PART : MATHEMATIC

1. Let the circles $C_1: (x-\alpha)^2+(y-\beta)^2 = r_1^2$ and $C_2: (x-8)^2 + \left(y-\frac{15}{2}\right)^2 = r_2^2$ touch each other externally at the point (6, 6). If the point (6, 6) divides the line segment joining the centres of the circles C_1 and C_2 internally in the ratio 2 : 1, then $(\alpha+\beta)+4(r_1^2+r_2^2)$ equals

(1) 145 (2) 110 (3) 125 (4) 130

Ans. **NTA (4)**
RESO (4)

Sol. (6, 6) lies on C_2

$r_2^2 = 4 + \frac{9}{4} \Rightarrow r_2 = \frac{5}{2}$

Now $r_1 = 2r_2 = 5$.

C_1 & C_2 touching externally

(α, β) — (6,6) — $\left(8, \frac{15}{2}\right)$ in the ratio 2 : 1

$\frac{\alpha+8}{3} = 6, \quad \frac{\beta+15}{3} = 6 \quad \Rightarrow \alpha = 2, \beta = 3$

Now $\alpha + \beta + 4(r_1^2 + r_2^2) = 2 + 3 + 4\left(25+\frac{25}{4}\right) = 130$

2. Let $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$. If $A^3 = 4A^2 - A - 21I$, where I is the identity matrix of order 3×3, then $2a + 3b$ is equal to

(1) –13 (2) – 9 (3) –12 (4) –10

Ans. **NTA (1)**
RESO (1)

Sol. Characteristic equation $|A - \lambda I| = 0$

$-\lambda^3 + (5 + b)\lambda^2 - (5b + 1 - a)\lambda + (6b - 10 - ab) = 0$

$\Rightarrow A^3 = (5 + b)A^2 - (5b + 1 - a)A + (6b - 10 - ab)I$

$b = -1, a = -5 \Rightarrow 2a + 3b = -13$

3. For the function $f(x)=(\cos x) - x+1$, $x \in$ IR, between the following two statements

(S1) f(x) = 0 for only one value of x in $[0, \pi]$.

(S2) f(x) is decreasing in $\left[0, \frac{\pi}{2}\right]$ and increasing in $\left[\frac{\pi}{2}, \pi\right]$.

(1) Only (S1) is correct.
(2) Both (S1) and (S2) are correct.
(3) Only (S2) is correct.
(4) Both (S1) and (S2) are incorrect.

Ans. **NTA (1)**
RES) (1)

Sol. $f'(x) = -\sin x - 1 \le 0, x \in [0, 2\pi]$

f(x) is monotonic decreasing

$\Rightarrow$ f(x) is one–one function $\Rightarrow f(0) = 2 > 0, f(\pi) = -1 - \pi + 1 = -\pi < 0$

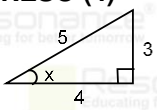
$\Rightarrow$ Only one solution

4. If $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$ then $80(\tan^2 x - \cos x)$ is equal to

(1) 108 (2) 18 (3) 19 (4) 109

Ans. **NTA (4)**
RESO (4)

Sol.



$\tan x = \frac{3}{4}, \cos x = -\frac{4}{5} \Rightarrow 80(\tan^2 x - \cos x) = 80\left(\frac{9}{16} + \frac{4}{5}\right) = 5 \times 9 + 16 \times 4 = 109$

5. The sum of all the solutions of the equation $(8)^{2x} - 16 \cdot (8)^x + 48 = 0$ is:

(1) $1 + \log_8(6)$ (2) $\log_8(4)$ (3) $1 + \log_6(8)$ (4) $\log_8(6)$

Ans. **NTA (1)**
RESO (1)

Sol. $8^x = t \quad t^2 - 16t + 48 = 0 \Rightarrow (t-8)^2 = 16$

$t = 4 \quad \text{or} \quad 12$

$x = \log_8^4, \log_8^{12} \Rightarrow \text{Sum} = \log_8(4 \times 12) = \log_8(8 \times 6) = 1 + \log_8(6)$

6. If the shortest distance between the lines

$L_1 : \vec{r} = (2 + \lambda)\hat{i} + (1 - 3\lambda)\hat{j} + (3 + 4\lambda)\hat{k}, \lambda \in R$

