PART : MATHEMATICS

1. Let $|\cos\theta\cos(60-\theta)\cos(60+\theta)| \le \frac{1}{8}, \theta \in [0,2\pi]$. Then, the sum of all $\theta \in [0,2\pi]$, where $\cos 3\theta$ attains its maximum value, is:

(1) 9π (2) 6π (3) 18π (4) 15π

NTA (2)
RESO (2)

Sol. $\left|\cos\theta\cos\left(\frac{\pi}{3}-\theta\right)\cos\left(\frac{\pi}{3}+\theta\right)\right| \le \frac{1}{8}$

$\left|\frac{1}{4}\cos 3\theta\right| \le \frac{1}{8}$

$|\cos 3\theta| \le \frac{1}{2}$

$-\frac{1}{2} \le \cos 3\theta \le \frac{1}{2}$

Maximum value of $\cos 3\theta = \frac{1}{2}$

$3\theta = 2n\pi \pm \frac{\pi}{3}$

$\theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}$

When $\theta \in [0,2\pi]$

$\theta = \frac{\pi}{9}, \frac{2\pi}{3} \pm \frac{\pi}{9}, \frac{4\pi}{3} \pm \frac{\pi}{9}, 2\pi - \frac{\pi}{9}$

Sum of value of $\theta = 6\pi$

2. Let a circle passing through (2, 0) have its centre at the point (h, k). Let (x_c, y_c) be the point of intersection of the lines $3x+5y=1$ and $(2+c)x + 5c^2y=1$. If $h = \lim\limits_{c\to 1} x_c$ and $k = \lim\limits_{c\to 1} y_c$, then the equation of the circle is :

(1) $25x^2+25y^2-20x+2y-60=0$ (2) $25x^2+25y^2-2x+2y-60=0$
(3) $5x^2+5y^2-4x-2y-12=0$ (4) $5x^2+5y^2-4x+2y-12=0$

NTA (1)
RESO (1)

Sol. $3x+5y=1$
$(2+c)x+2c^2y=1$
$(3c^2-c-2)x = (c^2-1)$

$x = \frac{c^2-1}{3c^2-c-2}$ and $y = \frac{1}{5}\left(1-\frac{3(c^2-1)}{3c^2-c-2}\right) = \frac{1-c}{5(3c^2-c-2)}$

$h = \lim\limits_{c\to 1}\frac{c^2-1}{3c^2-c-2} = \lim\limits_{c\to 1}\frac{c+1}{3c+2} = \frac{2}{5}$

$K = \lim\limits_{c\to 1}\frac{1-c}{5(c-1)(3c+2)} = \lim\limits_{c\to 1}\frac{-1}{5(3c+12)} = -\frac{1}{25}$

Centre of circle $= \left(\frac{2}{5}, -\frac{1}{25}\right)$

Now radius of circle = $\sqrt{\left(2-\frac{2}{5}\right)^2+\left(\frac{1}{25}\right)^2}=\sqrt{\frac{64}{25}+\frac{1}{625}}$

Equation of circle is

$\left(x-\frac{2}{5}\right)^2+\left(y+\frac{1}{25}\right)^2=\frac{64}{25}+\frac{1}{625}$

$x^2+y^2-\frac{4}{5}x+\frac{2}{25}y+\frac{4}{25}+\frac{1}{625}=\frac{64}{25}+\frac{1}{625}$

$25x^2+25y^2-20x+2y+4=64$

$25x^2+25y^2-20x+2y-60=0$

3. Let $f(x)=x^2+9$, $g(x)=\frac{x}{x-9}$ and $a = fog(10)$, $b = gof(3)$. If e and ℓ denote the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{a}+\frac{y^2}{b}=1$, then $8e^2+\ell^2$ is equal to.

(1) 16 (2) 12 (3) 6 (4) 8

NTA (4)
RESO (4)
Sol. $a=f(g(10))=f(10)=109$

and $b=gof(3)=g(f(3))=g(18)=2$

so curve is $\frac{x^2}{109}+\frac{y^2}{2}=1$ an ellipse

where $e^2=1-\frac{2}{109}=\frac{107}{109}$ and $\ell=\frac{2b}{\sqrt{a}}$

$\Rightarrow \ell=\frac{2(2)}{\sqrt{109}}$

$\Rightarrow \ell^2=\frac{16}{109}$

Now $8e^2+\ell^2=\frac{856+16}{109}=\frac{872}{109}=8$

4. Let $\lambda, \mu \in R$. If the system of equations

$3x+5y+\lambda z=3$
$7x+11y-9z=2$
$97x+155y-189z=\mu$

has infinitely many solutions, then $\mu+2\lambda$ is equal to:

(1) 22 (2) 24 (3) 27 (4) 25

NTA (4)
RESO (4)
Sol. For infinitely many solutions

$\Delta=\begin{vmatrix}3 & 5 & \lambda\\ 7 & 11 & -9\\ 97 & 155 & -189\end{vmatrix}=0$

$R_3 \to R_3-21R_2$

$$\begin{vmatrix} 3 & 5 & \lambda \\ 7 & 11 & -9 \\ -50 & -76 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 3 & 5 & \lambda \\ 7 & 11 & -9 \\ 25 & 38 & 0 \end{vmatrix} = 0$$

$\Rightarrow \lambda (266 - 275) + 9 (114 - 125) = 0$

$\Rightarrow -9\lambda - 99 = 0 \Rightarrow \lambda = -11$

$$\Delta_3 = \begin{vmatrix} 3 & 5 & 3 \\ 7 & 11 & 2 \\ 97 & 155 & \mu \end{vmatrix} = 0$$

$97 (10-33) -155 (6-21) + \mu (33-35) = 0$

$2\mu = 97 (-23) + 155 (15)$

$2\mu = -2231 + 2375$

$\mu = 47$

So, $\mu + 2\lambda = 47 - 22$

$= 25$

5. A variable line L passes through the point (3, 5) and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is the origin, is:

(1) 35 (2) 30 (3) 25 (4) 40

NTA (2)

RESO (2)

Sol. As a line passes through P(a, b) cuts the coordinate axis at A and B then area of ΔOAB will be minimum iff P is mid point of AB

$\Rightarrow$ Equation line AB is $\frac{x}{2a} + \frac{y}{2b} = 1$

Hence equation of required line is $\frac{x}{6} + \frac{y}{10} = 1$

$\Rightarrow 5x + 3y = 30$

and area of Δ is = $\frac{1}{2}(6)(10) = 30$

6. The shortest distance between the lines $\frac{x-3}{4} = \frac{y+7}{-11} = \frac{z-1}{5}$ and $\frac{x-5}{3} = \frac{y-9}{-6} = \frac{z+2}{1}$ is:

(1) $\frac{178}{\sqrt{563}}$ (2) $\frac{185}{\sqrt{563}}$

(3) $\frac{179}{\sqrt{563}}$ (4) $\frac{187}{\sqrt{563}}$

NTA (4)

RESO (4)

Sol. Given lines are

$\bar{r} = (3\hat{i} - 7\hat{j} + \hat{k}) + \lambda(4\hat{i} - 11\hat{j} + 5\hat{k}) = \vec{a}_1 + \lambda\vec{b}_1$ and $\bar{r} = (5\hat{i} + 9\hat{j} - 2\hat{k}) + \mu(3\hat{i} - 6\hat{j} + \hat{k}) = \vec{a}_2 + \mu\vec{b}_2$ form shortest distance between two lines

$$= \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$= \left| \frac{(2\hat{i} + 16\hat{j} - 3\hat{k}) \cdot (19\hat{i} + 11\hat{j} + 9\hat{k})}{\sqrt{(19)^2 + (11)^2 + (9)^2}} \right| \quad \text{here} \quad \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -11 & 5 \\ 3 & -6 & 1 \end{vmatrix} = 19\hat{i} + 11\hat{j} + 9\hat{k}$$

$$= \frac{38 + 176 - 27}{\sqrt{563}}$$

$$= \frac{187}{\sqrt{563}}$$

7. Let $\vec{OA} = 2\vec{a}$, $\vec{OB} = 6\vec{a} + 5\vec{b}$ and $\vec{OC} = 3\vec{b}$, where O is the origin. If the area of the parallelogram with adjacent sides $\vec{OA}$ and $\vec{OC}$ is 15 sq. units, then the area (in sq. units) of the quadrilateral OABC is equal to:

(1) 40 (2) 35 (3) 38 (4) 32

NTA (2)
RESO (2)

Sol. $15 = |\vec{OA} \times \vec{OC}| \Rightarrow \frac{5}{2} = |\bar{a} \times \bar{b}|$ _____(1)

Area of quadrilateral OABC = $\frac{1}{2}|\vec{OB} \times \vec{AC}| = \frac{1}{2}|(6\bar{a} + 5\bar{b}) \times (3\bar{b} - 2\bar{a})|$

$= 14|\bar{a} \times \bar{b}| = 14 \times \frac{5}{2} = 35$ Square units

8. The solution curve, of the differential equation $2y\frac{dy}{dx} + 3 = 5\frac{dy}{dx}$, passing through the point (0,1) is a conic, whose vertex lies on the line:

(1) 2x+3y=–9 (2) 2x+3y=6 (3) 2x+3y=9 (4) 2x+3y=–6

NTA (3)
RESO (3)

Sol. $2y\,dy + 3dx = 5dy$

$\Rightarrow y^2 + 3x = 5y + c$

passes through point (0, 1)

$1 = 5 + c \Rightarrow c = -4$

$y^2 + 3x = 5y - 4$

$y^2 - 5y = -3x - 4$

$$\Rightarrow \left(y - \frac{5}{2}\right)^2 = -3x - 4 + \frac{25}{4}$$

$$\Rightarrow \left(y - \frac{5}{2}\right)^2 = -3\left(x - \frac{3}{4}\right)$$

so vertex = $\left(\frac{3}{4}, \frac{5}{2}\right)$

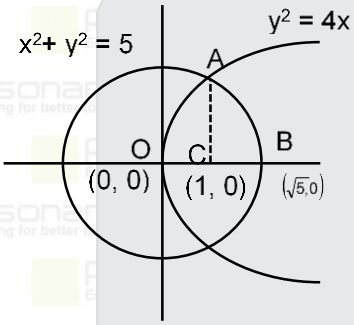
It lies on the line 2x+3y=9

9. The parabola $y^2 = 4x$ divides the area of the circle $x^2 + y^2 = 5$ in two parts. The area of the smaller part is equal to:

(1) $\frac{2}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (2) $\frac{1}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

(3) $\frac{2}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (4) $\frac{1}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

NTA (1)
RESO (1)
Sol.



The points of intersection of $y^2 = 4x$ and $x^2 + y^2 = 5$ are (1, 2) and (1, –2)
hence area of smaller region bounded by $y^2 = 4x$ and $x^2 + y^2 = 5$

$$= 2\{\text{area of OACO + Area of CABC}\} = 2\left[\int_0^1 2\sqrt{x}\,dx+\int_1^{\sqrt{5}}\sqrt{5-x^2}\,dx\right]$$

$$= 2\left[\left|\frac{4}{3}x^{\frac{3}{2}}\right|_0^1+\left(\frac{1}{2}x\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right)\Bigg|_1^{\sqrt{5}}\right]=2\left[\left(\frac{4}{3}-0\right)+\left(0+\frac{5\pi}{4}\right)-\left(1+\frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right)\right]$$

$$= 2\left[\frac{1}{3}+\frac{5\pi}{4}-\frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right]=\frac{2}{3}+\frac{5\pi}{2}-5\sin^{-1}\frac{1}{\sqrt{5}}=\frac{2}{3}+5\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)=\frac{2}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

10. Let three vectors $\vec{a} = \alpha\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b} = 5\hat{i}+3\hat{j}+4\hat{k}$, $\vec{c} = x\hat{i}+y\hat{j}+z\hat{k}$ from a triangle such that $\vec{c} = \vec{a} - \vec{b}$ and the area of the triangle is $5\sqrt{6}$. If α is a positive real number, then $\left|\vec{c}\right|^2$ is equal to

(1) 16 (2) 12 (3) 10 (4) 14

NTA (4)
RESO (4)
Sol. $\bar{c} = (\alpha-5)\hat{i}+\hat{j}-2\hat{k}$

Area of $\Delta ABC = \left|\frac{1}{2}(\bar{a}\times\bar{b})\right| = 5\sqrt{6} \Rightarrow \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \alpha & 4 & 2\\ 5 & 3 & 4\end{vmatrix} = 10\sqrt{6}$

$$\left|10\hat{i}-(4\alpha-10)\hat{j}+(3\alpha-20)\hat{k}\right| = 10\sqrt{6}$$

$\Rightarrow \quad \sqrt{100+(10-4\alpha)^2+(3\alpha-20)^2} = 10\sqrt{6}$

$\Rightarrow \quad 25\alpha^2 - 200\alpha + 600 = 600$

$\Rightarrow \quad 25\alpha(\alpha - 8) = 0$

$\Rightarrow \quad \alpha = 8 \qquad (\because \ \alpha \in R^+)$

$\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$

$\Rightarrow \quad |\vec{c}|^2 = 9+1+4 = 14$

11. Let the line L intersect the lines $x-2 = -y = z-1$, $2(x+1) = 2(y-1) = z+1$ and be parallel to the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{2}$

Then which of the following points lies on L?

