frac{(\sqrt{5} + \sqrt{2})(\sqrt{5} + 2\sqrt{2}) + (\sqrt{5} - \sqrt{2})(\sqrt{5} - 2\sqrt{2})}{-3}\right]$$
$$= 6$$

6. Let $A=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}$ and $P=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}, \theta>0$.

If $B=PAP^T, C=P^TB^{10}P$ and the sum of the diagonal elements of C is $\frac{m}{n}$, where $\gcd(m,n)=1$, then $m+n$ is:
(1) 127
(2) 65
(3) 2049
(4) 258

Answer (2)

Sol. $A=\begin{pmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{pmatrix}$

$$P=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$$
$$PP^T=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$$
$$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=I$$
$$C=P^TBB^9P$$
$$=P^TPAP^TB^9P$$
$$=AP^TPAP^TB^8P$$
$$=A^2P^TB^8P$$
$$\vdots$$
$$=A^{10}$$

Sum of diagonal elements of $A^{10}=\left(\frac{1}{\sqrt{2}}\right)^{10}+1$
$=\frac{1}{32}+1=\frac{33}{32}=\frac{m}{n}$

So, $m+n=32+33$
$=65$

7. If $\alpha+i\beta$ and $\gamma+i\delta$ are the roots of $x^2-(3-2i)x-(2i-2)=0, i=\sqrt{-1}$, then $\alpha\gamma+\beta\delta$ is equal to:
(1) 6
(2) -2
(3) 2
(4) -6

Answer (3)

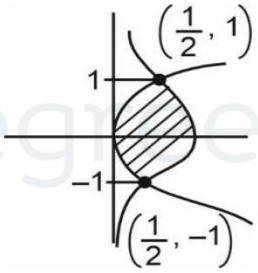
Sol.

$$x^2-(3-2i)x-(2i-2)=0\begin{matrix}\nearrow p\\ \searrow q\end{matrix}$$

$p+q=3-2i=1+(2-2i)$
$pq=2-2i=1(2-2i)$
$\Rightarrow p=1+0i, q=2-2i$
Comparing $\alpha\gamma=2\Rightarrow\alpha\gamma+\beta\delta=2\beta\delta=0(-2)$
8. The area of the region bounded by the curves $x\left(1+y^2\right)=1$ and $y^2=2x$ is:
(1) $\frac{\pi}{2}-\frac{1}{3}$
(2) $\frac{1}{2}\left(\frac{\pi}{2}-\frac{1}{3}\right)$
(3) $2\left(\frac{\pi}{2}-\frac{1}{3}\right)$
(4) $\frac{\pi}{4}-\frac{1}{3}$

Answer (1)

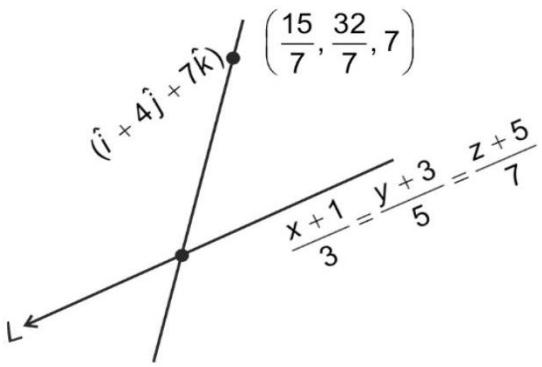
Sol.



$x=\frac{1}{1+y^2}, x=\frac{y^2}{2}$
Solving, $\frac{1}{1+y^2}=\frac{y^2}{2}\Rightarrow y=\pm 1$
Area $=\int_{-1}^{1}\left(\frac{1}{1+y^2}-\frac{y^2}{2}\right)dy=\tan^{-1}\ y-\frac{y^3}{6}\Big|_{-1}^{1}=\frac{\pi}{2}-\frac{1}{3}$
9. The square of the distance of the point $\left(\frac{15}{7},\frac{32}{7},7\right)$ from the line $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ in the direction of vector $\hat{i}+4\hat{j}+7\hat{k}$ is
(1) 66
(2) 54
(3) 41
(4) 44

Answer (1)

Sol.



$\Rightarrow \left(\lambda+\frac{15}{7}, 4\lambda+\frac{32}{7}, 7\lambda+7\right)$ for some λ lie on L.

$\Rightarrow \frac{\lambda+15/7+1}{3}=\frac{7\lambda+7+5}{7}$

$\Rightarrow \lambda=-1 \Rightarrow PQ=\sqrt{66}$

$\Rightarrow PQ^2=66$

10. Let $f:\mathbf{R}\to\mathbf{R}$ be a twice differentiable function such that $f(2)=1$. If $F(x)=xf(x)$ for all $x\in\mathbf{R}$, $\int_0^2 x\,F'(x)dx=6$ and $\int_0^2 x^2F''(x)dx=40$, then $F'(2)+\int_0^2 F(x)dx$ is equal to:

(1) 15
(2) 9
(3) 13
(4) 11

Answer (4)

Sol. $\int_0^2 xF'(x)=6$

$\left[F'(x)\cdot\frac{x^2}{2}\right]_0^2-\int_0^2 F''(x)\cdot\frac{x^2}{2}dx=6$

$2F'(2)=26\Rightarrow F'(2)=13$

Given : $F(x)=xf(x)$

$F'(x)=xf'(x)+f(x)$

Put $x=2$

$F'(2)=2f'(2)+f(2)$

$f'(2)=6$

As $\int_0^2 xF'(x)=6$

$\Rightarrow\ [xF(x)]_0^2-\int_0^2 F(x)dx=6$

$\Rightarrow \int_0^2 F(x)dx=-2$

$\therefore\ F'(2)+\int_0^2 F(x)dx=13-2=11$

11. If the components of $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ along and perpendicular to $\vec{b}=3\hat{i}+\hat{j}-\hat{k}$ respectively, are $\frac{16}{11}(3\hat{i}+\hat{j}-\hat{k})$ and $\frac{1}{11}(-4\hat{i}-5\hat{j}-17\hat{k})$, then $\alpha^2+\beta^2+\gamma^2$ is equal to:

(1) 23
(2) 18

(3) 26
(4) 16

Answer (3)

Sol. Component of $\vec{a}$ along $\vec{b}$ is $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\vec{b}$

$$\frac{(\alpha,\beta,\gamma)\cdot(3,1,-1)}{\sqrt{11}}(3\hat{\square}+\hat{\square}-\hat{k})$$

$$\frac{(3\alpha+\beta-\gamma)}{\sqrt{11}}(3\hat{\square}+\hat{\square}-\hat{k})$$

$$=\frac{16}{11}(3\hat{\square}+\hat{\square}-\hat{k})$$

$$\vec{a}-(\vec{a}\cdot\vec{b})\cdot b=\overrightarrow{v_2}$$

$$\vec{a}=\overrightarrow{v_1}+\overrightarrow{v_2}$$

$$\vec{a}=\frac{(44,11,-33)}{11}$$

$$\therefore\ \alpha=4,\beta=1,\gamma=-3.$$

12. If $f(x)=\int\frac{1}{x^{1/4}\left(1+x^{1/4}\right)}dx, f(0)=-6$, then $f(1)$ is equal to :
(1) $\log_e 2+2$
(2) $2-\log_e 2$
(3) $4\left(\log_e 2-2\right)$
(4) $4\left(\log_e 2+2\right)$

Answer (3)

Sol. Put $x^{1/4}=t\Rightarrow dx=4t^3dt$

$$\int\frac{4t^3dt}{t(t+1)}=4\int\left(\frac{t^2-1}{t+1}+\frac{1}{t+1}\right)dt$$

$$f(x)=4\left[\frac{x^{1/2}}{2}-x^{1/4}+\ln\left|x^{1/4}+1\right|\right]+C$$

$$f(0)=-6$$
$$\Rightarrow C=-6$$
$$\Rightarrow f(1)=4\left(\log_e 2-2\right)$$

13. If A and B are the points of intersection of the circle $x^2+y^2-8x=0$ and the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ and a point P moves on the line $2x-3y+4=0$, then the centroid of $\triangle PAB$ lies on the line:
(1) $4x-9y=12$
(2) $6x-9y=20$

(3) $9x - 9y = 32$
(4) $x + 9y = 36$

Answer (2)

Sol.
C : $x^2 + y^2 - 8x = 0$

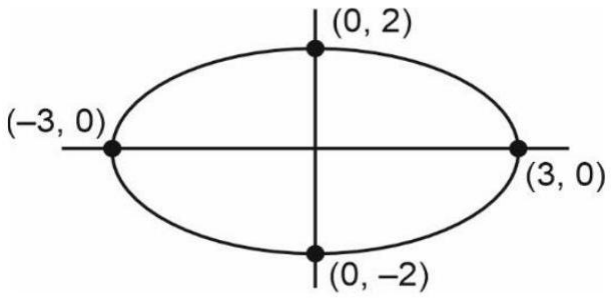
$$H: \frac{x^2}{9} - \frac{y^2}{4} = 1$$