$L_2 : \vec{r} = 2(1 + \mu)\hat{i} + 3(1 + \mu)\hat{j} + (5 + \mu)\hat{k}, \mu \in R$

is $\frac{m}{\sqrt{n}}$, where gcd(m, n) = 1, then the value of m + n equals

(1) 377 (2) 384 (3) 390 (4) 387

Ans. **NTA (4)**
RESO (4)

Sol. Let A = (2, 1,3), B = (2, 3, 5) : Points on lines.

$$\overline{AB} = 0\hat{i} + 2\hat{j} + 2\hat{k}, \quad \overline{n_1} \times \overline{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 4 \\ 2 & 3 & 1 \end{vmatrix}$$

$$\overline{n_1} \times \overline{n_2} = -15\hat{i} + 7\hat{j} + 9\hat{k}$$

$$\text{S.D.} = \frac{\left|\overline{AB} \cdot \left(\overline{n_1} \times \overline{n_2}\right)\right|}{\left|\overline{n_1} \times \overline{n_2}\right|} = \frac{0 + 14 + 18}{\sqrt{355}} = \frac{32}{\sqrt{355}} \Rightarrow m + n = 387$$

7. Let H:= $\frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the hyperbola, whose eccentricity is $\sqrt{3}$ and the length of the latus rectum is $4\sqrt{3}$. Suppose the point $(\alpha, 6)$, $\alpha > 0$ lies on H. If β is the product of the focal distances of the point $(\alpha, 6)$, then $\alpha^2 + \beta$ is equal to

(1) 169 (2) 172 (3) 171 (4) 170

Ans. **NTA (3)**
RESO (3)

Sol. $e = \sqrt{3}$

$3 = 1 + \frac{a^2}{b^2}$

$3b^2 = b^2 + a^2$

$2b^2 = a^2$ .....(i)

also L.R. $= \frac{2a^2}{b} = 4\sqrt{3}$

$b^2 = \sqrt{3}\, b$

$b = \sqrt{3}$

and $a^2 = 6$

Now, focus $(0,\pm 3)$

also $(\alpha, 6)$ lies on $\frac{-x^2}{6} + \frac{y^2}{3} = 1$

$-\frac{\alpha^2}{6} + \frac{36}{3} = 1$

$11 \times 6 = \alpha^2$

Product of focal distances of point $(\alpha, 6)$ is

$$e\left|y_1 - \frac{b}{e}\right| \times e\left|y_1 + \frac{b}{e}\right| = \left|e^2\left(y_1^2 - \frac{b^2}{e^2}\right)\right| = |3 \times 36 - 3|$$

$\beta = 105$

Now, $\alpha^2 + \beta = 66 + 105 = 171$

8. The set of all α, for which the vectors $\vec{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$ are inclined at an obtuse angle for all $t \in R$, is

(1) $(-2, 0]$ (2) $\left(-\frac{4}{3}, 0\right]$ (3) $\left(-\frac{4}{3}, 1\right]$ (4) $[0, 1)$

Ans. **NTA (2)**
RESO (2)

Sol. $\vec{a} \cdot \vec{b} < 0 \ \forall\ t \in R.$

$\alpha t^2 - 12 + 6\alpha t < 0, t \in R \Rightarrow \alpha < 0$ .....(i) and $D < 0$

$36\alpha^2 - 4\alpha(-12) < 0.$

$36\alpha^2 + 48\alpha < 0.$

$12\alpha(3\alpha + 4) < 0$

$\alpha \in \left(-\frac{4}{3}, 0\right)$.....(ii)

(i) $\cap$ (ii)

$\alpha \in -\left(\frac{4}{3}, 0\right)$

If $\alpha = 0 \Rightarrow 0 - 12 + 0 < 0 \Rightarrow \alpha \in \{0\} \Rightarrow \alpha \in \left(\frac{-4}{3}, 0\right]$

9. If the set $R = \{(a,b) : a + 5b = 42, a,b \in N\}$ has m elements and $\sum_{n=1}^{m}\left(1 - i^{n!}\right) = x + iy$, where $i = \sqrt{-1}$, then the value of m + x + y is

(1) 4 (2) 8 (3) 5 (4) 12

Ans. **NTA (4)**
RESO (4)