(1) $\left(-\frac{1}{3},1,1\right)$ (2) $\left(-\frac{1}{3},-1,1\right)$ (3) $\left(-\frac{1}{3},-1,-1\right)$ (4) $\left(-\frac{1}{3},1,-1\right)$

NTA (4)
RESO (4)

Sol. $L_1 : \frac{x-2}{1} = \frac{y}{-1} = \frac{z-1}{1}$

$L_2 : \frac{x-1}{\left(\frac{1}{2}\right)} = \frac{y-1}{\left(\frac{1}{2}\right)} = \frac{z+1}{1}$

Line L passes through points A $(\lambda+2, -\lambda, \lambda+1)$ and B $\left(\frac{1}{2}\mu-1, \frac{1}{2}\mu+1, \mu-1\right)$.

Dr's of line L = $\left(\frac{1}{2}\mu-\lambda-3, \frac{1}{2}\mu+\lambda+1, \mu-\lambda-2\right)$

So, $\frac{\frac{1}{2}\mu-\lambda-3}{3} = \frac{\frac{1}{2}\mu+\lambda+1}{1} = \frac{\mu-\lambda-2}{2}$

$\frac{1}{2}\mu-\lambda-3 = \frac{3}{2}\mu+3\lambda+3$ and $\mu+2\lambda+2 = \mu-\lambda-2$

$\mu + 4\lambda = -6$ $\quad 3\lambda = -4$

$\mu = -6 + \frac{16}{3}$ $\quad \lambda = -\frac{4}{3}$

$\mu = -\frac{2}{3}$ $\quad A\left(\frac{2}{3}, \frac{4}{3}, -\frac{1}{3}\right)$

Equation of line L is

$\frac{x-\frac{2}{3}}{3} = \frac{y-\frac{4}{3}}{1} = \frac{z+\frac{1}{3}}{2}$

Point $\left(-\frac{1}{3},1,-1\right)$ satisfying this line L.

12. Let $\int \frac{2-\tan x}{3+\tan x}dx = \frac{1}{2}(\alpha x + \log_e|\beta \sin x + \gamma \cos x|) + C$, where C is the constant of integration then $\alpha + \frac{\gamma}{\beta}$ is equal to:

(1) 7 (2) 1 (3) 4 (4) 3

NTA (3)
RESO (3)

Sol. $\int \frac{2\cos x - \sin x}{3\cos x + \sin x}dx$

Put $2\cos x - \sin x = a(-3\sin x + \cos x) + b(3\cos x + \sin x)$

$2 = a + 3b, \quad -1 = -3a + b \quad \Rightarrow a = \frac{1}{2} = b$

So $\int \frac{2\cos x - \sin x}{3\cos x + \sin x}dx = \frac{1}{2}\ell n|3\cos x + + \sin x| + \frac{1}{2}x + C$

$= \frac{1}{2}[x + \ell n|\sin x + 3\cos x|] + C$

So $\alpha = 1, \beta = 1, \gamma = 3$

$\alpha + \frac{\gamma}{\beta} = 1 + \frac{3}{1} = 4$

13. Let $f(x) = ax^3 + bx^2 + cx + 41$ be such that $f(1) = 40$, $f'(1) = 2$ and $f''(1) = 4$ then $a^2+b^2+c^2$ is equal to

(1) 73 (2) 54 (3) 51 (4) 62

NTA (3)
RESO (3)

Sol. $x=1 \Rightarrow a+b+c=-1 \quad ....(1)$

$f'(x) = 3ax^2+2bx+c$

$3a+2b+c=2 \quad ....(2)$

$f''(x) = 6ax+2b$

$6a+2b=4$

$3a+b=2 \quad ....(3)$

(1), (2) $\Rightarrow 2a+b=3 \quad ....(4)$

$a=-1 \Rightarrow b=5 \Rightarrow c=-5$

$a^2+b^2+c^2=51$

14. The coefficient of x^{70} in $x^2(1+x)^{98} + x^3(1+x)^{97} + x^4(1+x)^{96} + ...+ x^{54}(1+x)^{46}$ is ${}^{99}C_P - {}^{46}C_q$ then a possible value of p+q is

(1) 68 (2) 61 (3) 55 (4) 83

NTA (4)
RESO (4)

Sol. coefficient of x^{70} is ${}^{98}C_{68} + {}^{97}C_{67} + {}^{96}C_{66} + ----------+ {}^{46}C_{16}$

$= -{}^{46}C_{15} + {}^{46}C_{15} + {}^{46}C_{16} + {}^{47}C_{17} + {}^{48}C_{18} + ---------- + {}^{98}C_{68}$

$= -{}^{46}C_{15} + {}^{47}C_{16} + {}^{47}C_{17} + {}^{48}C_{18} + ---------- + {}^{98}C_{68}$

$= -{}^{46}C_{15} + {}^{99}C_{68}$

$= {}^{99}C_{68} - {}^{46}C_{15}$

So p = 68, q = 15

$\Rightarrow$ p + q = 83

15. If the sum of the series $\frac{1}{1.(1+d)}+\frac{1}{(1+d)(1+2d)}+\ldots\ldots\frac{1}{(1+9d)(1+10d)}$ is equal to 5, then 50d is equal to

(1) 20 (2) 15 (3) 5 (4) 10

NTA (3)
RESO (3)

Sol. $\frac{1}{1(1+d)}+\frac{1}{(1+d)(1+2d)}+\ldots\ldots\ldots\frac{1}{(1+9d)(1+10d)}=5$

$$\Rightarrow \sum_{r=1}^{10}\left\{\frac{1}{\{1+(r-1)d\}\{1+rd\}}\right\}=5$$

$$\Rightarrow \frac{1}{d}\sum_{r=1}^{10}\left\{\frac{1}{1+(r-1)d}-\frac{1}{1+rd}\right\}=5$$

$$\Rightarrow \left\{\frac{1}{1}-\frac{1}{1+d}\right\}+\left\{\frac{1}{1+d}-\frac{1}{1+2d}\right\}+\ldots..+\left\{\frac{1}{1+9d}-\frac{1}{1+10d}\right\}=5d$$

$$\Rightarrow 1-\frac{1}{1+10d}=5d$$

$$\Rightarrow \frac{10d}{1+10d}=5d$$

$\Rightarrow 2 = 1 + 10d$

$\Rightarrow 10d = 1$

$\Rightarrow 50\,d = 5$

16. Let α, β be the roots of the equation $x^2+2\sqrt{2}x-1=0$. The quadratic equation, whose roots are $\alpha^4+\beta^4$ and $\frac{1}{10}\left(\alpha^6+\beta^6\right)$ is

(1) $x^2-190x+9466=0$ (2) $x^2-195x+9506=0$
(3) $x^2-195x+9466=0$ (4) $x^2-180x+9506=0$

NTA (2)
RESO (2)

Sol. $x^2+2\sqrt{2}x-1=0$ $\begin{matrix}\nearrow\ \alpha\\ \searrow\ \beta\end{matrix}$

$\alpha+\beta=-2\sqrt{2}, \quad \alpha\beta=-1$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$= 8 + 2 = 10$

$\alpha^4+\beta^4=\left(\alpha^2+\beta^2\right)^2-2(\alpha\beta)^2$

$= 100 - 2 = 98$

and $\alpha^6+\beta^6=\left(\alpha^2+\beta^2\right)^3-3\alpha^2\beta^2\left(\alpha^2+\beta^2\right)$

$= 1000 - 3(10)$

$= 970$

New equation is

$x^2-(98+97)x+97\times98=0$

$x^2-195x+9506=0$

17. A ray of light coming from the point P(1,2) gets reflected from the point Q on the x-axis and then passes through the point R(4,3). If the point S(h,k) is such that PQRS is a parallelogram, then hk^2 is equal to:

(1) 80 (2) 90 (3) 60 (4) 70

NTA (4)

RESO (4)

Sol. P'(1, –2)

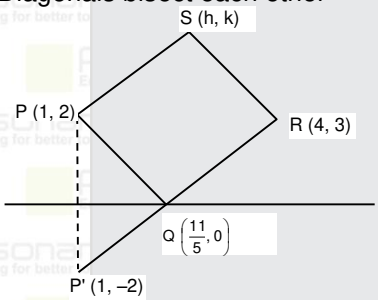
Equation of P' R is

$$\Rightarrow y - 3 = \frac{-2-3}{1-4}(x-4)$$

$$\Rightarrow 3y = 5x - 11$$

So Q $\left(\frac{11}{5}, 0\right)$

Diagonals bisect each other



$$\frac{1+4}{2} = \frac{h+\frac{11}{5}}{2} \Rightarrow h = \frac{14}{5}$$

$$\frac{2+3}{2} = \frac{0+k}{2} \Rightarrow k = 5 \qquad \Rightarrow hk^2 = \frac{14}{5} \times 25 = 70$$

18. The solution of the differential equation $(x^2+y^2)dx - 5xy\,dy = 0$, $y(1)=0$, is

(1) $\left|x^2-2y^2\right|^6 = x$ (2) $\left|x^2-4y^2\right|^5 = x^2$ (3) $\left|x^2-4y^2\right|^6 = x$ (4) $\left|x^2-2y^2\right|^5 = x^2$

NTA (2)

RESO (2)

Sol. $\left(x^2+y^2\right)dx = 5xy\,dy$

$$\frac{dy}{dx} = \frac{x^2+y^2}{5xy}$$

Let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\frac{1+v^2}{5v} = v + x\frac{dv}{dx}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{5v} - v$$

$$x\frac{dv}{dx} = \frac{1-4v^2}{5v}$$

$$\frac{5v}{1-4v^2}dv = \frac{dx}{x}$$

$$\int \frac{dx}{x} = -\frac{5}{8}\int \frac{-8v}{1-4v^2}dv$$

$$\ell nx = -\frac{5}{8}\ell n\left|1-4v^2\right| + \ell nc$$

$$\ell nx + \frac{5}{8}\ell n\left|1-\frac{4y^2}{x^2}\right| = \ell nc \qquad \because\ y(1) = 0$$

So $\ell nc = 0$

$$8\ell nx + 5\ell n\left|\frac{x^2-4y^2}{x^2}\right| = 0$$

$$8\ell nx + 5\ell n\left|x^2-4y^2\right| - 10\ell nx = 0$$

$$\ell n\left|x^2-4y^2\right|^5 - \ell nx^2 = 0$$

$$\left|x^2-4y^2\right|^5 = x^2$$

19. The frequency distribution of the age of students in a class of 40 students is given below.

Age	15	16	17	18	19	20
No. of Students	5	8	5	12	x	y

If the mean deviation about the median is 1.25, then 4x+5y is equal to

(1) 46 (2) 43 (3) 44 (4) 47

NTA (3)
RESO (3)
Sol.