By solving $\frac{x^2}{9} - \left(\frac{8x - x^2}{4}\right) = 1$
$4x^2 - 72x + 9x^2 = 36$
$\Rightarrow 13x^2 - 72x - 36 = 0$
$\Rightarrow 13x^2 - 78x + 6x - 36 = 0$
$\Rightarrow 13x(x - 6) + 6(x - 6) = 0 \Rightarrow x = 6$
or $-\frac{13}{6} \times$ neglected
$\Rightarrow y^2 = 8(6) - (6)^2$
$\Rightarrow y = \pm\sqrt{12}$
So, points A and B are $(6,\sqrt{12}),(6, -\sqrt{12})$
$P\left(h, \frac{2h + 4}{3}\right)$
Centroid of $\triangle PAB$ is $\left(\frac{12+h}{3}, \frac{2h+4}{9}\right)$
By options this centroid lies on the live $6x - 9y = 20$

14. If the midpoint of a chord of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is $\left(\sqrt{2}, \frac{4}{3}\right)$, and the length of the chord is $\frac{2\sqrt{\alpha}}{3}$, then α is
(1) 26
(2) 22
(3) 20
(4) 18

Answer (2)

Sol.



$E: \frac{x^2}{9}+\frac{y^2}{4}=1$

$T = S_1$

$\Rightarrow \frac{\sqrt{2}x}{9}+\frac{1}{4}\left(\frac{4}{3}y\right)-1=\frac{2}{9}+\frac{16}{9(4)}-1$

$\frac{\sqrt{2}x}{9}+\frac{y}{3}=\frac{2}{9}+\frac{4}{9}$

$\frac{\sqrt{2}x}{9}+\frac{y}{3}=\frac{2}{3}\Rightarrow \sqrt{2x}+3y=6$

Now point of intersection of chord and ellipse is

$\frac{(6-3y)^2}{18}+\frac{y^2}{4}=1$

$\frac{(2-y)^2}{2}+\frac{y^2}{4}=1$

$2\left(4+y^2-4y\right)+y^2=4$

$\Rightarrow 3y^2-8y+4=0$

$\Rightarrow y=2,\frac{2}{3}$

So, points are (0,2) are $\left(2\sqrt{2},\frac{2}{3}\right)$

Length of chord $=\sqrt{(2\sqrt{2})^2+\left(\frac{2}{3}-2\right)^2}$

DegreeFYD

$=\sqrt{8+\frac{16}{9}}$

$=\frac{\sqrt{88}}{3}=\frac{2\sqrt{22}}{3}$

On comparing $\alpha = 22$

15. Let $[x]$ denote the greatest integer less than or equal to x. Then the domain of $f(x) = \sec^{-1}(2[x]+1)$ is:

(1) $(-\infty,-1]\cup[1,\infty)$

(2) $(-\infty,-1]\cup[0,\infty)$

(3) $(-\infty,\infty)$

(4) $(-\infty,\infty)-\{0\}$

Answer (3)

Sol. $f(x)=\sec^{-1}(2[x]+1)$

$\Rightarrow 2[x]+1\geq 1$ or $2[x]+1\leq -1$

$\Rightarrow 2[x]\geq 0$ or $2[x]\leq -2$

$\Rightarrow [x]\geq 0$ or $[x]\leq -1$

$\Rightarrow x\geq 0$ or $x\leq 0$

Domain of $f(x)$ is $(-\infty,\infty)$

16. Let $f:[0,3] \to A$ be defined by $f(x) = 2x^3 - 15x^2 + 36x + 7$ and $g:[0,\infty) \to B$ be defined by $g(x) = \frac{x^{2025}}{x^{2025}+1}$. If both the functions are onto and $S = \{x \in \mathbf{Z} : x \in A \text{ or } x \in B\}$, then $n(S)$ is equal to :
(1) 36
(2) 30
(3) 29
(4) 31

Answer (2)
Sol. $f(x) = 2x^3 - 15x^2 + 36x + 7$
$f'(x) = 6x^3 - 30x + 36 = 0$
$\Rightarrow x^2 - 5x + 6 = 0$
$\therefore\ x = 1,5$
$f(0) = 7, f(2) = 35, f(3) = 34$
$A = [7,35]$
$g(x) = \frac{x^{2025}}{1 + x^{2025}}$
$B = [0,1)$
$\therefore\ S = [0,7,8,9....35]$
Number of elements = 30
17. Let A,B,C be three points in xy-plane, whose position vector are given by $\sqrt{3}\hat{i} + \hat{j}, \hat{i} + \sqrt{3}\hat{j}$ and $a\hat{i} + (1-a)\hat{j}$ respectively with respect to the origin O. If the distance of the point C from the line bisecting the angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is $\frac{9}{\sqrt{2}}$, then the sum of all the possible values of a is :
(1) 2
(2) 0
(3) 1
(4) $\frac{9}{2}$

Answer (3)

Sol. Equation of line in the internal bisector of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is $(\sqrt{3}+1)\hat{i} + (\sqrt{3}+1)\hat{j}$

$\Rightarrow$ line will be $y = x \Rightarrow x - y = 0$

$$D = \left|\frac{a-(1-a)}{\sqrt{a^2+(1-a)^2}}\right| = \frac{9}{\sqrt{2}}$$

$$(2a-1)^2 = \frac{81}{2}\left(a^2 + (1-a)^2\right)$$

$$\Rightarrow 2\left(4a^2 - 4a + 1\right) = 81a^2 + 81a^2 - 162a - 81$$

$$\Rightarrow 162a^2 - 162a + 81 - 8a^2 + 8a - 2 = 0$$

$$\Rightarrow 154a^2 - 154a + 79 = 0$$

Sum of values $= -\frac{(-154)}{154} = 1$
18. Let f be real valued continuous function defined on the positive real axis such that $g(x) = \int_0^x t f(t)dt$. If $g\left(x^3\right) = x^6 + x^7$, then value of $\sum_{r=1}^{15} f\left(r^3\right)$ is :
(1) 270

(2) 310
(3) 340
(4) 320

Answer (2)
Sol. $g(x) = x^2 + x^{7/3}$

$$g'(x) = 2x + \frac{7}{3}x^{4/3}$$
$$f(x) = \frac{g'(x)}{x}$$
$$f(x) = 2 + \frac{7}{3}x^{1/3}$$
$$f\left(r^3\right) = 2 + \frac{7}{3}r$$
$$\sum_{r=1}^{15}\left(2 + \frac{7}{3}r\right) = 2(15) + \frac{7}{3}\left(\frac{15(16)}{2}\right)$$
$$= 310$$

19. For positive integers n, if $4a_n = \left(n^2 + 5n + 6\right)$ and $s_n = \sum_{k=1}^{n}\left(\frac{1}{a_k}\right)$, then the value of $507S_{2025}$ is:
(1) 135
(2) 1350
(3) 675
(4) 540

Answer (3)

sol. $S_n = \sum_{k=1}^{n}\frac{4}{K^2+5k+6}$
$= \sum_{k=1}^{n}\frac{4}{(K+2)(K+3)} = 4\sum_{K=1}^{n}\left(\frac{1}{K+2} - \frac{1}{K+3}\right)$
$= 4\left[\frac{1}{3} - \frac{1}{4}\right]$
$= 4\left[\frac{1}{4} - \frac{1}{5}\right]$
$= 4\left[\frac{1}{n+2} - \frac{1}{n+3}\right]$
$S_n = 4\left[\frac{1}{3} - \frac{1}{n+3}\right]$
$S_{2025} = 4\left[\frac{1}{3} - \frac{1}{2028}\right]$
$S_{2025} = 4\left[\frac{675}{2028}\right]$
$507S_{2025} = 675$

20. Let $f: R - \{0\} \to (-\infty, 1)$ be a polynomial of degree 2 , satisfying $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$. If $f(K) = -2K$, then the sum of squares of all possible values of K is :
(1) 6
(2) 7

(3) 9
(4) 1

Answer (1)
Sol. Let $f(x) = ax^2 + bx + c$
Putting in given function at equation and comparing coefficients given
$c = 1, a = \pm 1$
Hence $f(x) = 1 - x^2$
$f(k) = -2k \Rightarrow k^2 - 2k - 1 = 0 \sim \begin{matrix}\alpha \\ \beta\end{matrix}$
$\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= 4 - 2(-1) = 6$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.
21. The interior angles of a polygon with n sides, are in an A.P, with common difference 6°. If the largest interior angle of the polygon is 219°, then n is equal to ____ .