Sol. $a + 5b = 2 + 40 \Rightarrow (a - 2) = -5b + 40 \Rightarrow \frac{a-2}{5} = \frac{b-8}{-1} = \lambda \Rightarrow a = 2 + 5\lambda,\ b = 8 - \lambda$

$\lambda = 0 \Rightarrow a = 2, b = 8$; ....., $\lambda = 7 \Rightarrow a = 37, b = 1$

$R = \{(37, 1), (32, 2), (27, 3), (22, 4), (17, 5), (12, 6), (7, 7), (2, 8)\}$

so m = 8

Now $\sum_{n=1}^{8}(1 - (i)^{n!}$

$= 8 - \{i^1 + i^{2!}\ i^{3!} + i^{4!} + \ldots\ldots\ldots\ldots\ i^{8!}\}$

$= 8 - [i - 1 - i + 1 + 1 + 1 + 1 + 1]$

$= 8 - 4 = 4 = 4 + 0i$

so $m + x + y = 8 + 4 + 0 = 12$

10. Let y = y(x) be the solution of the differential equation

$(1+y^2)e^{\tan x}\,dx + \cos^2 x\,(1 + e^{2\tan x})dy = 0$, $y(0) = 1$. Then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{2}{e}$ (2) $\frac{1}{e^2}$ (3) $\frac{1}{e}$ (4) $\frac{2}{e^2}$

Ans. **NTA (3)**
RESO (3)

Sol. $(1 + y^2)e^{\tan x}dx + \cos^2 x(1 + e^{2\tan x})dy = 0$

$$\frac{dy}{1+y^2} = -\frac{e^{\tan x}.\sec^2 x\,dx}{1+e^{2\tan x}}$$

$$\int\frac{dy}{1+y^2} = -\int\frac{e^{\tan x}.\sec^2 x\,dx}{1+e^{2\tan x}}$$

let $e^{\tan x} = t$

$e^{\tan x}.\sec^2 x\,dx = dt$

$$\int\frac{dy}{1+y^2} = -\int\frac{dt}{1+t^2}$$

$\tan^{-1} y = -\tan^{-1}(t) + C$

$\tan^{-1} y = -\tan^{-1}(e^{\tan x}) + C$

at x = 0, y = 1

$\frac{\pi}{4} = -\frac{\pi}{4} + C \qquad C = \frac{\pi}{2}$

$\tan^{-1} y = -\tan^{-1}(e^{\tan x}) + \frac{\pi}{2}$

Now at $x = \frac{\pi}{4}$

$\tan^{-1} y = -\tan^{-1}(e) + \frac{\pi}{2} = \cot^{-1} e = \tan^{-1}\frac{1}{e} \qquad y = \frac{1}{e}$

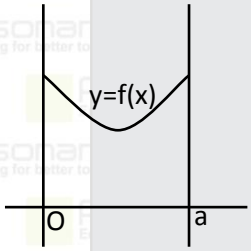
11. Let f(x) be a positive function such that the area bounded by y = f(x), y = 0 from x = 0 to x = a>0 is $e^{-a} + 4a^2 + a - 1$. Then the differential equation, whose general solution is $y = c_1 f(x) + c_2$, where c_1 and c_2 are arbitrary constants, is

(1) $(8e^x + 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (2) $(8e^x + 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

(3) $(8e^x - 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (4) $(8e^x - 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

Ans. **NTA (1)**
RESO (1)

Sol. $\int_0^a f(x)dx = e^{-a} + 4a^2 + a - 1$



$f(a) = -e^{-a} + 8a + 1$

$y = C_1\left(1 + 8x - \frac{1}{e^x}\right) + C_2$

$\frac{dy}{dx} = C_1\left(8 + \frac{1}{e^x}\right) \Rightarrow \frac{\left(\frac{dy}{dx}\right)}{\left(8 + \frac{1}{e^x}\right)} = C_1$

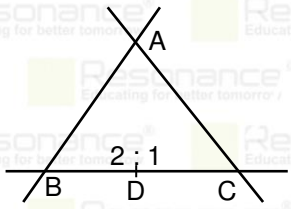
$\frac{d^2y}{dx^2}\left(8 + \frac{1}{e^x}\right) - \left(\frac{-1}{e^x}\right)\frac{dy}{dx} = 0$

12. The equations of two sides AB and AC of a triangle ABC are 4x + y = 14 and 3x – 2y = 5, respectively. The point $\left(2, -\frac{4}{3}\right)$ divides the third side BC internally in the ratio 2:1. the equation of the side BC is

(1) x – 3y – 6 = 0 (2) x + 3y + 2 = 0 (3) x – 6y – 10 = 0 (4) x + 6y + 6 = 0

Ans. **NTA (2)**
RESO (2)

Sol.