Age	**No. of student**	**C.f**	$\|x_{c^\circ} - 18\|$	$f_c\|x_c - 18\|$
15	5	5	3	15
16	8	13	2	16
17	5	18	1	5
18	12	30	0	0
19	x	30+x	1	x
20	y	30+x+y	2	2y

$N = \Sigma f_i = 30 + x + y = 40$

$x + y = 10$ .....(1)

$\frac{N}{2} = 20$

So, median of given observers is 18

$\sum f_i\,|x_i - 18| = 36 + x + 2y$

Mean deviation about median $= \frac{\Sigma f_i\,|x_i - 18|}{\Sigma f_i}$

$\Rightarrow \frac{36 + x + 2y}{40} = 1.25$

$\Rightarrow 36 + x + 2y = 50$

$\Rightarrow x + 2y = 14$ .....(2)

So, $y = 4,\ x = 6$

$4x + 5y = 24 + 20 = 44$

20. If the domain of the function $f(x)=\sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is $R-(\alpha,\beta)$, then $12\alpha\beta$ is equal to

(1) 36 (2) 40 (3) 24 (4) 32

NTA (4)
RESO (4)

Sol. $-1 \le \frac{x-1}{2x+3} \le 1$

$\frac{x-1}{2x+3}+1 \ge 0$ and $\frac{x-1}{2x+3}-1 \le 0$

$\frac{3x+2}{2x+3} \ge 0$ $\frac{-x-4}{2x+3} \le 0$

$x \in \left(-\infty,-\frac{3}{2}\right)\cup\left[-\frac{2}{3},\infty\right)$ $\frac{x+4}{2x+3} \ge 0$

$x \in (-\infty,-4]\cup\left(-\frac{3}{2},\infty\right)$

So domain of f(x) is $x \in (-\infty,-4]\cup\left[-\frac{2}{3},\infty\right)$

$x \in R-\left(-4,-\frac{2}{3}\right)$

$\alpha = -4$ and $\beta = \frac{-2}{3}$

$12\alpha\beta = 32$

21. The sum of the square of the modulus of the elements in the set $\{z=a+ib: a, b \in Z, z \in C, |z-1| \le 1, |z-5| \le |z-5i|\}$ is_____

NTA (9)
RESO (9)

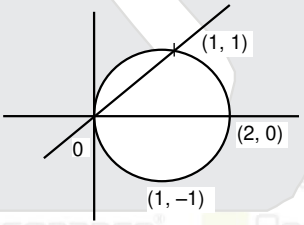
Sol. $|z-1| \le 1 \Rightarrow (a-1)^2 + b^2 \le 1$

$|z-5| \le |z-5i| \Rightarrow (a-5)^2 + b^2 \le a^2 + (b-5)^2 \quad \Rightarrow a \ge b$

$z = 0, 1, 2, 1+i, 1-i$

$|z| = 0, 1, 2, \sqrt{2}, \sqrt{2}$

sum of $|z|^2 = 0 + 1 + 4 + 2 + 2 = 9$



22. If a function f satisfies $f(m+n) = f(m) + f(n)$ for all $m, n \in N$ and $f(1)=1$, then the largest natural number λ such that $\sum_{k=1}^{2022} f(\lambda+k) \le (2022)^2$ is equal to____

NTA (1010)
RESO (1010)

Sol. $f(m+n) = f(m) + f(n)$

So $f(x) = kx \quad \because f(1) = 1 \Rightarrow k = 1$

$\therefore f(x) = x$

Now $\sum_{k=1}^{2022}(\lambda+k) \le (2022)^2$

$$2022\,\lambda + \frac{2022\times 2023}{2} \le (2022)^2$$

$$\lambda \le 2022 - \frac{2023}{2}$$

$$\lambda \le 2022 - 1011.5$$

$$\lambda \le 1010.5$$

$$\lambda_{max} = 1010$$

23. Let A be a non-singular matrix of order 3. If det (3adj(2adj((detA)A))=$3^{-13}\,2^{-10}$ and det(3adj(2A))=$2^m.3^n$, then |3m+2n| is equal to_____

NTA (14)
RESO (14)

Sol. det (3adj(2adj((detA)A))=$3^{-13}\,2^{-10}$

3^3 det (adj (2adj ((det A)A))) = $3^{-13}.\,2^{-10}$

$|2\text{adj}((\det A)A)|^2 = 3^{-16}.\,2^{-10}$

$(2^3\,|\text{adj}(|A|A)|)^2 = 3^{-16}.\,2^{-10}$

$(||A|\,A|^2)^2 = 3^{-16}.\,2^{-16}$

$||A|\,A|^4 = 3^{-16}.\,2^{-16}$

$(|A|^3\,|A|)^4 = 3^{-16}.\,2^{-16}$

$|A|^{12}\,|A|^4 = 3^{-16}.\,2^{-16}$

$|A| = 3^{-1}.\,2^{-1}$

Let C = 2A

$\Rightarrow$ det (3adjC) = $2^m.\,3^n \Rightarrow 3^3|C|^2 = 2^m.3^n$

$(2^3\,|A|)^2 = 2^m.\,3^{n-3} \Rightarrow |A|^2 = 2^{m-6}.\,3^{n-3}$

$3^{-2}\,.\,2^{-2} = 2^{m-6}.\,3^{n-3}$

$\Rightarrow$ m = 4, n = 1

|3m + 2n| = 12 + 2 = 14

24. Let f : $(0,\pi) \to R$ be a function given by $f(x)=\begin{cases}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0<x<\frac{\pi}{2}\\ a-8, & x=\frac{\pi}{2}\\ (1+|\cot x|)^{\frac{b}{a}|\tan x|}, & \frac{\pi}{2}<x<\pi\end{cases}$

where a, b$\in$Z. If f is continuous at $x=\frac{\pi}{2}$, then a^2+b^2 is equal to _____

NTA (81)
RESO (81)

Sol. As f(x) is continuous at $x=\frac{\pi}{2}$

$$\Rightarrow Lf\left(\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right) = Rf\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{x\to\frac{\pi}{2}}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}} = a-8 = \lim_{x\to\frac{\pi}{2}}(1+|\cot x|)^{\frac{b}{a}|\tan x|}$$

$$\Rightarrow \left(\frac{8}{7}\right)^{0} = a-8 = e^{\frac{b}{a}}$$

$\Rightarrow a = 9$ and $b = 0$

Hence $a^2 + b^2 = 81$

25. Let the set of all positive values of λ, for which the point of local minimum of the function $\left(1+x\left(\lambda^2-x^2\right)\right)$ satisfies $\frac{x^2+x+2}{x^2+5x+6}<0$, be (α,β). Then $\alpha^2+\beta^2$ is equal to ____

NTA (39)

RESO (39)

Sol. $f(x) = 1 + x(\lambda^2 - x^2)$

$f(x) = 1 + \lambda^2 x - x^3$

$f'(x) = \lambda^2 - 3x^2$

$= -(\sqrt{3}x+\lambda)(\sqrt{3}x-\lambda)$

Local minima of for is $x = -\frac{\lambda}{\sqrt{3}}$

$$\frac{x^2+x+2}{x^2+5x+6}<0$$

$$\frac{1}{(x+2)(x+3)}<0$$

$x \in (-3,-2)$

So, $-3 < -\frac{\lambda}{\sqrt{3}} < -2$

$2\sqrt{3} < \lambda < 3\sqrt{3}$

$\lambda \in (2\sqrt{3}, 3\sqrt{3}) = (\alpha, \beta)$

$\alpha^2+\beta^2 = 12 + 27 = 39$

26. Let a, b and c denote the outcome of three independent rolls of a fair tetrahedral die, whose four faces are marked 1,2,3,4. If the probability that $ax^2+bx+c=0$ has all real roots is $\frac{m}{n}$, gcd(m,n)=1, then m+n is equal to ____

NTA (19)

RESO (19)

Sol. $n(S) = 4 \times 4 \times 4 = 64$

For real roots $b^2 \geq 4ac$

	b	a	c	
(i)	2	1	1	1 way
(ii)	3	1	1	3 ways
		1	2	
		2	1	
(iii)	4	1	1	8 ways
		1	2	
		2	1	
		3	1	
		1	3	
		2	2	
		1	4	
		4	1	

So probability of real roots $= \frac{1+3+8}{64} = \frac{12}{64} = \frac{3}{16} = \frac{m}{n}$

So $m + n = 3 + 16 = 19$

27. Let $\lim\limits_{n\to\infty}\left(\frac{n}{\sqrt{n^4+1}} - \frac{2n}{(n^2+1)\sqrt{n^4+1}} + \frac{n}{\sqrt{n^4+16}} - \frac{8n}{(n^2+4)\sqrt{n^4+16}} + \ldots + \frac{n}{\sqrt{n^4+n^4}} - \frac{2n.n^2}{(n^2+n^2)\sqrt{n^4+n^4}}\right)$ be $\frac{\pi}{k}$, using only the principal values of the inverse trigonometric functions. Then k^2 is equal to ____

NTA (32)

RESO (32)

Sol. $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\left(\frac{n}{\sqrt{n^4+r^4}} - \frac{2nr^2}{(n^2+r^2)\sqrt{n^4+r^4}}\right)$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\left\{\frac{1}{\sqrt{1+\left(\frac{r}{n}\right)^4}} - \frac{2\left(\frac{r}{n}\right)^2}{\left(1+\left(\frac{r}{n}\right)^2\right)\sqrt{1+\left(\frac{r}{n}\right)^4}}\right\}\frac{1}{n}$$

$\frac{r}{n} = x$

$\frac{1}{n} = dx$

r	1	n
x	0	1

$$= \int_0^1 \left(\frac{1}{\sqrt{1+x^4}} - \frac{2x^2}{(1+x^2)\sqrt{1+x^4}} \right) dx$$

$$= \int_0^1 \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}} dx$$

$$= -\int_0^1 \frac{1-\frac{1}{x^2}}{\left(\frac{1}{x}+x\right)\sqrt{\frac{1}{x^2}+x^2}} dx$$

$$x + \frac{1}{x} = t \qquad \Rightarrow \left(1-\frac{1}{x^2}\right)dx = dt$$

$$= -\int_{\infty}^{2} \frac{1}{t\sqrt{t^2-2}} dt$$

$$= \int_2^{\infty} \frac{1}{t\sqrt{t^2-2}} dt = \left[\frac{1}{\sqrt{2}} \sec^{-1} \frac{t}{\sqrt{2}} \right]_2^{\infty} = \frac{\pi}{2\sqrt{2}} - \frac{\pi}{4\sqrt{2}} = \frac{\pi}{4\sqrt{2}} = \frac{\pi}{k}$$

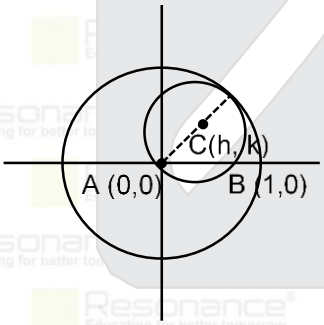
So, $k^2 = (4\sqrt{2})^2 = 32$

28. Let the centre of a circle , passing through the points (0,0),(1,0) and touching the circle $x^2+y^2=9$, be (h,k). Then for all possible values of the coordinates of the centre (h,k) , $4(h^2+k^2)$ is equal to ____

NTA (9)

RESO (9)

Sol.



Diameter of interior circle = 3

$$r = \frac{3}{2}$$

$$AC = \sqrt{h^2+k^2} = \frac{3}{2}$$

$$\Rightarrow h^2 + k^2 = \frac{9}{4}$$

$$\Rightarrow 4\left(h^2+k^2\right) = 9$$

29. Let A={2,3,6,7} and B ={4,5,6,8}. Let R be a relation defined on A ×B by (a_1 , b_1) R (a_2 , b_2) if and only if $a_1 + a_2 = b_1 + b_2$. Then the number of elements in R is ____

NTA (25)
RESO (25)
Sol. $A = \{2, 3, 6, 7\}$
$B = \{4, 5, 6, 8\}$
$(a_1, b_1)\ R\ (a_2, b_2) \Rightarrow a_1 + a_2 = b_1 + b_2$

$$\left.\begin{array}{ll}(2,4)\ R\ (6,4) & (3,6)\ R\ (7,4)\\ (2,4)\ R\ (7,5) & (3,5)\ R\ (7,5)\\ (2,5)\ R\ (7,4) & (6,5)\ R\ (7,8)\\ (3,4)\ R\ (6,5) & (6,8)\ R\ (7,5)\\ (3,5)\ R\ (6,4) & (7,6)\ R\ (7,8)\\ (3,4)\ R\ (7,6) & (6,4)\ R\ (6,8)\end{array}\right] \times 2$$

$(6, 6)\ R\ (6, 6)$
Total number of elements in R = $12 \times 2 + 1 = 25$

30. The remainder when 428^{2024} is divided by 21 is ____
NTA (1)
RESO (1)
Sol. $21 \times 20 = 420 \quad \Rightarrow (428)^{2024} = (420+8)^{2024}$

$= {}^{2024}C_0 (420)^{2024} + \ldots + {}^{2024}C_{2023} (420) 8^{2023} + 8^{2024}$

$= 420$ [some integer] $+ (64)^{1012}$; $\quad 63 = 21 \times 3$

$= 21 \times 20$ [some integer] $+ (63+1)^{1012}$

$= 21 \times 20$ [some integer] $+ 63$[some other integer] $+ 1$

Hence remainder is 1.