Answer (20)

Sol. $\frac{n}{2}[2a + (n-1)6] = (n-2)180^\circ$

and $an + 3n^2 - 3n = 3n(n-2)180^\circ$
$\therefore$ Given $a + (n-1)6^\circ = 219^\circ$
$\Rightarrow a = 225^\circ - 6n^\circ$
Putting value of a in (i)
We get $\left(225 - 6n^2\right) + 3n^2 - 3n = 180n - 360^\circ$
$\Rightarrow 2n^2 - 42n - 360 = 0$
$\Rightarrow n^2 - 14n - 120 = 0$
$\Rightarrow (n-20)(n+6) = 0 \Rightarrow n = 20, -6$
(Rejected)
$\therefore n = 20$
22. If $f(x) = \lim_{n\to\infty} \sum_{r=0}^{n} \left(\frac{\tan\left(x/2^{r+1}\right) + \tan^3\left(x/2^{r+1}\right)}{1 - \tan^2\left(x/2^{r+1}\right)} \right)$. Then $\lim_{x\to 0} \frac{e^x - e^{f(x)}}{(x - f(x))}$ is equal to ____ .

Answer (01)

Sol. $f(x) = \lim_{x\to\infty}\left(\frac{\tan\left(x/2^{r+1}\right)+\tan^3\left(x/2^{r+1}\right)}{1-\tan^2\left(x/2^{r+1}\right)}\right)$

$= \lim_{x\to\infty}\frac{\tan\left(\frac{x}{2^{r+1}}\right)}{\cos\left(\frac{x}{2^r}\right)}$

$= \lim_{x\to\infty}\frac{\sin\left(x/2^{r+1}\right)}{\cos\left(\frac{x}{2^{r+1}}\right)\cos\left(\frac{x}{2^r}\right)}$

$= \lim_{x\to\infty}\frac{\sin\left(\frac{x}{2^r}-\frac{x}{2^{r+1}}\right)}{\cos\left(\frac{x}{2^{r+1}}\right)\cos\left(\frac{x}{2^r}\right)}$

$= \lim_{x\to\infty}\tan\left(\frac{x}{2^r}\right)-\tan\left(\frac{x}{2^{r+1}}\right)$

From condition given question

$\therefore \lim_{n\to\infty}\sum_{r=0}^{n}\left[\tan\left(\frac{x}{2^r}\right)-\tan\left(\frac{x}{2^{r+1}}\right)\right] = \tan x$

$\therefore \lim_{x\to 0}\left(\frac{e^x - e^{\tan x}}{x-\tan x}\right)$

$\Rightarrow \lim_{x\to 0}e^{\tan x}\left(\frac{e^{x-\tan x}-1}{x-\tan x}\right)$

$\Rightarrow \lim_{x\to 0}e^{\tan x}\lim_{x\to 0}\left(\frac{e^{x-\tan x}-1}{x-\tan x}\right)$

$\Rightarrow 1\cdot 1\left(\because \lim_{x\to 0}\frac{e^{x-1}}{x}=1\right)$

$= 1$

23. If $y = y(x)$ is the solution of the differential equation,

$\sqrt{4-x^2}\frac{dy}{dx} = \left(\left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2 - y\right)\sin^{-1}\left(\frac{x}{2}\right), -2\le x\le 2, y(2) = \frac{\pi^2-8}{4}$

, then $y^2(0)$ is equal to ____ .

Answer (4)

Sol. $\frac{dy}{dx}+\frac{\left(\sin^{-1}\frac{x}{2}\right)}{\sqrt{4-x^2}}y = \frac{\left(\sin^{-1}\frac{x}{2}\right)^3}{\sqrt{4-x^2}}$

$ye^{\left(\frac{\sin^{-1}\frac{x}{2}}{2}\right)^2} = \int\frac{\left(\sin^{-1}\frac{x}{2}\right)^3}{4-x^2}e^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}dx$

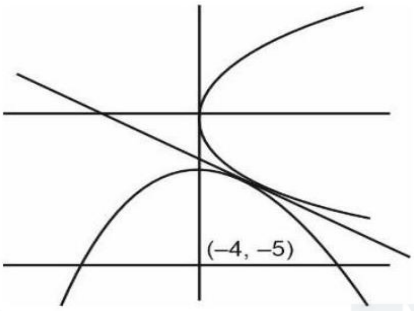
$y = \left(\sin^{-1}\frac{x}{2}\right)^2 - 2 + c\cdot e^{\frac{-\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}$

$$y(2) = \frac{\pi^2}{4} - 2 \Rightarrow c = 0$$
$$y(0) = -2$$

24. Let A and B be the points of intersection of the line $y + 5 = 0$ and the mirror image of the parabola $y^2 = 4x$ with respect to the line $x + y + 4 = 0$. If d denotes the distance between A and B, and a denotes the area of ΔSAB, where S is the focus of the parabola $y^2 = 4x$, then the value of $(a + d)$ is ____ .

Answer (14)

Sol.



To find image of $P\left(t^2, 2t\right)$

$$\frac{x - t^2}{1} = \frac{y - 2t}{1} = \frac{-2\left(t^2 + 2t + 4\right)}{1^2 + 1^2} = -(t+1)^2 - 3$$
$$x = t^2 - (t+1)^2 - 3 = -2t - 4$$
$$y = 2t - (t+1)^2 - 3 = -t^2 - 4$$
$$t = \frac{-x - 4}{2}$$
$$\Rightarrow\ y + 4 = -\left(\frac{-x-4}{2}\right)^2$$
$$\Rightarrow (y+4) = -\frac{(x+4)^2}{4}$$
$$\Rightarrow\ x^2 = -4y$$
$\Rightarrow$ Focus $(-4, -5)$

Also, $y = -5$ intersect

$$\therefore\ (-4)(-1) = (x+4)^2$$
$$4 = (x+4)^2$$
$$x + 4 = \pm 2$$
$$x = -2, -6$$
$$\Rightarrow d = 4$$
$$a = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ -2 & -5 & 1 \\ -6 & -5 & 1 \end{vmatrix}$$
$$= \frac{1}{2}|[1(-5+5) + 1(10-30)]|$$

$= \frac{1}{2}(20)$
$a = 10$
$\therefore a + d = 14$
25. The number of natural numbers, between 212 and 999 , such that sum of their digits is 15 , is ____ .

Answer (64)
Sol. Let the number be
$2ab, a + b = 13$
$\Rightarrow a,b \in \{0,9\} \Rightarrow 6$
numbers $\{(9,4),(8,5)...(4,9)\}$
Similarly, for $3ab, a + b = 12 \Rightarrow 7$ numbers
For 4ab, $a + b = 11 \Rightarrow$ Numbers
For 5ab, $a + b = 10 \Rightarrow 9$ numbers
For 6ab, $a + b = 9 \Rightarrow 10$ numbers
For 7ab, $a + b = 8 \Rightarrow 9$ numbers
For 8ab, $a + b = 7 \Rightarrow 8$ numbers
For 9ab, $a + b = 6 \Rightarrow 7$ numbers
$\therefore$ Total ways = 64.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer :

26. A body of mass 4 kg is placed on a plane at a point P having coordinates (3,4)m. Under the action of force $\vec{F} = (2\hat{i} + 3\hat{j})$N, it moves to a new point Q having coordinates (6,10)m in 4 sec . The average power and instantaneous power at the end of 4 sec are in the ratio of
(1) 1:2
(2) 6:13
(3) 4:3
(4) 13:6

Answer (2)

Sol. $\vec{F} = (2\hat{i} + 3\hat{j})$N; displacement $\vec{S} = (3\hat{i} + 6\hat{j})$m Acceleration, $\vec{a} = \frac{\vec{F}}{m} = \left(\frac{2\hat{i}+3\hat{j}}{4}\right)$ m/s^2

$$\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\Rightarrow 3\hat{i} + 6\hat{j} = 4\vec{u} + \frac{1}{2}\left(\frac{2\hat{i} + 3\hat{j}}{4}\right) \cdot 16 = 4\vec{u} + 4\hat{i} + 6\hat{j}$$

$$\text{or } \vec{u} = \frac{1}{4}(3\hat{i} + 6\hat{j} - 4\hat{i} - 6\hat{j})$$

$$\vec{u} = -\frac{1}{4}\hat{i}$$

$$\text{Average Power} = \vec{F} \cdot \frac{\vec{S}}{t}$$

$$= \frac{(2\hat{i} + 3\hat{j}) \cdot (3\hat{i} + 6\hat{j})}{4} \text{ watt}$$

$$= 6 \text{ watt}$$

$$\text{Instantaneous Power} = \vec{F} \cdot \vec{v}$$

$$= \vec{F} \cdot (\vec{u} + \vec{a}t)$$

$$= (2\hat{i} + 3\hat{j}) \cdot \left(-\frac{1}{4}\hat{i} + 2\hat{i} + 3\hat{j}\right)$$

$$= (2\hat{i} + 3\hat{j}) \cdot \left(+\frac{7}{4}\hat{i} + 3\hat{j}\right)$$

$$= (3.5 + 9) \text{ watt}$$

$$= 12.5 \text{ watt}$$

27. A 400 g solid cube having an edge of length 10 cm floats in water. How much volume of the cube is outside the water?
(Given : density of water = 1000 kg m^{-3})
(1) 600 cm^3
(2) 400 cm^3
(3) 4000 cm^3
(4) 1400 cm^3