$B = (x_2, 14 - 4x_2)$

$C = \left(x_3, \frac{3x_3 - 5}{2}\right)$

Let D = $\left(2,\frac{-4}{3}\right)$

$\frac{2x_3+x_2}{3}=2 \rightarrow (1)$ $\frac{3x_3-5+14-4x_2}{3}=\frac{-4}{3} \rightarrow (2)$

Solving (1), (2) $x_3 = 1 \Rightarrow C\ (1, -1)$

slope of BC = $\frac{-1+\frac{4}{3}}{1-2}=\frac{-1}{3}$

13. Let $f(x) = 4\cos^3x + 3\sqrt{3}\cos^2x - 10$. The number of points of local maxima of f in interval $(0, 2\pi)$ is

(1) 2 (2) 3 (3) 4 (4) 1

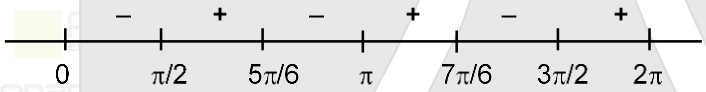
Ans. **NTA (1)**

RESO (1)

Sol. $f'(x) = -12\cos^2 x \sin x + 6\sqrt{3}\cos x \times (-\sin x)$

$= -6\sin x [2\cos^2 x + \sqrt{3}\cos x]$

$-6\sin x \cos x (2\cos x + \sqrt{3})$



local maxima at $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$ so number of local maxima points are 2

14. Let the sum of two positive integers be 24. If the probability, that their product is not less than $\frac{3}{4}$ times their greatest possible product, is $\frac{m}{n}$, where gcd(m, n) = 1, then n – m equals

(1) 11 (2) 10 (3) 9 (4) 8

Ans. **NTA (2)**

RESO (2)

Sol. $a + b = 24$, $\frac{a+b}{2} \geq \sqrt{ab} \Rightarrow 12 \geq \sqrt{ab}$

$ab \leq 144$. Now $ab \geq \frac{3}{4}(144)$

$ab \geq 108$

a = 6, b = 18; a = 7, b = 17 ; a = 8, b = 16 ; ..... , upto a = 18, b = 6

Favourable = 13, Total = 23

Required probability = $\frac{13}{23} \Rightarrow m = 13, n = 23$

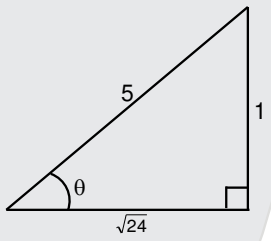
15. Let z be a complex number such that $|z + 2| = 1$ and $\text{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$. Then the value of $\left|\text{Re}\left(\overline{z+2}\right)\right|$ is

(1) $\frac{2\sqrt{6}}{5}$ (2) $\frac{\sqrt{6}}{5}$ (3) $\frac{1+\sqrt{6}}{5}$ (4) $\frac{24}{5}$

Ans. **NTA (1)**
RESO (1)

Sol. Let $z + 2 = e^{i\theta} \Rightarrow \left(\frac{z+1}{z+2}\right) = \frac{-2+e^{i\theta}+1}{e^{i\theta}} = 1 - \frac{1}{e^{i\theta}}$

$\Rightarrow \left(\frac{z+1}{z+2}\right) = 1 - \cos\theta + i\sin\theta$



$\sin\theta = \frac{1}{5}$

$\text{Re}(z + 2) = \cos\theta \Rightarrow \left|\text{Re}\left(\overline{z+2}\right)\right| = \frac{2\sqrt{6}}{5}$

16. Let P(x, y, z) be a point in the first octant, whose projection in the xy-plane is the point Q. Let OP = γ; the angle between OQ and the positive x-axis be θ; and the angle between OP and the positive z-axis be ϕ, where O is the origin. Then the distance of P from the x-axis is