PART : PHYSICS

31. A sample of 1 mole gas at temperature T is adiabatically expanded to double its volume. If adiabatic constant for the gas is $\gamma = \frac{3}{2}$, then the work done by the gas in the process is :

T तापमान पर एक मोल गैस का नमूना रूद्धोष्म प्रक्रम के द्वारा आयतन के दो गुने तक प्रसारित किया जाता है। यदि गैस के लिए रूदोष्म नियतांक $\gamma = \frac{3}{2}$ हो, तब इस प्रक्रम में गैस द्वारा किया गया कार्य है :

(1) $RT\left[2+\sqrt{2}\right]$

(2) $\frac{T}{R}\left[2+\sqrt{2}\right]$

(3) $RT\left[2-\sqrt{2}\right]$

(4) $\frac{R}{T}\left[2-\sqrt{2}\right]$

Ans. **(3)**

Sol. $V \rightarrow 2V$

$TV^{\gamma-1} = \text{const}$

$T(V)^{\frac{3}{2}-1} = T_f(2V)^{\frac{3}{2}-1}$

$T_f = \frac{T}{\sqrt{2}}$

$W = \frac{-nR\Delta T}{\gamma-1} = -R\frac{\left(\frac{T}{\sqrt{2}}-T\right)}{1/2}$

$W = RT\left(2-\sqrt{2}\right)$

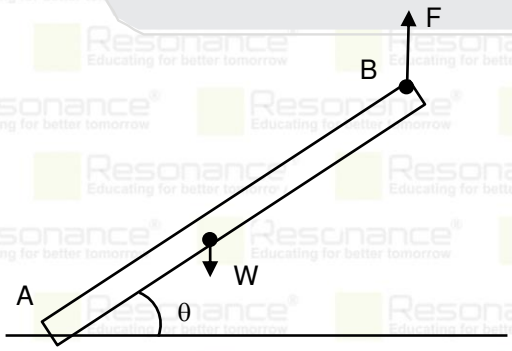
32. A heavy iron bar, of weight W is having its one end on the ground and the other on the shoulder of a person. The bar makes an angle θ with the horizontal. The weight experienced by the person is :

W भार की एक भारी लोहे की छड़ का एक सिरा पृथ्वी सतह पर तथा दूसरा सिरा एक व्यक्ति के कँधे पर रखा है। छड़ क्षैतिज के साथ θ कोण बनाती है। व्यक्ति द्वारा अनुभव किया गया भार है :

(1) $W\cos\theta$ (2) W (3) $\frac{W}{2}$ (4) $W\sin\theta$

Ans. **(3)**

Sol.



$\tau_F = \tau_W$

$\Rightarrow F\,L\cos\theta = W\,\frac{L}{2}\cos\theta \Rightarrow F = \frac{W}{2}$

33. The dimensional formula of latent heat is :
गुप्त ऊष्मा का विमीय सूत्र है :

(1) $[M^0LT^{-2}]$ (2) $[ML^2T^{-2}]$ (3) $[M\,LT^{-2}]$ (4) $[M^0L^2T^{-2}]$

Ans. **(4)**

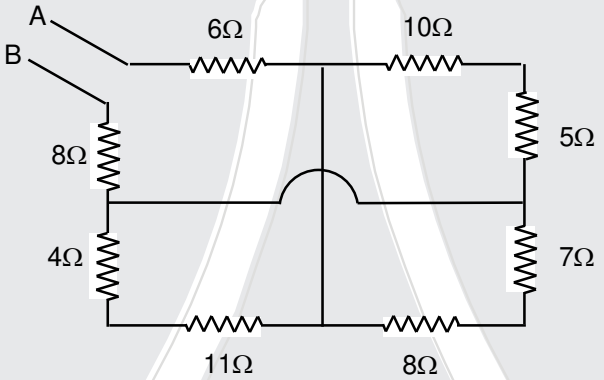
Sol. $Q = Lm$

$$\text{Latent heat} = \frac{Q}{\text{mass}}$$

$$= \frac{[M^1L^2T^{-2}]}{[M^1L^0T^0]}$$

Latent Heat = $[L^2T^{-2}]$

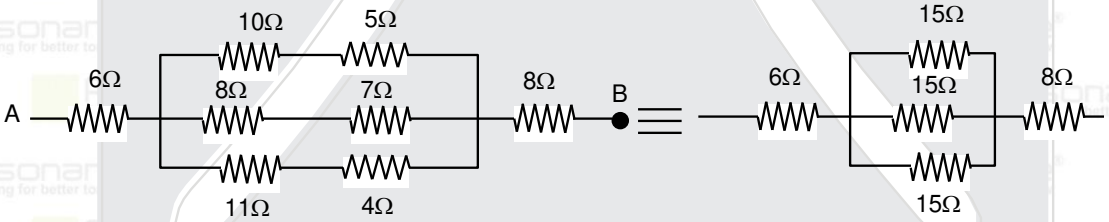
34. The equivalent resistance between A and B is :
A व B के बीच समतुल्य प्रतिरोध है :



(1) 27 Ω (2) 19 Ω (3) 18 Ω (4) 25 Ω

Ans. **(2)**

Sol.



$$R_{eq} = 6 + 8 + \frac{15}{3} = 19\ \Omega$$

35. One main scale division of a vernier caliper is equal to m units. If n^{th} division of main scale coincides with $(n+1)^{th}$ division of vernier scale, the least count of the vernier caliper is

(1) $\frac{1}{(n+1)}$ (2) $\frac{n}{(n+1)}$ (3) $\frac{m}{(n+1)}$ (4) $\frac{m}{n(n+1)}$

Ans. **(3)**

Sol. $1VSD = \frac{n}{(n+1)} \times m$

$LC = 1MSD - 1VSD$

$$= m - \frac{n}{(n+1)}.m$$

$$= \frac{m(n+1-n)}{n+1},\ LC = \frac{m}{n+1}$$

36. A plane EM wave is propagating along x direction. It has a wavelength of 4 mm. If electric field is in y direction with the maximum magnitude of 60 Vm^{-1}, the equation for magnetic field is :
एक समतुल्य विद्युत चुंबकीय तरंग x अक्ष के अनुदिश संचरित हो रही है। इसकी तरंग दैर्ध्य 4mm है। यदि विद्युत क्षेत्र 60 Vm^{-1} के अधिकतम परिमाण के साथ y दिशा में हो तो चुम्बकीय क्षेत्र के लिए समीकरण है :

(1) $B_z = 60\sin\left[\frac{\pi}{2}\left(x-3\times10^8t\right)\right]\hat{k}\,T$ (2) $B_x = 60\sin\left[\frac{\pi}{2}\left(x-3\times10^8t\right)\right]\hat{i}\,T$

(3) $B_z = 2\times10^{-7}\sin\left[\frac{\pi}{2}\left(x-3\times10^8t\right)\right]\hat{k}\,T$ (4) $B_z = 2\times10^{-7}\sin\left[\frac{\pi}{2}\times10^3\left(x-3\times10^8t\right)\right]\hat{k}\,T$

Ans. **(4)**

Sol. $\hat{B} = \hat{K}\times\hat{E} = \hat{i}\times\hat{j} = \hat{k}$

$$B_0 = \frac{E_0}{C} = \frac{60}{3\times10^8} = 2\times10^{-7}T$$

$$\vec{B} = B_0\sin[k(x-vt)]\hat{B}\,T$$

$$\vec{B} = 2\times10^{-7}\sin\left[\frac{2\pi}{4\times10^{-3}}\left(x-3\times10^8t\right)\right]\hat{k}\,T$$

$$\vec{B} = 2\times10^{-7}\sin\left[\frac{\pi}{2}\times10^3\left(x-3\times10^8t\right)\right]\hat{k}\,T$$

37. A galvanmeter has a coil of resistance 200 Ω with a full scale deflection at 20 μA. The value of resistance to be added to use it as an ammeter of range (0-20) mA is :
200 Ω प्रतिरोध की कुंडली वाले एक धारामापी में 20 μA पर पूर्ण पैमाने का विक्षेप उत्पन्न होता है। उसको (0-20) mA परास के अमीटर के रूप में प्रयोग करने के लिए जोड़े गये प्रतिरोध का मान है :

(1) 0.10 Ω (2) 0.20 Ω (3) 0.40 Ω (4) 0.50 Ω

Ans. **(2)**

Sol. $I_GR_G = I_sR_s$

$$I_GR_G = (I-I_G)R_s$$

$I = 20$ mA , $I_G = 20$ μA

$$R_s = \left(\frac{20\times10^{-6}}{20\times10^{-3}-20\times10^{-6}}\right)\times200$$

$$R_s = \frac{200}{1000-1}$$

$$R_s = 0.2\Omega$$

38. An astronaut takes a ball of mass m from earth to space. He throws the ball into a circular orbit about earth at an altitude of 318.5 km. From earth's surface to the orbit, the change in total mechanical energy of the ball is $x\frac{Gm_em}{21R_e}$. The value of x is (take R_e = 6370 km) :
एक अंतरिक्ष यात्री m द्रव्यमान की एक गेंद को पृथ्वी से अंतरिक्ष में ले जाता है। वह 318.5 किमी. की ऊँचाई पर पृथ्वी के परितः एक वृत्तीय कक्षा में गेंद को फेंकता है। पृथ्वी सतह से कक्षा तक गेंद की कुल यात्रिक ऊर्जा में परिवर्तन $x\frac{Gm_em}{21R_e}$ है। X का मान है : (R_e = 6370 किमी.)

(1) 9 (2) 12 (3) 11 (4) 10

Ans. **(3)**

Sol. $$\Delta ME = \frac{-GM_em}{2\left(R_e + \frac{R_e}{20}\right)} - \left(-\frac{GM_em}{R_e}\right)$$

$$= \frac{GM_em}{R_e}\left[1 - \frac{10}{21}\right]$$

$$\Delta ME = \frac{11GM_em}{21R_e}$$

x = 11

39. Given below are two statements :
Statement (I) : When currents vary with time, Newton's third law is valid only if momentum carried by the electromagnetic field is taken into account.
Statement (II) : Ampere's circuital law does not depend on Biot-Savart's law.
In the light of the above statements, choose the correct answer from the options given below :
(1) Both Statement I and Statement II are false
(2) Statement I is false but Statement II is true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are true
नीचे दो कथन दिये गये है :
कथन (I) : जब धारा समय के साथ बदलती है, न्यूटन का तीसरा नियमन केवल तभी मान्य होगा यदि इस स्थिति में विद्युत चुंबकीय क्षेत्र द्वारा आरोपित संवेग को ध्यान रखा जाये।
कथन (II) : ऐम्पियर का परिपथीय नियम बायो–सैबर्ट के नियम पर निर्भर नहीं करता है।
उपर्युक्त कथनों के आलोक में नीचे दिए गए विकल्प में **सही उत्तर** का चयन कीजिए।
(1) कथन I सत्य है परन्तु कथन II असत्य है।
(2) कथन I तथा कथन II दोनों सत्य है।
(3) कथन I असत्य है लेकिन कथन II सत्य है।
(4) कथन I तथा कथन II दोनों असत्य है।

Ans. **(3)**
Sol. Statement - I is correct
Statement - II is wrong , ampere's circuital law can be derived from Biot-Savart law and is not independent of Biot-Savart's law.

40. Given below are two statements :
Statement (I) : When an object is placed at the centre of curvature of a concave lens, image is formed at the centre of curvature of the lens on the other side.
Statement (II) : Concave lens always forms a virtual and erect image.
In the light of the above statements, choose the correct answer from the options given below :
(1) Both Statement I and Statement II are false
(2) Statement I is true but Statement II is false
(3) Statement I is false but Statement II is true
(4) Both Statement I and Statement II are true
नीचे दो कथन दिये गये है :
कथन (I) : जब एक वस्तु को एक अवतल लैंस के वक्रता केन्द्र पर रखा जाता है तो लैंस के दूसरी ओर वक्रता केन्द्र पर प्रतिबिम्ब प्राप्त होता है।
कथन (II) : अवतल लेंस हमेशा सीधा तथा आभासी प्रतिबिम्ब बनाता है।
उपर्युक्त कथनों के आलोक में नीचे दिए गए विकल्प में **सही उत्तर** का चयन कीजिए।
(1) कथन I असत्य है लेकिन कथन II सत्य है।
(2) कथन I तथा कथन II दोनों असत्य है।
(3) कथन I सत्य है परन्तु कथन II असत्य है।
(4) कथन I तथा कथन II दोनों सत्य है।

Ans. **(3)**

Sol. **Note :** Option 1 will be correct if we also consider virtual objects.

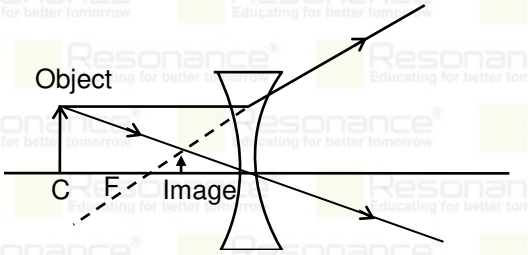


Image → erect, virtual, same side of lens

Statement (1) is incorrect

Statement (2) is correct if we consider only real objects, if we also consider virtual objects then statement -II will be incorrect.