Answer (1)

Sol. Volume of cube inside water

$$= \left(\frac{\text{Density of cube}}{\text{Density of water}}\right) \times \text{ Volume of cube}$$

$$= \frac{\text{Mass of cube}}{\text{Density of water}}$$

$$= \frac{400\text{gm}}{1\text{gm/cm}^3}$$

$$= 400 \text{ cm}^3$$

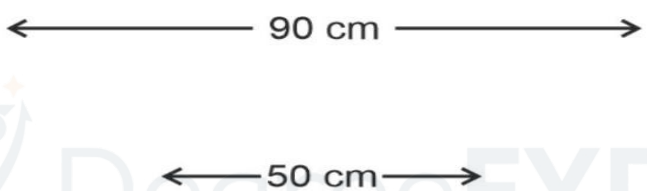
Volume of cube outside water

= Volume of cube – Volume of cube inside water
= 1000 cm^3 – 400 cm^3
= 600 cm^3

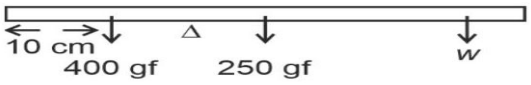
28. A uniform rod of mass 250 g having length 100 cm is balanced on a sharp edge at 40 cm mark. A mass of 400 g is suspended at 10 cm mark. To maintain the balance of the rod, the mass to be suspended at 90 cm mark, is
(1) 290 g
(2) 300 g
(3) 200 g
(4) 190 g

Answer (4)

Sol.



← 40 cm →



(400gf)(30 cm) = (250gf)(10 cm) + (wgf)(50 cm) (balancing torques)
w = 190
29. Match List-I with List-II.
(A)
(B)
(C)

List-I		List-II
Angular Impulse	(I)	$[M^0 L^2 T^{-2}]$
Latent Heat	(II)	$[ML^2 T^{-3} A^$
Electrical resistivity	(III)	$[ML^2 T^{-1}]$
Electromotive	(IV)	[

force		$ML^3 T^{-3} A^{-1}$]

Choose the correct answer from the options given below.
(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
(2) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
(3) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(4) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

Answer (1)

Sol. [Angular Impulse] = $ML^2 T^{-1}$
[Latent Heat] = $M^0 L^2 T^{-2}$
[Electrical resistivity] = $ML^3 T^{-3} A^{-2}$
[Electromotive force] = $ML^2 T^{-3} A^{-1}$
30. In a long glass tube, mixture of two liquids A and B with refractive indices 1.3 and 1.4 respectively, forms a convex refractive meniscus towards A. If an object placed at 13 cm from the vertex of the meniscus in A forms an image with a magnification of ' -2 ' then the radius of curvature of meniscus is
(1) 1 cm
(2) $\frac{4}{3}$ cm
(3) $\frac{2}{3}$ cm
(4) $\frac{1}{3}$ cm

Answer (3)

Sol. | A

$$\mu_A = 1.3 \; \mu_B = 1.4$$
$$u = -13 \text{ cm}$$
$$m = \frac{\frac{v}{\mu_B}}{\frac{u}{\mu_A}} = -2$$
$$\Rightarrow v = -2u\left(\frac{\mu_B}{\mu_A}\right)$$
$$= -2(-13 \text{ cm})\left(\frac{1.4}{1.3}\right)$$
$$= +28 \text{ cm}$$

Also,

$$\frac{\mu_B}{v} - \frac{\mu_A}{u} = \frac{\mu_B - \mu_A}{R}$$
$$\frac{1.4}{28} - \frac{1.3}{-13} = \frac{0.1}{R}$$
$$\Rightarrow R = \frac{2}{3} \text{ cm}$$

31. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): Knowing initial position x_0 and initial momentum p_0 is enough to determine the position and momentum at any time t for a simple harmonic with a given angular frequency ω.
Reason (R): The amplitude and phase can be expressed in terms of x_0 and p_0.

In the light of the above statements, choose the correct answer from the options given below:
(1) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)
(2) (A) is false but (R) is true
(3) Both (A) and (R) are true and (R) is the correct explanation of (A)
(4) (A) is true but (R) is false

Answer (3)

Sol. If we express position $x(t) = A\sin(\omega t + \phi)$
then $x_0 = A\sin\phi$
$v_0 = A\omega\cos\phi$
$\Rightarrow \tan\phi = \frac{\omega x_0}{v_0}$
$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$
Hence both position and linear momentum of a particle can be expressed as a function of time if we know initial momentum and position
32. Earth has mass 8 times and radius 2 times that of a planet. If the escape velocity from the earth is 11.2 km/s, the escape velocity in km/s from the planet will be:
(1) 8.4
(2) 2.8
(3) 11.2
(4) 5.6

Answer (4)
Sol. $v_{\text{escape}} = \sqrt{\frac{2GM}{R}}; M_{\text{earth}} = 8M_{\text{planet}}; R_{\text{earth}} = 2R_{\text{planet}}$

$$\frac{v_{\text{planet}}}{v_{\text{earth}}} = \sqrt{\frac{M_{\text{planet}}}{M_{\text{earth}}} \times \frac{R_{\text{earth}}}{R_{\text{planet}}}}$$

$$v_{\text{planet}} = (11.2 \text{ km/s}) \sqrt{\frac{1}{8} \times 2}$$

$$= 5.6 \text{ km/s}$$

33. A uniform magnetic field of 0.4 T acts perpendicular to a circular copper disc 20 cm in radius. The disc is having a uniform angular velocity of $10\pi \text{rads}^{-1}$ about an axis through its centre and perpendicular to the disc. What is the potential difference developed between the axis of the disc and the rim? ($\pi = 3.14$)
(1) 0.0628 V
(2) 0.5024 V
(3) 0.1256 V
(4) 0.2512 V

Answer (4)
Sol. $E = \frac{1}{2} B\omega R^2$

$$= \frac{1}{2}(0.4)(10\pi)(0.2)^2 \text{volt}$$

$$= 0.2512 \text{ V}$$

34. Which of the following phenomena can not be explained by wave theory of light?
(1) Refraction of light
(2) Reflection of light
(3) Diffraction of light
(4) Compton effect

Answer (4)

Sol. Compton effect refers to scattering of a photon by free electrons. This phenomenon provides an evidence for particle nature of light.

35. The ratio of vapour densities of two gases at the same temperature is $\frac{4}{25}$, then the ratio of r.m.s. velocities will be:
(1) $\frac{25}{4}$
(2) $\frac{4}{25}$
(3) $\frac{2}{5}$
(4) $\frac{5}{2}$

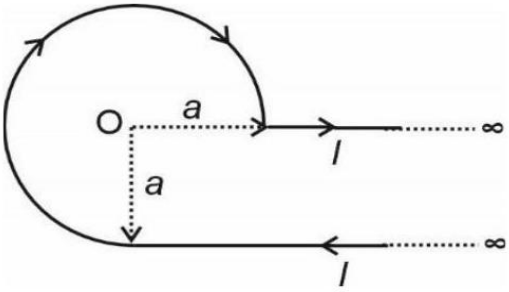
Answer (4)

Sol. (Vapor density) $= \left(\frac{\text{Molar mass}}{2}\right)$, i.e, $vd = \frac{M}{2}$

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$\frac{v_2}{v_1} = \sqrt{\frac{M_1}{M_2}} = \sqrt{\frac{25}{4}} = \frac{5}{2}$

36.