(1) $\gamma\sqrt{1+\cos^2\theta\sin^2\phi}$ (2) $\gamma\sqrt{1-\sin^2\phi\cos^2\theta}$

(3) $\gamma\sqrt{1-\sin^2\theta\cos^2\phi}$ (4) $\gamma\sqrt{1+\cos^2\phi\sin^2\theta}$

Ans. **NTA (2)**
RESO (2)

Sol. Q (x,y,0), $\sqrt{x^2+y^2+z^2} = \gamma$, $\cos\theta = \frac{x}{\sqrt{y^2+x^2}} = \frac{x}{\sqrt{\gamma^2-z^2}}$

$\cos\phi = \frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{z}{\gamma}$, $OP = \sqrt{y^2+z^2} = \sqrt{\gamma^2-x^2}$

$OP = \sqrt{\gamma^2-\left(\gamma^2-z^2\right)\cos^2\theta} = \sqrt{\gamma^2-\gamma^2\left(1-\cos^2\phi\right)\cos^2\theta}$

17. Let $I(x) = \int \frac{6}{\sin^2 x(1-\cot x)^2}dx$. If $I(0) = 3$, then $I\left(\frac{\pi}{12}\right)$ is equal to

(1) $2\sqrt{3}$ (2) $6\sqrt{3}$ (3) $\sqrt{3}$ (4) $3\sqrt{3}$

Ans. **NTA (4)**
RESO (4)

Sol. $I(x) = \int \frac{6\,\text{cosec}^2 x}{(1-\cot x)^2}dx$, $\frac{d}{dx}(1-\cot x) = \text{cosec}^2 x$

Put $t = 1 - \cot x$

$I(x) = \int \frac{6dt}{t^2} = \frac{-6\sin x}{(\sin x - \cos x)} + C$

$x = 0 \Rightarrow 3 = 0 + C$

$x = \frac{\pi}{12} \Rightarrow I\left(\frac{\pi}{12}\right) = \frac{-6\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}{\left(\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)-\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\right)} + 3 = 3\sqrt{3}$

18. The number of critical points of the function $f(x) = (x-2)^{2/3}(2x+1)$ is

(1) 0 (2) 3 (3) 2 (4) 1

Ans. **NTA (3)**
RESO (3)

Sol. $f'(x) = \frac{2}{3}(x-2)^{\frac{-1}{3}}(2x+1) + (x-2)^{\frac{2}{3}}(2)$

$= \frac{2}{3}\left(\frac{(2x+1)+3(x-2)}{(x-2)^{\frac{1}{3}}}\right) = \frac{10(x-1)}{3(x-2)^{\frac{1}{3}}}$

$x = 1, 2$ are critical points.

19. Let [t] be the greatest integer less than or equal to t. Let A be the set of all prime factors of 2310 and $f : A \to Z$ be the function $f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$. The number of one-to-one functions from A to the range of f is

(1) 25 (2) 120 (3) 24 (4) 20

Ans. **NTA (2)**
RESO (2)

Sol. $2310 = 2 \times 3 \times 5 \times 7 \times 11 \Rightarrow A = \{2, 3, 5, 7, 11\}$

$y = x^2$ is increasing for $x \in A$

$y = x^2 + \left[\frac{x^3}{5}\right]$ is also increasing for $x \in A$

$\Rightarrow y = f(x)$ is increasing function.

Hence images are different. $\Rightarrow$ Range contains 5 elements

$\Rightarrow$ there will be 5! = 120 one-one functions.

20. The value of $k \in N$ for which the integral $I_n = \int_0^1 (1-x^k)^n dx$, $n \in N$, satisfies $147I_{20} = 148\, I_{21}$ is

(1) 8 (2) 10 (3) 14 (4) 7

Ans. **NTA (4)**
RESO (4)

Sol. n=21

$$I_n = \int_0^1 (1-x^k)^n \, dx$$

$$= x\left(.(1-x^k)^n\right)_0^1 - \int_0^1 x.n.(1-x^k)^{n-1}\left(-k.x^{k-1}\right) dx$$

$$I_n = -nk \int_0^1 (1-x^k)^{n-1} \, (1 - x^k - 1) \, dx$$

$I_n = -nk\, I_n + nk\, I_{n-1}$

$(1+nk)\, I_n = nk\, I_{n-1}$

$n = 21$

$1 + 21k = 148,\ 21k = 147 \quad \Rightarrow \quad k = 7$

21. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. If the sum of the diagonal elements of A^{13} is 3^n, then n is equal to _____ .