For u = f/2 (virtual object)

$$V = \frac{u.f}{u+f}$$

$$V = \frac{f/2(-f)}{f/2+(-f)} = -\frac{f^2}{-f}$$

$V = f \Longrightarrow$ real image

41. The energy equivalent of 1 g of substance is :

पदार्थ के 1 ग्राम के समतुल्य ऊर्जा है :

(1) 5.6×10^{12} MeV (2) 5.6×10^{26} MeV (3) 5.6 eV (4) 11.2×10^{24} MeV

Ans. **(2)**

Sol. $E = mc^2$

$$E = 1\times10^{-3}\times\left(3\times10^{8}\right)^2 \text{ Joules}$$

$$E = \frac{9\times10^{13}\,eV}{1.6\times10^{-19}}$$

$E = 5.6 \times 10^{26}$ MeV

42. A light unstretchable string passing over a smooth light pulley connects two blocks of masses m_1 and m_2. If the acceleration of the system is $\frac{g}{8}$, then the ratio of the masses $\frac{m_2}{m_1}$ is :

एक हल्की घिरनी के ऊपर से जाती हुई एक हल्की न खिंचन वाली डोरी m_1 व m_2 द्रव्यमान के दो गुटकों को जोड़ती है। यदि निकाय का त्वरण $\frac{g}{8}$ हो तब द्रव्यमानों का अनुपात $\frac{m_2}{m_1}$ है :

(1) 5 : 3 (2) 4 : 3 (3) 9 : 7 (4) 8 :1

Ans. **(3)**

Sol.

$$\left(\frac{m_2-m_1}{m_2+m_1}\right)g = \frac{g}{8}$$

$$\frac{m_2-m_1+m_2}{m_2-m_1-m_2-m_1} = \frac{1+8}{1-8}$$

$$\frac{2m_2}{2m_1} = \frac{9}{-7}$$

$$\frac{m_2}{m_1} = \frac{9}{7}$$

$m_2 : m_1 = 9 : 7$

43. A proton, an electron and an alpha particle have the same energies. Their de-Broglie wavelengths will be compared as :
एक प्रोटॉन, एक इलैक्ट्रॉन तथा एक एल्फा कण की ऊर्जा एक समान है। इनकी डी–ब्राग्ली तरंगदैर्ध्य की निम्न प्रकार तुलना की जायेगी:
(1) $\lambda_p < \lambda_e < \lambda_\alpha$ (2) $\lambda_\alpha < \lambda_p < \lambda_e$ (3) $\lambda_p > \lambda_e > \lambda_\alpha$ (4) $\lambda_e > \lambda_\alpha > \lambda_p$

Ans. **(2)**

Sol. $\lambda = \frac{h}{P} = \frac{h}{\sqrt{2m(KE)}}$

$\lambda \propto \frac{1}{\sqrt{m}}$

$m_\alpha > m_p > m_{e^-}$

$\lambda_\alpha < \lambda_p < \lambda_{e^-}$

44. A light emitting diode (LED) is fabricated using GaAs semiconducting material whose band gap is 1.42 eV. The wavelength of light emitted from the LED is :
एक प्रकाश उत्सर्जित डायोड (LED) को GaAs अर्द्धचालकीय पदार्थ का प्रयोग करके बनाया जाता है जिसका बैंड अन्तराल 1.42 eV है। LED से उत्सर्जित प्रकाश की तरंग दैर्ध्य है :
(1) 1400 nm (2) 650 nm (3) 875 nm (4) 1243 nm

Ans. **(3)**

Sol. $\lambda = \frac{1240\ nm\ eV}{E}$, E = band gap

$\lambda = \frac{1240\ nm}{1.42} = 873.23\ nm$

$\lambda = 875\ nm$

45. The volume of an ideal gas ($\lambda = 1.5$) is changed adiabatically from 5 liters to 4 litres. The ratio of initial pressure to final pressure is :
एक आदर्श गैस ($\lambda = 1.5$) का आयतन 5 लीटर से 4 लीटर तक रुद्धोष्म प्रक्रम के द्वारा बदला जाता है। प्रारंभिक दाब तथा अन्तिम दाब में अनुपात है :
(1) $\frac{8}{5\sqrt{5}}$ (2) $\frac{16}{25}$ (3) $\frac{4}{5}$ (4) $\frac{2}{\sqrt{5}}$

Ans. **(1)**

Sol. PV^γ = Constant

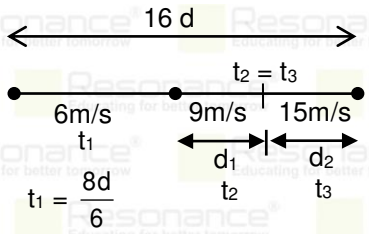
$P_i (5)^{3/2} = P_f (4)^{3/2}$

$\frac{P_i}{P_f} = \left(\frac{4}{5}\right)^{3/2} = \left(\frac{2}{\sqrt{5}}\right)^3 = \frac{8}{5\sqrt{5}}$

46. A particle moving in a straight line covers half the distance with speed 6 m/s. The other half is covered in two equal time intervals with speed 9 m/s and 15 m/s respectively. The average speed of the particle during the motion is :
एक सरल रेखा में गतिमान एक कण आधी दूरी 6 मी./से की चाल से तय करता है। शेष आधी दूरी दो समान अन्तरालों में क्रमशः 9 मी/से व 15 मी./से की चालों से तय करता है। गति के दौरान कण की औसत चाल है :
(1) 9.2 m/s (2) 8 m/s (3) 10 m/s (4) 8.8 m/s

Ans. **(2)**

Sol.



$t_1 = \frac{8d}{6}$

$t_2 = t_3$ = for same time, distance will be in ratio of velocity

$$\frac{d_1}{d_2} = \frac{9}{15} = \frac{3}{5} = \frac{3d}{5d}$$

$$t_2 = t_3 = \frac{3d}{9} = \frac{5d}{15}$$

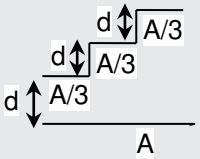
$$\text{Avg. velocity} = \frac{\text{Total distance}}{\text{Total Time}}$$

$$\frac{16d}{\frac{8d}{6}+\frac{3d}{9}+\frac{5d}{15}} = \frac{16}{\frac{4}{3}+\frac{2}{3}} = 8 \text{ m/s}$$

$v_{avg} = 8$ m/s

47. A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in figure. If the area of each stair is $\frac{A}{3}$ and the height is d, the capacitance of the arrangement is :

एक संधारित्र को A क्षेत्रफल की एक प्लेट तथा सीढ़ीनुमा आकृति वाली दूसरी प्लेट को लेकर बनाया गया है, जैसा कि चित्र में दर्शाया गया हैं यदि प्रत्येक सीढ़ी का क्षेत्रफल $\frac{A}{3}$ तथा ऊँचाई d हो तो इस व्यवस्थ की धारिता है :



(1) $\frac{11\epsilon_0 A}{20d}$ (2) $\frac{18\epsilon_0 A}{11d}$ (3) $\frac{11\epsilon_0 A}{18d}$ (4) $\frac{13\epsilon_0 A}{17d}$

Ans. **(3)**

Sol. In Parallel Combination :- $C_{eq} = C_1 + C_2 + C_3$

$$C = \frac{\frac{A}{3}\varepsilon_0}{d} + \frac{\frac{A}{3}\varepsilon_0}{2d} + \frac{\frac{A}{3}\varepsilon_0}{3d}$$

$$C = \frac{A\varepsilon_0}{3d}\left(1+\frac{1}{2}+\frac{1}{3}\right)$$

$$C = \frac{11A\varepsilon_0}{18d}$$

48. A bulb and a capacitor are connected in series across an ac supply. A dielectric is then placed between the plates of the capacitor. The glow of the bulb :
एक बल्ब तथा एक संधारित्र को एक प्रत्यावर्ती स्त्रोत से श्रेणीक्रम में जोड़ा गया है। जब संधारित्र की प्लेटों के बीच एक परावैद्युत रखा जाता है तब बल्ब की चमक :
(1) increases बढ़ती है
(2) becomes zero शून्य हो जाती है
(3) decreases घटती है
(4) remains same समान रहती है

Ans. **(1)**

Sol. Increases

$C\uparrow \Rightarrow x_c\downarrow \Rightarrow Z\downarrow \Rightarrow I_{rms}\uparrow$

$\Rightarrow P_{bulb} = I_{rms}^2 R_{bulb}$

$P_{bulb}\uparrow \Rightarrow$ Brightness$\uparrow$

49. A sphere of relative density σ and diameter D has concentric cavity of diameter d. The ratio of $\frac{D}{d}$, if it just floats on water in a tank is :
व्यास D व सापेक्ष घनतव σ के एक गोले में d व्यास का एक संकेन्द्रीय कोश (cavity) है। यदि यह एक टैंक में पानी की सतह पर केवल तैरता हो तो अनुपात $\frac{D}{d}$ का मान क्या है

(1) $\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{3}}$ (2) $\left(\frac{\sigma-1}{\sigma}\right)^{\frac{1}{3}}$ (3) $\left(\frac{\sigma-2}{\sigma+2}\right)^{\frac{1}{3}}$ (4) $\left(\frac{\sigma+1}{\sigma-1}\right)^{\frac{1}{3}}$

Ans. **(1)**

Sol. for equilibrium

$$\sigma\frac{4}{3}\pi(D^3-d^3)g = 1\frac{4}{3}\pi D^3 g$$

$$\Rightarrow \frac{D^3-d^3}{D^3} = \frac{1}{\sigma}$$

$$\Rightarrow 1-\frac{d^3}{D^3} = \frac{1}{\sigma} \Rightarrow \frac{d^3}{D^3} = 1-\frac{1}{\sigma}$$

$$\Rightarrow \frac{D}{d} = \left(\frac{\sigma}{\sigma-1}\right)^{1/3}$$

50. A particle of mass m moves on a straight line with its velocity increasing with distance according to the equation $v = \alpha\sqrt{x}$, where α is a constant. The total work done by all the forces applied on the particle during its displacement from x =0 to x = d, will be :
m द्रव्यमान का एक कण एक सीधी रेखा में अपने वेग से गति करता है जो $v = \alpha\sqrt{x}$ समीकरण के अनुसार दूरी के साथ बढ़ता है। जहाँ α एक नियतांक है। कण पर आरोपित सभी बलों द्वारा x =0 से x = d तक कण के विस्थापन के दौरान कृत कार्य होगा :

(1) $2m\alpha^2 d$ (2) $\frac{md}{2\alpha^2}$ (3*) $\frac{m\alpha^2 d}{2}$ (4) $\frac{m}{2\alpha^2 d}$

Ans. **(3)**

Sol. $W = k_f - k_i$

$\Rightarrow W = \frac{1}{2}m\left(\alpha\sqrt{d}\right)^2 - 0$

$\Rightarrow W = \frac{m\alpha^2 d}{2}$

51. Two Persons pull a wire towards themselves. Each person exerts a force of 200 N on the wire. Young's modulus of the material of wire is $1 \times 10^{11}\ Nm^{-2}$. Original length of the wire is 2 m and the area of cross section is $2\ cm^2$. The wire will extend in length by .....μm.

दो व्यक्ति एक तार को अपनी–अपनी ओर खींचते हैं। प्रत्येक व्यक्ति इस तार पर 200 N का एक बल लगाते हैं। तार के पदार्थ का यंग प्रत्यास्थता गुणांक $1 \times 10^{11}\ Nm^{-2}$. है। यदि तार की प्रारम्भिक लम्बाई 2 मीटर तथा अनुप्रस्थ परिच्छेद क्षेत्रफल 2 सेमी.^{2} है। . तार की लम्बाई में .....μm. द्वारा खिंचता है।

Ans. **20**

Sol. $Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} \Rightarrow \Delta l = \frac{Fl}{AY} = \frac{200\times 2}{2\times 10^{-4}\times 10^{11}}$

$\Delta l = 2\times 10^{-5}\ m$

$\Delta l = 20\mu m$

52. If $\vec{a}$ and $\vec{b}$ makes an angle $\cos^{-1}\left(\frac{5}{9}\right)$ with each other, then $|\vec{a}+\vec{b}| = \sqrt{2}|\vec{a}-\vec{b}|$ for $|\vec{a}| = n|\vec{b}|$ The integer value of n is _____ .

यदि $\vec{a}$ व $\vec{b}$ एक दूसरे के साथ $\cos^{-1}\left(\frac{5}{9}\right)$ का कोण बनाते है, तब $|\vec{a}| = n|\vec{b}|$ के लिए $|\vec{a}+\vec{b}| = \sqrt{2}|\vec{a}-\vec{b}|$ है। n का पूर्णांक मान _____ है

Ans. **3**

Sol. n = 3

$\sqrt{2}\ \sqrt{a^2+b^2-2ab\cos\theta} = \sqrt{a^2+b^2+2ab\cos\theta}$

$2\ (n^2b^2+b^2-\frac{10}{9}nb^2) = n^2b^2+b^2+\frac{10}{9}nb^2$

$(n^2+1)\ b^2\ \frac{-30}{9}\ nb^2 = 0$

$9n^2 - 30n + 9 = 0$

$9n^2 - 27\ n - 3n + 9 = 0$

$9n\ (n-3) - 3\ (n-3) = 0$

$n = 3\ ,\ n = 1/3$

$n = 3$

53. A string is wrapped around the rim of a wheel of moment of inertia $0.40\ kgm^2$ and radius 10 cm. The wheel is free to rotate about its axis, initially the wheel is at rest. The string is now pulled by a force of 40 N. The angular velocity of the wheel after 10 s is x rad/s, where x is _____ .