An infinite wire has a circular bend or radius a, and carrying a current I as shown in figure. The magnitude of magnetic field at the origin O of the arc is given by:

(1) $\frac{u_0}{4\pi}\frac{I}{a}\left[\frac{3\pi}{2}+1\right]$

(2) $\frac{u_0}{2\pi}\frac{I}{a}\left[\frac{\pi}{2}+2\right]$

(3) $\frac{u_0}{4\pi}\frac{I}{a}\left[\frac{3\pi}{2}+2\right]$

(4) $\frac{u_0}{4\pi}\frac{I}{a}\left[\frac{\pi}{2}+1\right]$

Answer (1)

Sol. $B_{\text{net}} = \frac{\mu_0 I}{4\pi a} + \frac{3}{8}\frac{\mu_0 I}{a}$

$= \frac{\mu_0 I}{4\pi a}\left(1+\frac{3\pi}{2}\right)$

37 A concave mirror produces an image of an object such that the distance between the object and image is 20 cm . If the magnification of the image is $'-3'$, then the magnitude of the radius of curvature of the mirror is:

(1) 30 cm

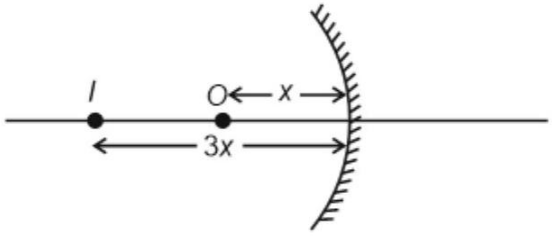
(2) 7.5 cm

(3) 3.75 cm

(4) 15 cm

Answer (4)

Sol.



$$m = -3 = -\frac{u}{v}$$
$$u = -x$$
$$v = -3x$$
$$\Rightarrow 2x = 20 \text{ cm}$$

$x = 10$ cm

$f = \frac{uv}{u+v} = \frac{(-10)(-30)}{(-10)+(-30)} = -7.5$ cm

$f = -\frac{R}{2}$

$R = -2f = 15$ cm

38. The frequency of revolution of the electron in Bohr's orbit varies with n , the principal quantum number as

(1) $\frac{1}{n^2}$

(2) $\frac{1}{n^3}$

(3) $\frac{1}{n}$

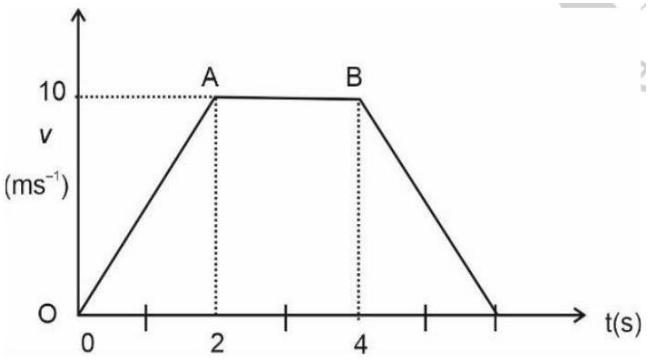
(4) $\frac{1}{n^4}$

Answer (2)

Sol. $f = \frac{v}{2\pi r} = \left(\frac{v_0}{n}\right)\left(\frac{1}{2\pi r_0 n^2}\right)$

$f = \frac{v_0}{2\pi r_0} = \left(\frac{1}{n^3}\right) = f_0\left(\frac{1}{n^3}\right)$

39. The velocity a time graph of an object moving along a straight line is shown in figure. What is the distance covered by the object between t = 0 to t = 4s?



(1) 10 m

(2) 30 m

(3) 11 m

(4) 13 m

Answer (2)

Sol. Distance travelled = displacement when direction of velocity remains constant

⇒ Distance = Area

$$= \frac{1}{2}(2\text{ s} + 4\text{ s})(10\text{ m/s})$$
$$= 30\text{ m}$$

40. A balloon and its content having mass M is moving up with an acceleration ' a '. The mass that must be released from the content so that the ballon starts moving up with an acceleration '3a' will be
(Take ' g ' as acceleration due to gravity)
(1) $\frac{3Ma}{3a-g}$
(2) $\frac{3Ma}{2a-g}$
(3) $\frac{2Ma}{3a+g}$
(4) $\frac{3Ma}{2a+g}$

Answer (3)

Sol. Let us assure buoyancy force on balloon be B.

$$B - Mg = Ma\#(i)$$
$$B - (M-m)g = (M-m)3a\#(ii)$$

Subtracting (ii) from (i)

$$(B - Mg) - (B - (M-m)g) = Ma - 3(M-m)a$$

$-mg = (3m - 2M)a$
or $m = \frac{2Ma}{3a+g}$

41. The magnetic field of an E.M. wave is given by $\vec{B} = \left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{k}\right) 30\sin\left[\omega\left(t - \frac{z}{c}\right)\right]$ (S.I. Units). The corresponding electric field in S.I. units is
(1) $\vec{E} = \left(\frac{3}{4}\hat{i} + \frac{1}{4}\hat{k}\right) 30c\cos\left[\omega\left(t - \frac{z}{c}\right)\right]$
(2) $\vec{E} = \left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{k}\right) 30c\sin\left[\omega\left(t + \frac{z}{c}\right)\right]$
(3) $\vec{E} = \left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{k}\right) 30c\sin\left[\omega\left(t - \frac{z}{c}\right)\right]$
(4) $\vec{E} = \left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{k}\right) 30c\sin\left[\omega\left(t + \frac{z}{c}\right)\right]$

Answer (3)

Sol. $\hat{c} = \hat{k}$

$\hat{B} = \frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}$

$\hat{E} = \hat{B} \times \hat{C} = \frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}$

$|\vec{E}| = |\vec{B}|c = 30c$

$\Rightarrow \vec{E} = \left(\frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}\right) 30c \sin\ \omega\left(t - \frac{z}{c}\right)$

42. A parallel plate capacitor of capacitance 1μ F is charged to a potential difference of 20 V . The distance between plates is 1μ m. The energy density between plates of capacitor is.

(1) 1.8×10^5 J/m^3
(2) 1.8×10^3 J/m^3
(3) 2×10^{-4} J/m^3
(4) 2×10^2 J/m^3

Answer (2)

Sol. Energy density $= \frac{1}{2}\varepsilon_0 E^2$

$$= \frac{1}{2}\varepsilon_0 \left(\frac{V}{d}\right)^2$$

$$= \frac{1}{2}\left(8.85 \times 10^{-12}\right)\left(\frac{20}{10^{-6}}\right)^2 \text{ J/m}^3$$

$$\simeq 1.8 \times 10^3 \text{ J/m}^3$$

43. The kinetic energy of translation of the molecules in 50 g of CO_2 gas at 17°C is
(1) 3582.7 J
(2) 3986.3 J
(3) 4102.8 J
(4) 4205.5 J

Answer (3)

Sol. Kinetic energy of translation $= \frac{3}{2}nRT$

$$n = \frac{50\text{ g}}{44\text{ g}} = \frac{25}{22} \text{ mol}$$

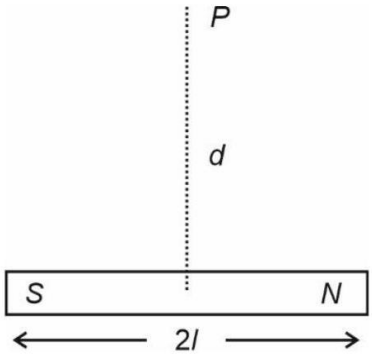
$$T = 17^\circ\text{C} = 290 \text{ K}$$

⇒ Kinetic energy of translation

$$= \frac{3}{2}\left(\frac{25}{22}\right)(8.3)(290)\text{J}$$

$$= 4102.8 \text{ J}$$

44.



A bar magnet has total length $2l = 20$ units and the field point P is at a distance $d = 10$ units from the centre of the magnet. If the relative uncertainty of length measurement is 1%, then uncertainty of the magnetic field at point P is

(1) 10%

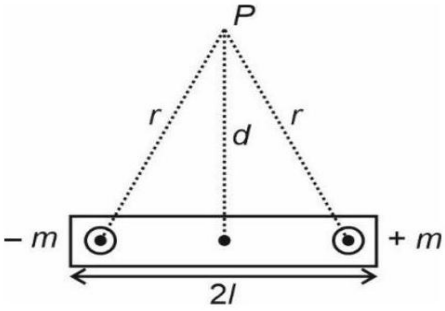
(2) 5%

(3) 3%

(4) 4%

Answer (4)

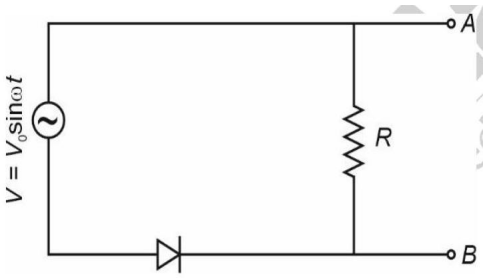
Sol.