Ans. **NTA (7)**
RESO (7)

Sol. $A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix} = = 3\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$

$A^4 = 9\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} = 9\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$

$A^8 = 81\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} = 81\begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}$

$A^{12} = 9 \times 81\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} = 3^6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{13} = 3^6\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

Trace = $3^6\,(2+1) = 3^7$

Alternate

By Characteristic equation, Caley-Hamilton theorem

$A^2 - 3A + 3I = 0$

$A^4 = 3^2(A - I)^2 = 9(A - 2I) \Rightarrow A^8 = 81\,(-A + I)$

$A^4.A^8 = 3^6(-A^2 + A + 2A - 2I) = 3^6\,(I)$

$A^{13} = A^{12}A = 3^6\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow$ Sum of diagonal elements = $3^6.2 + 3 = 3^7 \Rightarrow n = 7$

22. If the range of $f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}$, $\theta \in R$ is $[\alpha, \beta]$, then the sum of the infinite G.P., whose first term is 64 and the common ratio is $\frac{\alpha}{\beta}$, is equal to _____ .

Ans. **NTA (96)**
RESO (96)

Sol. Let $t = \sin^2 x \Rightarrow y$ $y = \frac{t^2 - 3t + 3}{t^2 - t + 1}, 0 \le t \le 1$

$y = 3 - \frac{2t^2}{t^2 - t + 1}$

if $t = 0 \Rightarrow y = 3$

If $t \pm 0$ $\quad y = 3 - \frac{2}{\left(1 - \frac{1}{t} + \frac{1}{t^2}\right)}$

Let $f(t) = 1 - \frac{1}{t} + \frac{1}{t^2}, \quad t \in (0, 1]$

$f'(t) = \frac{t-2}{t^3} < 0$ Maximum $f(t) \to \infty$

Maximum $y = 3 - \frac{2}{\infty} = 3$

Minimum $y = 3 - \frac{2}{1} = 1$

Range $= [1, 3] \Rightarrow \alpha = 1, \beta = 3 \quad \Rightarrow$ Common ratio $= \frac{1}{3}$

Sum $= \frac{a}{1-r} = \frac{64}{1 - \frac{1}{3}} = 96$

23. Three balls are drawn at random from a bag containing 5 blue and 4 yellow balls. Let the random variables X and Y respectively denote the number of blue and yellow balls. If $\bar{X}$ and $\bar{Y}$ are the means of X and Y respectively, then $7\bar{X} + 4\bar{Y}$ is equal to _____ .

Ans. **NTA (17)**
RESO (17)

Sol.

X = 0, Y = 3	X = 1, Y = 2	X = 2, Y = 1	X = 3, Y = 0
$^5C_0 . ^4C_3$	$^5C_1 . ^4C_2$	$^5C_2 . ^4C_1$	$^5C_3 . ^4C_0$

$\bar{X} = \frac{0 + 1.^5C_1.^4C_2 + 2.^5C_2.^4C_1 + 3.^5C_3.^4C_0}{^9C_3} = \frac{5}{3}$

$\bar{Y} = \frac{3.^5C_0.^4C_3 + 2.^5C_1.^4C_2 + 1.^5C_2.^4C_1 + 0}{^9C_3} \quad \frac{4}{3}$

$7\bar{X} + 4\bar{Y} = \frac{35 + 16}{3} = 17$

24. The number of 3-digit numbers, formed using the digits 2, 3, 4, 5 and 7, when the repetition of digits is not allowed, and which are not divisible by 3, is equal to _____ .

Ans. **NTA (36)**
RESO (36)

Sol. Total three digit numbers = $5 \times 4 \times 3 = 60$

Total three digit numbers divisible by 3 are $\begin{pmatrix} 2,3,4 & 2,3,7 \\ 3,4,5 & 3,5,7 \end{pmatrix}$

$4 \times 3! = 24$

Total three digit numbers not divisible by 3 = 60– 24 = 36

25. Let $\alpha = \sum_{r=0}^{n}\left(4r^2+2r+1\right)\ {}^{n}C_r$ and $\beta = \left(\sum_{r=0}^{n}\frac{{}^{n}C_r}{r+1}\right)+\frac{1}{n+1}$. If $140<\frac{2\alpha}{\beta}<281$, then the value of n is _____ .