0.40 किग्रा मी.^{2} जड़त्व आघूर्ण व 10 सेमी त्रिज्या के एक पहिए के रिम पर एक डोरी लपेटी गई है। पहिया अपनी अक्ष में परितः घूमने के लिए स्वतंत्र है। प्रारम्भ में पहिया विरामवस्था में है। अब डोरी की 40 N बल से खींचा गया है। 10 s बाद पहिए का कोणीय वेग x रेडियन/ से है, जहाँ x _____ है।

Ans. **100**

Sol. $\Gamma = I\alpha \Rightarrow FR = I\alpha$

$40 \times 10^{-1} = 40 \times 10^{-2}\alpha$

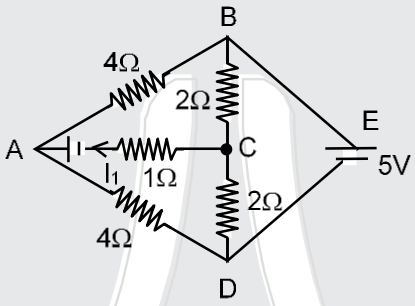
$\alpha = 10$ rad/s^2 = constant

$\omega = \omega_0 + \alpha t$

$\omega = 0 + 10\,(10) = 100$ rad/sec

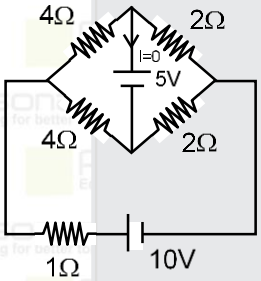
54. The current flowing through the 1 Ω resistor is $\frac{n}{10}$ A. The value of n is _____.

1 Ω प्रतिरोध से बहने वाली धारा $\frac{n}{10}$ A है। n का मान _____ है।



Ans. **25**

Sol.



$I_{1\Omega} = \frac{10}{3+1} = 2.5 = \frac{25}{10}$ A

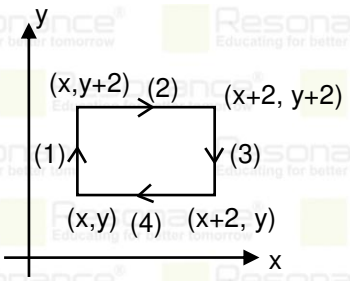
n = 25

55. A square loop of edge length 2 m carrying current of 2 A is placed with its edges parallel to the x-y axis. A magnetic field is passing through the x-y plane and expressed as $\vec{B} = B_0(1+4x)\hat{k}$, where $B_0 = 5T$. The net magnetic force experienced by the loop is _____ N.

2 मी लम्बी भुजा के एक वर्गाकार लूप को इस प्रकार रखा गया है कि इसकी भुजा x-y अक्ष के समान्तर है जबकि इसमें प्रवाहित धारा 2A है। x-y तल से एक चुम्बकीय क्षेत्र गुजरता है जिसे $\vec{B} = B_0(1+4x)\hat{k}$ द्वारा प्रदर्शित किया गया है, जहाँ $B_0 = 5T$। लूप द्वारा अनुभव किये जाने वाला कुल बल _____ N है।

Ans. **160**

Sol.



$\vec{F}_{loop} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$ $\qquad F_{wire} = i[\vec{\ell} \times \vec{B}]$

$\vec{F}_1 = 2[2\hat{j} \times 5(1+4x)\hat{k}) = (20+80x)\hat{i}$

$\vec{F}_2 = -\vec{F}_4 \Rightarrow \vec{F}_2 + \vec{F}_4 = 0$

$\vec{F}_3 = 2[-2\hat{j} \times 5(1+4(x+2))\hat{k}$

$= [-20-80x-160]\hat{i} = (-180-80x)\hat{i}$

$\vec{F}_{loop} = (20+80x)\hat{i} + 0 - (180+80x)\hat{i} = -160\hat{i}$ Newton's

$F_{loop} = 160\,N$

56. The position, velocity and acceleration of a particle executing simple harmonic motion are found to have magnitudes of 4m, 2 ms^{-1} and 16 m/s^2 at a certain instant. The amplitude of the motion is $\sqrt{x}$, m where x is

किसी क्षण सरल आवर्त गति करते हुए किसी कण की स्थिति, वेग तथा त्वरण के परिमाण क्रमशः 4 मी, 2 मी/से$^{-1}$ तथा 16 मी/से^{2} है। गति का आयाम $\sqrt{x}$ है जहाँ x _____ है।

Ans. **17**

Sol. $a = \omega^2 x \quad \Rightarrow \quad 16 = \omega^2(4)$

$\omega = 2$ rad/sec

$v = \omega\sqrt{A^2 - x^2} \Rightarrow \quad 4 = (2)^2 (A^2 - 4^2)$

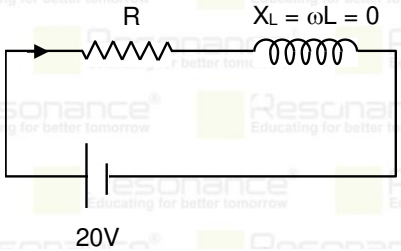
$A^2 = 17,\ A = \sqrt{17}\,m,\ x = 17$

57. When a coil is connected across a 20 V dc supply, it draw a current of 5 A. When it is connected across 20 V, 50 Hz ac supply, it draws a current of 4 A. The self inductance of the coil is _____ mH.
(take $\pi = 3$)

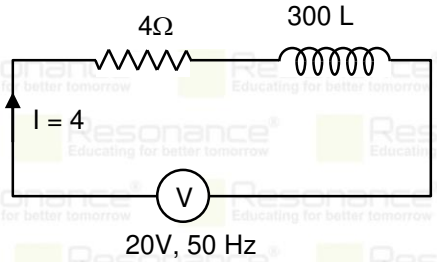
जब एक कुंडली को 20 V दिष्ट धारा स्त्रोत से जोड़ा जाता है इसमें 5A की धारा प्रवाहित है। जब इसे 20 V, 50 Hz प्रत्यावर्ती स्त्रोत से जोड़ा जाता है तो इस कुंडली में 4A की धारा बहती है। कुंडली का स्वप्रेरकत्व _____mHहै।
(दिया है $\pi = 3$)

Ans. **10**

Sol.



$R = \frac{20}{5} = 4\Omega$



$I = \frac{20}{Z} = 4 \quad \Rightarrow \quad Z = 5$

$Z = \sqrt{4^2 + (300\,L)^2} = 5$

$300\,L = 3$

$L = 10$ mH

58. At the centre of a half ring of radius R =10 cm and linear charge density 4n C m^{-1}, the potential is x π V. The value of x is.

R =10 सेमी त्रिज्या तथा 4n C m^{-1} रेखीय आवेश घनत्व के एक अद्ध छल्ले के केन्द्र पर विभव x π V है। x का मान _____ है।

Ans. **36**

Sol. $V_C = \frac{KQ}{R} = \frac{K\pi R\lambda}{R} = 9\times10^9 \times \pi \times 4\times10^{-9}$

$V_C = 36\pi$, x = 36

59. A star has 100 % helium composition. It starts to convert three ^4He into one ^{12}C via triple alpha process as $^4He + {}^4He + {}^4He \rightarrow {}^{12}C + Q$. The mass of the star is 2.0×10^{32} kg and it generates energy at the rate 5.808×10^{30} W. the rate of converting these ^4He to ^{12}C is n $\times 10^{42}$ s^{-1}, where n is _____

[Take, mass of ^4He = 4.0026 u, mass of ^{12}C = 12u]

एक तारा 100 % हीलियम से बना हैं यह तीन एल्फा प्रक्रम द्वारा तीन ^4He को एक ^{12}C में $^4He + {}^4He + {}^4He \rightarrow {}^{12}C + Q$ के अनुसार परिवर्तित करना प्रारम्भ करता है। तारे का द्रव्यमान 2.0×10^{32} किग्रा है तथा यह 5.808×10^{30} W की दर से ऊर्जा उत्पन्न करता है। तीन ^4He के एक ^{12}C में बदलने की दर n $\times 10^{42}$ s^{-1} है जहाँ n _____ है।

[दिया गया है : ^4He का द्रव्यमान = 4.0026 u, ^{12}C का द्रव्यमान = 12u]

Ans. **5**

Sol. $Q = \Delta mc^2 = 78 \times 10^{-4} \times 1.66 \times 10^{-27} \times [3 \times 10^8]^2$

$= 78 \times \frac{5}{3} \times 9 \times 10^{-15}$ W

$Q = 117 \times 10^{-14}$ W

Reactions per second $= \frac{5.808\times10^{30}}{117\times10^{-14}}$

$= 4.96 \times 10^{42}\ s^{-1}$

n = 5

60. In a Young's double slit experiment, the intensity at a point is $\left(\frac{1}{4}\right)^{th}$ of the maximum intensity, the minimum distance of the point from the central maximum is _____ μm.

[Given : l = 600 nm, d = 1.0 mm, D = 1.0 m]

एक यंग द्विझिर्री प्रयोग में, एक बिन्दु पर तीव्रता की एक चौथाई है, उस बिन्दु की केन्द्रीय उच्चिष्ठ से न्यूनतम दूरी _____ μm है।

[दिया गया है: l = 600 nm, d = 1.0 mm, D = 1.0 m]

Ans. **(200)**

Sol. $I_{max} = 4I_0$, when $I_1 = I_2 = I_0$

$$I = \frac{1}{4}(4I_0) = I_0$$

$$4I_0 \cos^2 \frac{\phi}{2} = I_0$$

$$\cos \frac{\phi}{2} = \pm \frac{1}{2}$$

$$\frac{\phi}{2} = \frac{\pi}{3}$$

$$\phi = \frac{2\pi}{3}$$

$$\Delta\phi = k\Delta x$$

$$\frac{2\pi}{3} = \frac{2\pi}{\lambda}\Delta x \Rightarrow \Delta x = \frac{\lambda}{3}$$

$d << D, \quad \lambda << d$

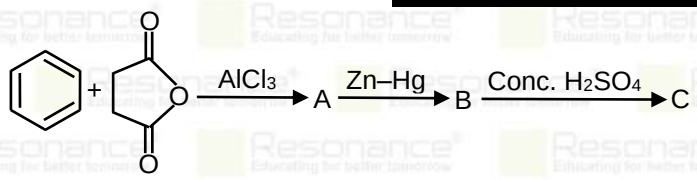
$$\Delta x = \frac{dy}{D}$$

$$y = \frac{\lambda}{3}\frac{D}{d}$$

$$\Rightarrow y = \frac{6\times10^{-7}\times1}{3\times10^{-3}} = 2\times10^{-4}\text{m} = 200\mu\text{m}$$

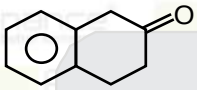
PART : CHEMISTRY

61.

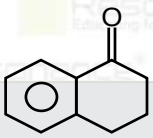


What is the structure of C?

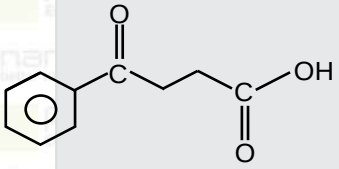
(1)



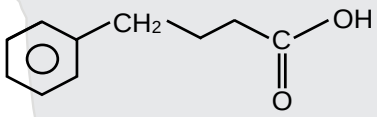
(2)



(3)

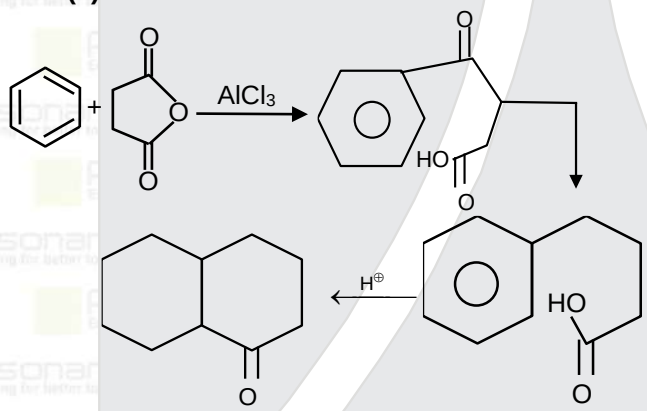


(4)



Ans. **NTA (2)**

Sol.



62. Methods used for purification of organic compounds are based on:
(1) nature of compound only.
(2) presence of impurity only.
(3) nature of compound and presence of impurity.
(4) neither on nature of compound nor on the impurity present.

Ans. **NTA (3)**

Sol. fact

63. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reasons (R)**.

Assertion (A) : S_N2 reaction of $C_6H_5CH_2Br$ occurs more radily than the S_N2 reaction of CH_3CH_2Br.