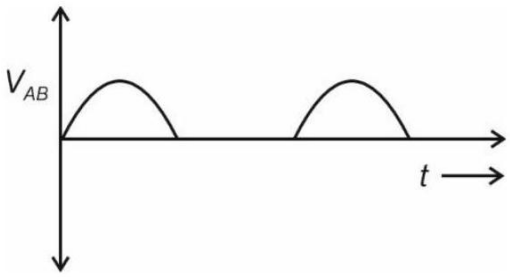
Magnetic field at $P, B = \frac{\mu_0}{4\pi}\frac{m(2l)}{r^3}$, where m is the pole strength

$$\Rightarrow \left(\frac{\Delta B}{B} \times 100\right) = \left(\frac{\Delta l}{l}\right) \times 100 + 3\left|\frac{\Delta r}{r}\right| \times 100$$
$$= 1\% + 3\% = 4\%$$

45. In the circuit shown here, assuming threshold voltage of diode is negligibly small, then voltage V_{AB} is correctly represented by

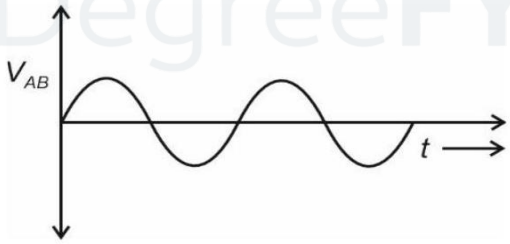


(1)

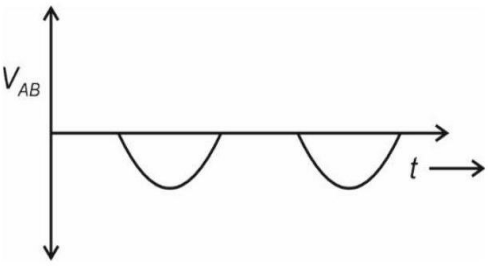


(2) V_{AB} would be zero at all times

(3)



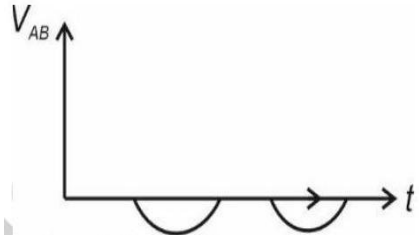
(4)



Answer (4)

Sol. Due to the diode the current will be allowed only along B to A through the resistor.
Hence V_{AB} will have only negative values i.e., the following graph represents the output

voltage V_{AB}.



SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. An electric dipole of dipole moment 6×10^{-6}Cm is placed in uniform electric field of magnitude 10^6V/m. Initially, the dipole moment is parallel to electric field. The work that needs to be done on the dipole to make its dipole moment opposite to the field, will be ____ J.

Answer (12)

Sol. Work done in rotating a dipole = ΔU

$$\text{or } W = \left(-PE\cos\theta_f\right) - \left(-PE\cos\theta_i\right)$$
$$= 2PE \quad \left(\because \theta_f = 180^\circ \text{ and } \theta_i = 0^\circ\right)$$
$$= \left(2 \times 6 \times 10^{-6} \times 10^6\right) \text{J} = 12 \text{ J}$$

47. The volume contraction of a solid copper cube of edge length 10 cm , when subjected to a hydraulic pressure of 7×10^6 Pa, would be ____ mm^3.
(Given bulk modulus of copper = $1.4 \times 10^{11} Nm^{-2}$)

Answer (50)

Sol. $B = -\frac{VP}{\Delta V}$

$$\Rightarrow \Delta V = -V\frac{\Delta P}{B}$$
$$= -(10 \text{ cm})^3 \times \frac{7 \times 10^6 \text{ Pa}}{1.4 \times 10^{11} \text{ N/m}^2} = 50 \text{ mm}^3$$

48. A thin transparent film with refractive index 1.4, is held on circular ring of radius 1.8 cm . The fluid in the film evaporates such that transmission through the film

at wavelength 560 nm goes to a minimum every 12 seconds. Assuming that the film is flat on its two sides, the rate of evaporation is ____ $\pi \times 10^{-13}$ m^3/s.

Answer (54)

Sol. For a thin film interference, a fringe for transmission is formed.

When

$$2\mu x = n\lambda$$

$$\Rightarrow \frac{dx}{dt} = \left(\frac{dn}{dt}\right)\frac{\lambda}{2\mu}$$

$$= \left(\frac{1}{12}\right)\frac{560 \times 10^{-9}}{2 \times 1.4} = \frac{5}{3} \times 10^{-8} \text{ m/s}$$

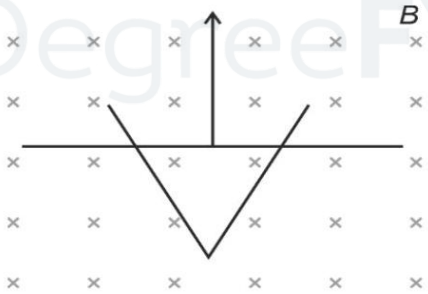
$V =$ Volume of film $= \pi R^2 \times$

$$\frac{dV}{dt} = \pi R^2 \frac{dx}{dt}$$

$$= \pi\left(1.8 \times 10^{-2}\right)^2 \frac{5}{3} \times 10^{-8} \text{ m}^3/\text{s}$$

$$= 54\pi \times 10^{-13} \text{ m}^3/\text{s}$$

49.



A conducting bar moves on two conducting rails as shown in the figure. A constant magnetic field B exists into the page. The bar starts to move from the vertex at time $t = 0$ with a constant velocity. If the induced EMF is $E \propto t^n$, then value of n is ____ .

Answer (1)

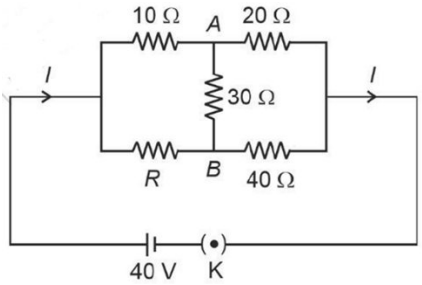
Sol. As the bar moves without change in orientation, the length of bar will be proportional to its distance from the vertex.
i.e. $l = c(vt)$
induced emf $E = Blv$

$$= cBv^2t$$

$\Rightarrow n = 1$

50. The value of current I in the electrical circuit as given below, when potential at A is

equal to the potential at B, will be ____ A.



Answer (2)

Sol. Since $V_A = V_B$, the given combination is a wheatstone bridge

i.e. $\frac{R}{10\Omega} = \frac{40\Omega}{20\Omega} \Rightarrow R = 20\Omega$

$$R_{eff} = \frac{(10\Omega + 20\Omega)(40\Omega + 20\Omega)}{(10\Omega + 20\Omega) + (40\Omega + 20\Omega)} = 20\Omega$$

$$\Rightarrow I = \frac{V}{R_{\text{eff}}} = \frac{40\text{ V}}{20\Omega} = 2\text{ A}$$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

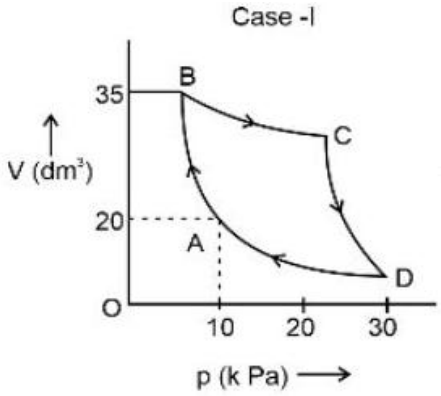
Choose the correct answer :

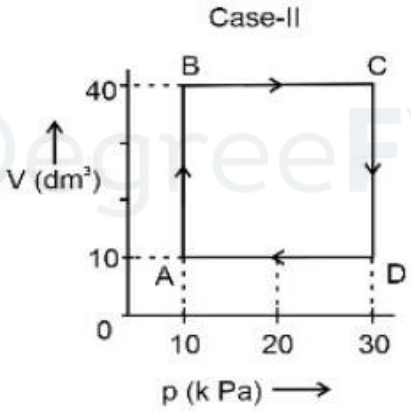
51. Identify the inorganic sulphides that are yellow in colour:
    (A) $(NH_4)_2$ S
    (B) PbS
    (C) CuS
    (D) $As_2 S_3$
    (E) $As_2 S_5$

Choose the correct answer from the options given below:
(1) (A), (D) and (E) only
(2) (A) and (B) only
(3) (A) and (C) only
(4) (D) and (E) only

Answer (1)

Sol. $(NH_4)_2$ S : colourless to yellow
PbS and CuS : Black
As_2 S_3 and As_2 S_5 : Yellow
52.