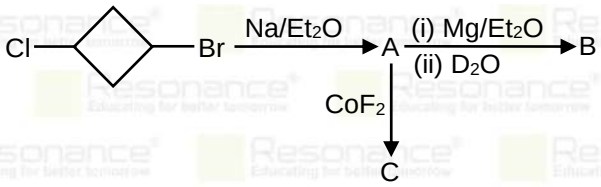
Reasons (R) : The partially bonded unhybridized p-orbital that develops in the trigonal bipyramidal transition state is stabilized by conjugation with the phenyl ring.

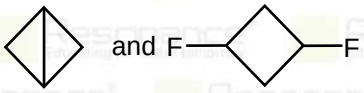
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Both **(A)** and **(R)** are correct but **(R)** is **not** the correct explanation of **(A)**.
(2) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**.
(3) **(A)** is not correct but **(R)** is correct.
(4) **(A)** is correct but **(R)** is not correct.

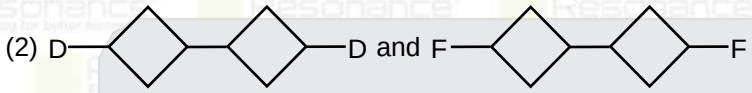
Ans. **NTA (2)**

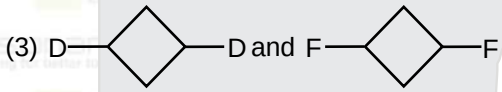
Sol. C = C, C = N, C = O stabilizes TBP transition state by –M effect

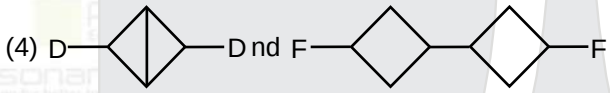
64. In the following sequence of reaction, the major products B and C respectively are:



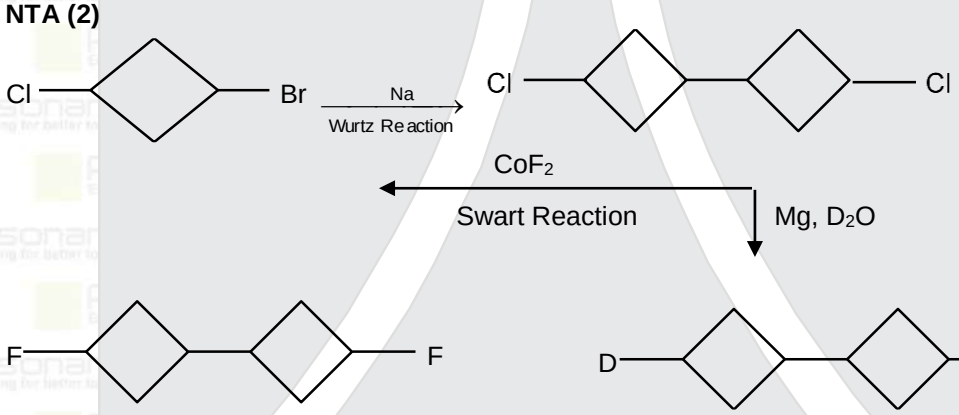
(1) 

(2) 

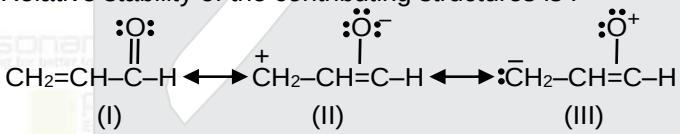
(3) 

(4) 

Ans. **NTA (2)**

Sol. 

65. Relative stability of the contributing structures is :

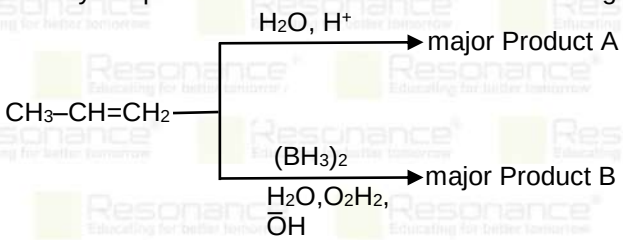


(1) (II) > (I) > (III) (2) (I) > (II) > (III) (3) (I) > (III) > (II) (4) (III) > (II) > (I)

Ans. **NTA (2)**

Sol. I structure is most stable due to more π bonds

66. Identify the product A and Product B in the following set of reactions.



(1) A - $CH_3CH_2CH_2–OH$ B - $CH_3\underset{\underset{OH}{|}}{C}H–CH_3$

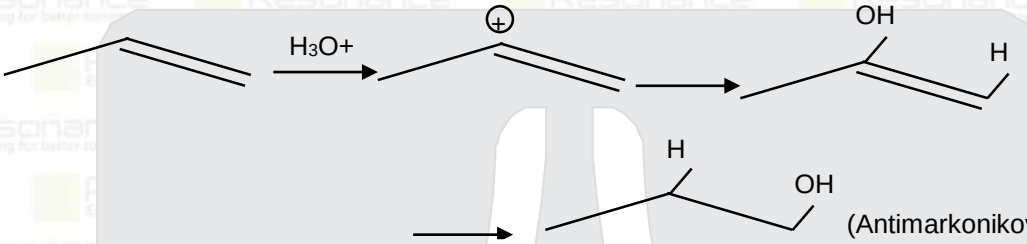
(2) A - $CH_3CH_2CH_3$ B - $CH_3CH_2CH_3$

(3) A - $CH_3CH_2CH_2–OH$ B - $CH_3CH_2CH_2–OH$

(4) A - $CH_3\underset{\underset{OH}{|}}{C}H–CH_3$ B - $CH_3CH_2CH_2–OH$

Ans. **NTA (4)**

Sol.



(Antimarkonikov rule)

67. 0.05M $CuSO_4$ when treated with 0.01M $K_2Cr_2O_7$ gives green colour solution of $Cu_2Cr_2O_7$. The two solutions are separated as shown below:

[SPM : Semi Permeable Membrane]

$K_2Cr_2O_7$	$CuSO_4$
Side X	SPM Side Y

Due to osmosis:

(1) Green colour formation observed on side X. (2) Green colour formation observed on side Y.

(3) Molarity of $CuSO_4$ solution is lowered. (4) Molarity of $K_2Cr_2O_7$ solution is lowered.

Ans. **NTA (3)**

Sol. Osmosis is the spontaneous net movement of solvent molecules from dilute solution (low conc.) to concentrated solution (high conc.) hence molarity of $CuSO_4$ solution is lowered.

68. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reasons (R)**.

Assertion (A) : Both rhombic and monoclinic sulphur exist as S_8 while oxygen exists as O_2.

Reasons (R) : Oxygen forms $p\pi–p\pi$ multiple bonds with itself and other elements having small size and high electronegativity like C, N, which is not possible for sulphur.

In the light of the above statements, choose the most **appropriate** answer from the options given below:

(1) Both **(A)** and **(R)** are correct but **(R)** is **not** the correct explanation of **(A)**.

(2) **(A)** is not correct but **(R)** is correct.

(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**.

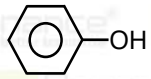
(4) **(A)** is correct but **(R)** is not correct.

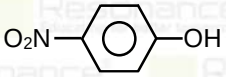
Ans. **NTA (4)**

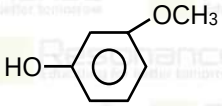
Sol. Both rhombic & monoclinic sulphur exists as S_8 but oxygen exists as O_2.

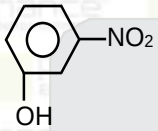
Oxygen form $p\pi–p\pi$ multiple bond due to small size & small bond length but $p\pi–p\pi$ bonding is not possible in sulphur due to its larger atomic size than oxygen both assertion & reason are correct explanation of the assertion.

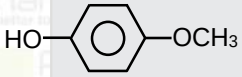
69. For the given compounds, the correct order in increasing pKa value:

(A) 

(B) 

(C) 

(D) 

(E) 

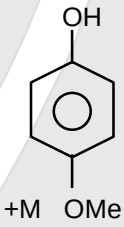
Choose the **correct** answer from the options given below:

(1) (B) < (D) < (A) < (C) < (E)
(2) (E) < (D) < (B) < (A) < (C)
(3) (E) < (D) < (C) < (B) < (A)
(4) (D) < (E) < (C) < (B) < (A)

Ans. **NTA (1)**

Sol. $pKa \propto \frac{1}{k_a}$

+ M effect will decrease Ka value



70. Given below are two statements:

Statement - (I) : The oxidation state of an element in a particular compound is the charge acquired in the molecule.

Statement - (II) : $p\pi$–$p\pi$ bond formation is more prevalent in second period elements over other periods.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) **Statement I** is correct but **Statement II** is incorrect.
(2) **Statement I** is incorrect but **Statement II** is correct.
(3) Both **Statement I** and **Statement II** are incorrect.
(4) Both **Statement I** and **Statement II** are correct.

Ans. **NTA (2)**

Sol. (A) The oxidation state of an element in a particular compound can be defined as the charge acquired by its atom on the basis of electronegative consideration from other atoms in the molecule.

(B) The second period elements of p-block display greater ability to form multiple bonds than the other members of their groups due to their smaller atomic size & higher electronegativity.

71. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reasons (R)**.

Assertion (A) : The total number fo geometrical isomers shown by $[Co(en)_2Cl_2]^+$ complex ion is three.

Reasons (R) : $[Co(en)_2Cl_2]^+$ complex ion has an octahedral geometry.

In the light of the above statements, choose the most **appropriate** answer from the options given below:

(1) Both **(A)** and **(R)** are correct but **(R)** is **not** the correct explanation of **(A)**.
(2) **(A)** is not correct but **(R)** is correct.
(3) **(A)** is correct but **(R)** is not correct.
(4) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**.

Ans. **NTA (2)**

Sol. The total number of geometrical isomers of $[Co(en)_2Cl_2]^+$ complex ion is two geometrical isomers cis & trans $[Co(en)_2Cl_2]^+$ has six coordination number & show octahedral geometry. the trans isomer of $[Co(en)_2Cl_2]^+$ is not optically active because of (Sp^3d^2) plane of symmetry but cis exists in d & ℓ forms.

72. On reaction of Lead Sulphide with dilute nitric acid which of the following is **not** formed?

(1) Lead nitrate (2) Nitric oxide

(3) Sulphur (4) Nitrous oxide

Ans. **NTA (4)**

Sol. $Pbs + \underset{dil}{HNO_3} \longrightarrow Pb(NO_3)_2 + NO + S + H_2O$

NO_2 is not formed.

73. Compare the energies of following sets of quantum numbers for multielectron system.

(A) n = 4, ℓ = 1

(B) n = 4, ℓ = 2

(C) n = 3, ℓ = 1

(D) n = 3, ℓ = 2

(E) n = 4, ℓ = 0

Choose the **correct** answer from the options given below:

(1) (E) < (C) < (D) < (A) < (B) (2) (E) < (C) < (A) < (D) < (B)

(3) (C) < (E) < (D) < (A) < (B) (4) (B) < (A) < (C) < (E) < (D)

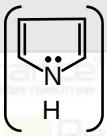
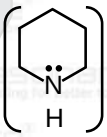
Ans. **NTA (3)**

Sol.

	$n + \ell$
(A) n = 4, ℓ = 1 $\Rightarrow$ 4p	5
(B) n = 4, ℓ = 2 $\Rightarrow$ 4d	6
(C) n = 3, ℓ = 1 $\Rightarrow$ 3p	4
(D) n = 3, ℓ = 2 $\Rightarrow$ 3d	5
(E) n = 4, ℓ = 0 $\Rightarrow$ 4s	4

$n + \ell$ rule $\rightarrow$ In neutral isolated atom, the lower value of $(n + \ell)$ for an orbital, lower is its energy order of energies -

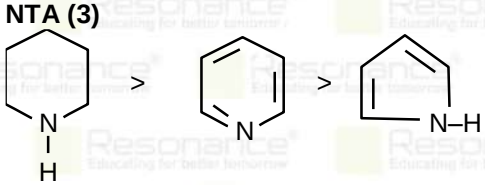
C < E < D < A < B

74. Correct order of basic strength of Pyrrole  Pyridien  and Piperidien  is:

(1) Pyrrole > Piperidine > Pyridine (2) Pyridine > Piperidine > Pyrrole

(3) Piperidine > Pyridine > Pyrrole (4) Pyrrole > Pyridine > Piperidine

Ans. **NTA (3)**

Sol. 

sp^3 sp^2 Aromatic

75. In which of the following pair the central atoms exihibit sp^2 hybridization?

(1) NH_2^- and H_2O (2) NH_2^- and BF_3 (3) H_2O and NO_2 (4) BF_3 and NH_2^-

Ans. **NTA (4)**

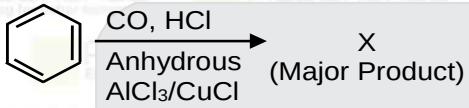
Sol. $BF_3 \rightarrow sp^2$ hybridization

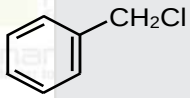
$NH_2^- \rightarrow sp^2$ hybridization

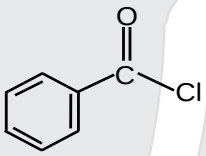
$H_2O \rightarrow sp^3$ hybridization

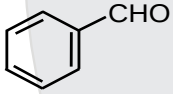
$NH_2^- \rightarrow sp^3$ hybridization

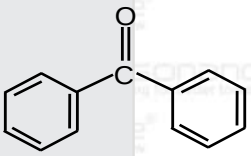
76. Identify major product "X" formed in the following reaction:



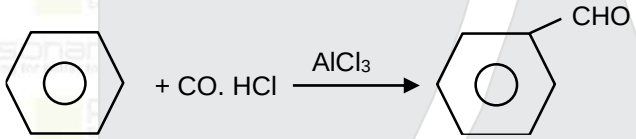
(1) 

(2) 

(3) 

(4) 

Ans. **NTA (3)**

Sol. 

77. The F^- ions make the enamel on teeth much harder by hydroxyapatite (the enamel on the surface of teeth) into much harder fluoroapatite having the formula.

(1) $[3(Ca_3(PO_4)_2).CaF_2]$ (2) $[3(Ca_2(PO_4)_2).Ca(OH)_2]$

(3) $[3(Ca_3(PO_4)_3).CaF_2]$ (4) $[3(Ca_3(PO_4)_2).Ca(OH)_2]$

Ans. **NTA (1)**

Sol. The F^- ions make the enamel on teeth much harder by converting hydroxyapatite $[3(Ca_3(PO_4)_2).Ca(OH)_2]$ the enamel on the surface of the teeth into much harder fluoroapatite $[3(Ca_3(PO_4)_2).CaF_2]$.

78. The electronic configuration of Cu(II) is $3d^9$ whereas that of Cu(I) is $3d^{10}$. Which of the following is correct.

(1) Cu(II) is less stable.

(2) Cu(II) is more stable.

(3) Stability of Cu(I) and Cu(II) depends on nature of copper salts.

(4) Cu(I) and (Cu(II) are equally stable.

Ans. **NTA (2)**

Sol. $Cu^+ \rightarrow Cu^{2+} + Cu$

Cu^+ undergoes disproportionation due to high hydration energy of Cu^{2+}. The stability of Cu^{2+} is more than Cu^+ as stability depend on the hydration energy, Cu^{2+} ion has a greater charge density than Cu^+ ion and thus forms much stronger bonds releasing more energy.

79. Identify the **incorrect** statements regarding primary standard of titrimetric analysis.

(A) It should be purely available in dry form.

(B) It should not undergo chemical change in air.

(C) It should be hygroscopic and should react with answer chemical instantaneously and stoichiometrically.

(D) It should be readily soluble in water.

(E) $KMnO_4$ & NaOH can be used as primary standard.

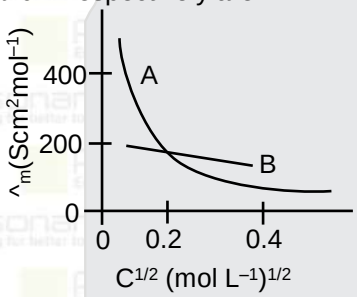
Choose the **correct** answer from the options given below:

(1) (C) and (E) only (2) (C) and (D) only (3) (B) and (E) only (4) (A) and (B) only

Ans. **NTA (1)**

Sol. $KMnO_4$ & NaOH can be used as secondary standard. It is difficult to obtain the pure state of $KMnO_4$ as it is not free form MnO_2.
Primary standard → It is typically a reagent which can be weighted easily & which is so pure that its weight is truly representative of the number of moles of substance contained.
Primary standard has
(i) High purely
(ii) Stability (low reactivity)
(iii) low hygroscopicity

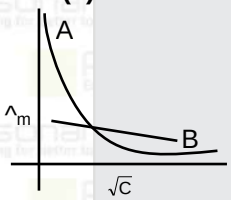
80. The major conductivity for electrolytes A and B are plotted against $C^{1/2}$ as shown below. Electrolytes A and B respectively are:



	A	B
(1)	weak electrolyte	weak electrolyte
(2)	strong electrolyte	weak electrolyte
(3)	weak electrolyte	strong electrolyte
(4)	strong electrolyte	strong electrolyte

Ans. **NTA (3)**

Sol.



$\wedge_m$ of strong electrolyte varies linearly with $\sqrt{C}$ according to Debye —Huckel —Onsager etc.

$$\wedge_m = \wedge_m^0 - \sqrt{C}$$

while $\wedge_m$ of weak electrolyte varies non-linearly with $\sqrt{C}$ due to change in percentage with concentration.

81. Total number of essential amino acid among the given list of amino acids is _____.
Arginine, Phenylalanine, Aspartic acid, Cysterine, Histidine, Valine, Proline

Ans. **NTA (4)**

Sol. Fact based

82. The standard reduction potentials at 298 K for the following half cells are given below:

$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$, $E^o = 1.33V$

$Fe^{3+}(aq) + 3e^- \rightarrow Fe$ $E^o = -0.04V$
$Ni^{2+}(aq) + 2e^- \rightarrow Ni$ $E^o = -0.25V$
$Ag^+(aq) + e^- \rightarrow Ag$ $E^o = 0.80V$
$Au^{3+}(aq) + 3e^- \rightarrow Au$ $E^o = 1.40V$

Consider the given electrochemical reactions,
The number of metal(s) which will be oxidized be $Cr_2O_7^{2-}$, in aqueous solution is _____.

Ans. **NTA (3)**

Sol. $Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$, $E^o = 1.33V$

Oxidation potential of Au is –1.40V so it is not oxidized by $Cr_2O_7^{2-}$ but Fe, Ni & Ag will be oxidized so the number of metals will be oxidized by $Cr_2O_7^{2-}$ in aqueous solution is 3.

83. When equal volume of 1M HCl and 1M H_2SO_4 are separately neutralised by excess volume of 1M NaOH solution. x and y kJ of heat is liberated respectively. The value of y/x is _____.

Ans. **NTA (2)**

Sol. When 1 mole of H^+ ions is neutralised by 1 mole of OH^- than 13.7 KCal/mol energy is released.

$HCl \longrightarrow H^+ + Cl^-$

1 mol 1 mol

$H_2SO_4 \longrightarrow 2H^+ + SO_4^{-2}$

1 mol 1 mol

During neutralisation of 1M HCl & 1M H_2SO_4 by NaOH solution x kJ & y kJ heat is released respectively than y = 2x hence $\frac{y}{x} = 2$

84. Number of colourless lanthanoid ions among the following is _____.

Eu^{3+}, Lu^{3+}, Nd^{3+}, La^{3+}, Sm^{3+}

Ans. **NTA (2)**

Sol. $Lu^{3+} \rightarrow$ colourless $[Xe](4f^{14})$

$Eu^{3+} \rightarrow$ pink $[Xe]4f^6$

$La^{3+} \rightarrow$ colourless $[Xe] \rightarrow 4f^0$

$Sm^{3+} \rightarrow$ yellow $[Xe]4f^5$

For the transition to occur there should be unpaired electrons present in the f-orbital in Lu^{3+} f–f transition is not possible so Lu^{3+} & La^{3+} will be colourless Lu^{3+} & La^{3+} lanthanoid ions are colourless.

85. Molarity (M) of an aqueous solution containing x g of anhyd. $CuSO_4$ in 500 mL solution at 32°C is 2×10^{-3} M. Its molarity will be _____ $\times 10^{-3}$ m. (nearest integer).

[Given density of the solution = 125g/mL]

Ans. **NTA (81)**

RESO (164)

Sol. $M = \frac{\eta_{CuSO_4}}{v(\ell)}$

$0.2 = \frac{\eta_{CuSO_4}}{\frac{500}{1000}}$

$\eta_{CuSO_4} = 0.1$

Mass of solution = 500 × 1.25 = 625 gm

Mass of solution = 0.1 × 159.6 = 15.96 gm

Mass of solvent = 625 –15.96 = 609.04 gm

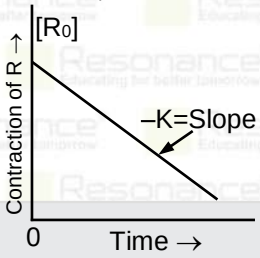
$m = \frac{\eta_{CuSO_4}}{W_{solvent}}$

$m = \frac{0.1}{609.04} \times 1000 = 0.16419 = 164.19 \times 10^{-3}$

86. Given below are two statements:

Statement I : The rate law for the reaction A + B → C is rate (r) = $k[A]^2[B]$. When the concentration of both A and B is doubled, the reaction rate is increased "x" times.

Statement II :



The figure is showing "the variation in concentration against time plot" for a "y" order reaction.

The value of x + y is _____ .

Ans. **NTA (8)**

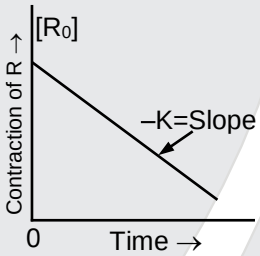
Sol. **Statement I :** A + B → C (r) = $k[A]^2[B]$.

when conc. of both A & B is doubled than rate becomes

$r^1 = k[2A]^2[2B]$

$r^1 = 8r$ than x = 8

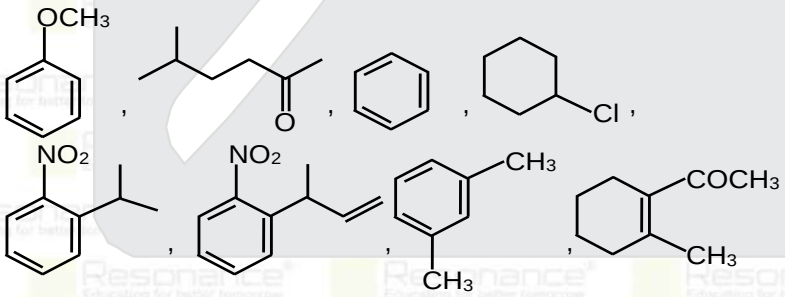
Statement II :



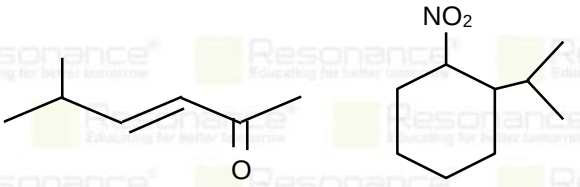
for zero order reaction $[A]_t = [A]_0 - kt$ graph is for zero order reaction hence value of y = 0

So x + y = 8 + 0 = 8

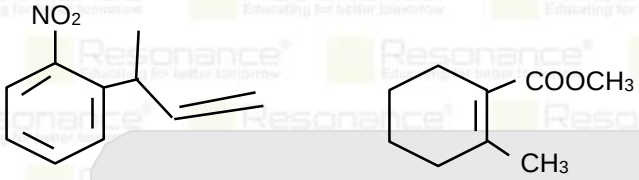
87. How many compounds among the following compounds show inductive, mesomeric as well as hyper conjugation effects?



Ans. **NTA (4)**



Sol.



88. The total number of species from the following in which one unpaired electron is present, is _____.

N_2, O_2, C_2^-, O_2^-, O_2^{2-}, H_2^+, CN^-, He_2^+

Ans. **NTA (4)**

Sol.

		Unpaired electron
H_2^+	$\sigma 1s^1$	1
He_2^+	$\sigma 1s^2 \sigma^* 1s^1$	1
O_2^-	$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_2^2 \Pi 2p_x^2 \Pi 2p_y^2 \Pi^* 2p_x^2 \Pi^* 2p_x^1$	1
C_2^-	$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \Pi 2p_x^2 \Pi 2p_y^2 \sigma 2p_z^1$	1

89. Number of ambidentate ligands among the following is _____.

NO_2^-, SCN^-, $C_2O_4^{2-}$, NH_3, CN^-, SO_4^{2-}, H_2O.

Ans. **NTA (3)**

Sol. Ambidentate ligands are those ligands that can attach themselves to the other central metal atoms through two different atoms NO_2^-, SCN^- & CN^- are ambidentate ligands.

For example

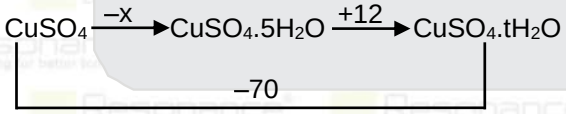
NO_2^- group can attach itself through nitrogen & oxygen.

90. The heat of solution of anhydrous $CuSO_4$ and $CuSO_4.5H_2O$ are –70 kJ mol^{-1} and +12 kJ mol^{-1} respectively.

The heat of hydration of $CuSO_4$ to $CuSO_4.5H_2O$ is –x kJ. The value of x is _____.
(nearest integer).

Ans. **NTA (82)**

Sol.



$-x + 12 = -70$

$x = 82$ kJ/mol.