An ideal gas undergoes a cyclic transformation starting from the point A and coming back to the same point by tracing the path $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$ as shown in the three cases above.

Choose the correct option regarding ΔU :
(1) ΔU(Case-I) = ΔU(Case-II) = ΔU(Case-III)
(2) ΔU(Case-III) > ΔU(Case-II) > ΔU(Case-I)
(3) ΔU(Case-I) > ΔU(Case-III) > ΔU(Case-II)
(4) ΔU(Case-I) > ΔU(Case-II) > ΔU(Case-III)

Answer (1)

Sol. As all the three case are cyclic process. Change in state functions will be zero.

Hence, $\Delta U = 0$ for all.

53. Concentrated nitric acid is labelled as 75% by mass. The volume in mL of the solution which contains 30 g of nitric acid is ____ .
Given: Density of nitric acid solution is 1.25 g/mL.
(1) 55
(2) 32
(3) 45
(4) 40

Answer (2)

Sol. 75% by mass means :
$\because$ 75 gHNO_3 in 100 g solution.
$\therefore$ 30 gHNO_3 in $\frac{100}{75} \times 30$ g solution.

$$m_{\text{solution}} = \frac{100 \times 30}{75} g$$

$$v_{\text{sol}} = \frac{m_{\text{sol}}}{d_{\text{sol}}} = \frac{100 \times 30}{75 \times 1.25} = 32 \text{ mL}$$

54. Identify correct conversion during acidic hydrolysis from the following:
(A) starch gives galactose.
(B) cane sugar gives equal amount of glucose and fructose.
(C) milk sugar gives glucose and galactose.
(D) amylopectin gives glucose and fructose.
(E) amylose gives only glucose.

Choose the correct answer from the option given below
(1) (B), (C) and
(D) only
(2) (A), (B) and (C) only
(3) (C), (D) and (E) only
(4) (B), (C) and (E) only

Answer (4)

Sol. Starch gives glucose on hydrolysis amylose and amylopectin are components of starch.
Hence, (B), (C) and (E) are correct.

55. Identify product [A],[B] and [C] in the following reaction sequence.

$$CH_3 - C \equiv CH \xrightarrow[H_2]{Pd/C} [A] \xrightarrow[(ii)\ Zn,\ H_2O]{(i)\ O_3} [B] + [C]$$

(1) [A]:$CH_3CH_2CH_3$,[B]:CH_3CHO,[C]:HCHO
(2) [A]:$CH_2 = CH_2$,[B] :

O

[C] : HCHO
(3) [A]:$CH_3 - CH = CH_2$,[B]:CH_3CHO,[C]:
HCHO
(4) [A]:$CH_3 - CH = CH_2$,[B]:CH_3CHO,[C]:CH_3CH_2OH

Answer (3)

Sol.

$$CH_3 - C \equiv CH \xrightarrow[H_2]{Pd/C} CH_3 - CH = CH_2$$

$$\xrightarrow[(ii)\ Zn,\ H_2O]{(i)\ O_3} CH_3 - CHO + HCHO$$

56. Given below are two statements:

Statement (I): According to the Law of Octaves, the elements were arranged in the increasing order of their atomic number.

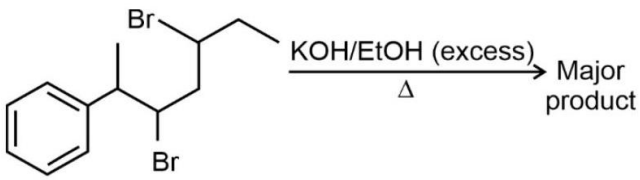
Statement (II): Meyer observed a periodically repeated pattern upon plotting physical properties of certain elements against their respective atomic numbers.

In the light of the above statements, choose the correct answer from the options given below:
(1) Both Statement I and Statement II are true
(2) Statement I is false but Statement II is true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are false

Answer (4)

Sol. In Law of Octaves, atomic mass is used for arrangement.

Mayer observed the pattern against atomic mass.
57. The major product of the following reaction is :



(1) 2-Phenylhepta-2,4-diene
(2) 6-Phenylhepta-3,5-diene
(3) 6-Phenylhepta-2,4-diene
(4) 2-Phenylhepta-2,5-diene

Answer (1)

Sol.



58. Which of the following is/are not correct with respect to energy of atomic orbitals of hydrogen atom?
(A) 1 s < 2p < 3 d < 4 s
(B) 1 s < 2 s = 2p < 3 s = 3p
(C) 1 s < 2 s < 2p < 3 s < 3p
(D) 1 s < 2 s < 4 s < 3 d

Choose the correct answer from the options given below:
(1)
(B) and (D) only
(2) (A) and (
(C) only
(3)
(C) and (D) only
(4) (A) and (
(B) only

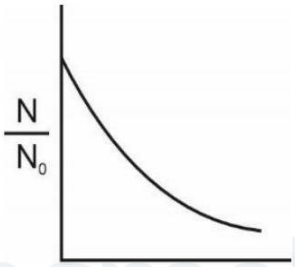
Answer (3)

Sol. For H-atom:
Energy of $1\text{ s} < 2\text{ s} = 2p < 3\text{ s} = 3p = 3d < 4s = 4p = 4d = 4f$.

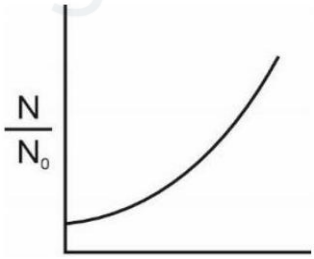
Hence (C) and (D) are incorrect.

59. For bacterial growth in a cell culture, growth law is very similar to the law of radioactive decay. Which of the following graphs is most suitable to represent bacterial colony growth?
Where N - Number of Bacteria at any time, N_0 Initial number of Bacteria.

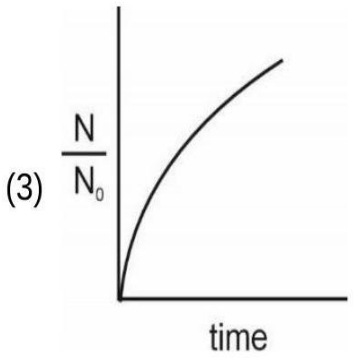
(1)



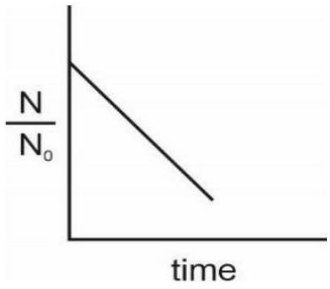
(2)



(3)



(4)



Answer (2)

Sol. For Bacterial growth following 1st order:
$\frac{N}{N_0}$ will increase with time exponentially.

60. Arrange the following in increasing order of solubility product:
$Ca(OH)_2$,AgBr,PbS,HgS

(1) HgS < PbS < AgBr < $Ca(OH)_2$
(2) PbS < HgS < $Ca(OH)_2$ < AgBr
(3) $Ca(OH)_2$ < AgBr < HgS < PbS
(4) HgS < AgBr < PbS < $Ca(OH)_2$

Answer (1)

Sol. Solubility of HgS < PbS < AgBr < $Ca(OH)_2$.

61. Match List - I with List - II.

	List - I (Complex)		List - I (Hybridisation of central metal ion)
(A)	$[CoF_6]^{3-}$	(I)	d^2sp^3
(B)	$[NiCl_4]^{2-}$	(II)	sp^3
(C)	$[Co(NH_3)_6]^{3}$	(III)	sp^3d^2
(D)	$[Ni(CN)_4]^{2-}$	(IV)	dsp^2

Choose the correct answer from the options given below:
(1) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
(2) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(3) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)
(4) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

Answer (2)

Sol. $[CoF_6]^{3-}$:Co^{3+} with WFL – sp^3d^2
$[NiCl_4]^{2-}$:Ni^{2+} with WFL – sp^3
$[Co(NH_3)_6]^{3+}$:Co^{3+} with SFL – d^2sp^3
$[Ni(CN)_4]^{2-}$:Ni^{2+} with SFL - dsp^2

62. Assume a living cell with 0.9%(w/w) of glucose solution (aqueous). This cell is immersed in another solution having equal mole fraction of glucose and water. (Consider the data upto first decimal place only)
The cell will :
(1) Shrink since solution is 0.45%(w/w) as a result of association of glucose molecules (due to hydrogen bonding)
(2) Show no change in volume since solution is 0.9%w/w
(3) Shrink since solution is 0.5%(w/w)
(4) Swell up since solution in 1%(w/w)

Answer (No option is correct)

Sol. Living cell has 0.9 g glucose in 100 g solution.
Which is dipped in a solution with $X_{H_2O} = \frac{1}{2}$.
Weight of solution $= \frac{1}{2} \times 180 + \frac{1}{2} \times 18 = 99$ g.
So, the solution has 90 g glucose in 99 g solution or %w/w = 90.9%

63. Given below are two statements:

Statement (I) :

and

are isomeric compounds.

Statement (II) :



and



are functional group isomers.

In the light of the above statements, choose the correct answer from the options given below:
(1) Statement I is false but Statement II is true
(2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are false
(4) Both Statement I and Statement II are true

Answer (4)

Sol.

and

are ring chain isomers.

Primary and secondary amines are functional group isomers.
64. Consider an elementary reaction
$A(g) + B(g) \rightarrow C(g) + D(g)$
If the volume of reaction mixture is suddenly reduced to $\frac{1}{3}$ of its initial volume, the reaction rate will become ' x ' times of the original reaction rate. The value of x is :
(1) $\frac{1}{9}$
(2) 9
(3) 3
(4) $\frac{1}{3}$

Answer (2)

Sol. Since, the reaction is elementary
Rate = $k[A]^1[B]^1$
When V is reduced to $\frac{1}{3}$ V, concentration will be tripled.

Hence, rate = $9 \times (\text{ rate })_{\text{initial}}$
x = 9
65. Identify correct statements:
(A) Primary amines do not give diazonium salts when treated with $NaNO_2$ in acidic condition.
(B) Aliphatic and aromatic primary amines on heating with $CHCl_3$ and ethanolic KOH form carbylamines.
(C) Secondary and tertiary amines also give carbylamine test.
(D) Benzenesulfonyl chloride is known as Hinsberg's reagent.
(E) Tertiary amines reacts with benzenesulfonyl chloride very easily.

Choose the correct answer from the options given below:
(1)
(D) and (E) only
(2) (A) and (B) only
(3)
(B) and (D) only
(4) (B) and
(C) only

Answer (3)

Sol. - Primary aromatic amines gives diazonium salt when treated with $NaNO_2$ in acidic medium.

- Only primary amines give carbylamine test.
- Tertiary amines do not react with benzenesulfonyl chloride.

66. The purification method based on the following physical transformation is:

$$\underset{(x)}{\text{Solid}} \xrightarrow{\text{Heat}} \underset{(x)}{\text{Vapour}} \xrightarrow{\text{Cool}} \underset{(x)}{\text{Solid}}$$

(1) Crystallization
(2) Sublimation
(3) Extraction
(4) Distillation

Answer (2)

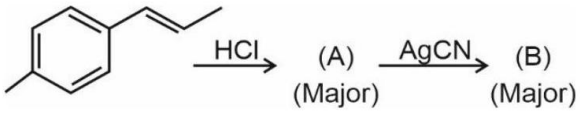
Sol. Phase transfer from solid to vapour directly is known as sublimation.

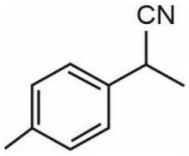
67. The amphoteric oxide among V_2O_3, V_2O_4 and V_2O_5, upon reaction with alkali leads to formation of an oxide anion. The oxidation state of V in the oxide anion is :
(1) +7
(2) + 3
(3) +4
(4) +5

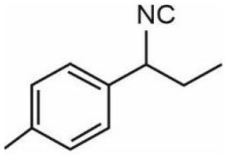
Answer (4)

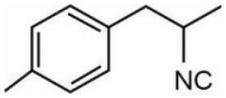
Sol. V_2O_5 is amphoteric oxide.
V_2O_5 gives VO_4^{3-} on reaction with alkali oxidation state of V in $VO_4^{3-} = +5$

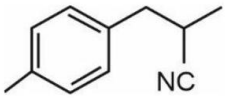
68. The product B formed in the following reaction sequence is :



(1) 

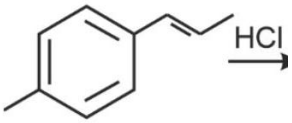
(2) 

(3) 

(4) 

Answer (2)

Sol.



AgCN





(Major)

69. The total number of compounds from below when treated with hot $KMnO_4$ giving benzoic acid is :

OH

I

(1) 4
(2) 3
(3) 6
(4) 5

Answer (4)
Sol. The compounds having benzylic hydrogen will give benzoic acid on treatment with hot $KMnO_4$.

Which are following:

DegreeFYD

70. Match List- I with List-II.

	List-I (Saccharides)		List-II (Glycosidic-linkages found)
(A)	Sucrose	(I)	α 1 - 4
(B)	Maltose	(II)	α 1-4 and α 1-6
(C)	Lactose	(III)	$\alpha 1 - \beta 2$
(D)	Amylopectin	(IV)	β1-4

Choose the correct answer from the options given below:
(1) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
(2) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
(3) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(4) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

Answer (2)

Sol.
(A) Sucrose : $\alpha - C_1 - \beta - C_2$
(B) Maltose : $\alpha - C_1 - C_4$
(C) Lactose : $\beta - C_1 - C_4$
(D) Amylopectin : $\alpha - C_1 - C_4$ and $\alpha - C_1 - C_6$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.
71. Electrolysis of 600 mL aqueous solution of NaCl for 5 min changes the pH of the solution to 12 . The current in Amperes used for the given electrolysis is ____ . (Nearest integer).

Answer (2)

Sol. For electrolysis of $NaCl_{(aq)}$:
At cathode: $2H_2O + 2e^- \rightarrow H_2 + 2OH^-$
So, 2 moles of $OH^-$is produced by 2 F charge.

$$\text{As pH} = 12 \Rightarrow [OH^-] = 10^{-2}$$

$$n_{OH^-} = \frac{10^{-2} \times 600}{1000} \text{ moles } = 6 \times 10^{-3} \text{ mol.}$$

6×10^{-3} moles of OH^- will be produced by 6×10^{-3} F charge

So, $i \times 5 \times 60 = 6 \times 10^{-3} \times 96500$

$$i = \frac{6 \times 96500}{5 \times 60 \times 1000} = 1.93 \simeq 2 \text{ A}$$

72. Total number of molecules/species from following which will be paramagnetic is

$\underline{\quad\quad}$.

$O_2, O_2^+, O_2^-, NO, NO_2, CO, K_2[NiCl_4], [Co(NH_3)_6]Cl_3, K_2[Ni(CN)_4]$

Answer (6)

Sol. Molecules/species having unpaired $e^{\ominus}$ are paramagnetic which are
$O_2, O_2^+, O_2^-, NO, NO_2, K_2[NiCl_4]$

73. Consider the following data :

Heat of formation of $CO_2(g) = -393.5$ kJ mol^{-1}
Heat of formation of $H_2O(l) = -286.0$ kJ mol^{-1}
Heat of combustion of benzene $= -3267.0$ kJ mol^{-1}
The heat of formation of benzene is ____ kjmol^{-1}
(Nearest integer)

Answer (48)

Sol. (i) $C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)}, \Delta H_1 = -393.5$ kJ/mol
(ii) $H_{2(g)} + \frac{1}{2}O_{2(g)} \rightarrow H_2O_{(l)}, \Delta H_2 = -286.0$ kJ/mol
(iii) $C_6H_6 + \frac{15}{2}O_{2(g)} \rightarrow 6CO_2 + 3H_2O + \Delta H_3 = -3267.0$ kJ/mol
(iv) $6C_{(s)} + 3H_{2(g)} \rightarrow C_6H_6 \Delta H_4 = ??$

We can get the required eqn by (i) $\times 6$ + (ii) $\times 3$ – (iii)
So, $\Delta H_4 = 6 \times (-393.5) + 3 \times (-286.0) - (-3267.0)$
$= 48$ kJ/mol

74. The spin only magnetic moment (μ) value (B.M.) of the compound with strongest oxidising power among Mn_2O_3, TiO and VO is ____ B.M. (Nearest integer).

Answer (5)

Sol. Mn_2O_3 is strongest oxidising agent among the given.

$$Mn_2O_3: Mn^{3+}: [Ar]4\ s^0 3d^4$$

$$\mu = \sqrt{4(4+2)} = \sqrt{24} = 4.89$$

$$\sim 5$$

75. A group 15 element forms $d\pi - d\pi$ bond with transition metals. It also forms hydride, which is a strongest base among the hydrides of other group members that form $d\pi - d\pi$ bond. The atomic number of the element is ____ .

Answer (15)

Sol. Phosphorus of group is element forms hydride PH_3 which is strongest base among the hydrides of other group members that form $d\pi - d\pi$ bond.