Ans. **NTA (5)**
RESO (5)

Sol. $\alpha = \sum_{r=0}^{n} r^2\ {}^{n}C_r + 2\sum_{r=0}^{n} r\ {}^{n}C_r + {}^{n}C_r$

$\Sigma(4.r(r-1) + 6r + 1)\ {}^{n}C_r$

$4.n\ (n-1)2^{n-2} + 6.n.2^{n-1} + 2^n$

$= 2^n(n+1)^2$

$\beta = \sum_{r=0}^{n}\frac{{}^{n}C_r}{r+1} + \frac{1}{n+1} = \frac{1}{n+1}.\left[2^{n+1}-1\right] + \frac{1}{n+1} = \frac{2^{n+1}}{n+1}$

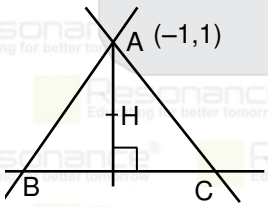
Now $140 < \frac{2\alpha}{\beta} < 248$ $\qquad 140 < (n+1)^3 < 281$

$n + 1 = 6$ $\qquad n = 5$

26. If the orthocentre of the triangle formed by the lines $2x+3y-1 = 0$, $x + 2y-1=0$ and $ax + by - 1 = 0$, is the centroid of another triangle, whose circumcentre and orthocentre respectively are (3, 4) and (–6, –8), then the value of $|a-b|$ is _____ .

Ans. **NTA (16)**
RESO (16)

Sol. Centroid of other triangle is H(0,0) which is orthocentre



Let AB ≡ $2x + 3y = 1$

BC ≡ $ax + by = 1$

Solving $\Rightarrow$ B $\left(\frac{3-b}{3a-2b}, \frac{a-2}{3a-2b}\right)$

Slope of AH = $\frac{1-0}{-1-0}$, Slope of BC = $\frac{-a}{b}$

Product = –1 $\qquad \Rightarrow a = -b \rightarrow (1)$

Slope of BH = $\frac{a-2}{3-b}$, Slope of AC = $\frac{-1}{2}$

Product = –1 $\Rightarrow \frac{a-2}{(3-b)} = 2 \Rightarrow b = 8, a = -8$ $\quad |a-b| = |-8-8| = 16$

27. The value of $\lim\limits_{x\to 0} 2\left(\frac{1-\cos x\sqrt{\cos 2x}\sqrt[3]{\cos 3x}.....\sqrt[10]{\cos 10x}}{x^2}\right)$ is _____ .

Ans. **NTA (55)**
RESO (55)

Sol. $\underset{x\to 0}{Lt}\frac{2}{x^2}\left(1-\left(1-\frac{x^2}{2}\right)\left(1-\frac{2^2x^2}{2}\right)^{\frac{1}{2}}\left(1-\frac{3^2x^2}{2}\right)^{\frac{1}{3}}.....\left(1-\frac{10^2x^2}{2}\right)^{\frac{1}{10}}\right)$

$= \underset{x\to 0}{Lt}\frac{2}{x^2}\left(1-\left(1-\frac{x^2}{2}\right)\left(1-\frac{2^2x^2}{2.2}\right)\left(1-\frac{3^2.x^2}{3.2}\right).....\left(1-\frac{10^2x^2}{2.10}\right)\right)$

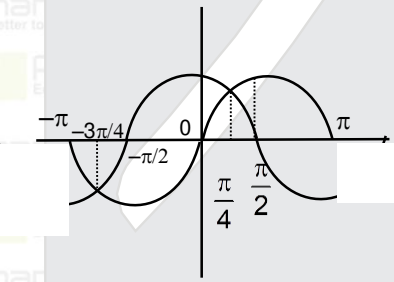
$= \underset{x\to 0}{Lt}\frac{2}{x^2}\left(1-\left(1-\frac{x^2}{2}-\frac{2x^2}{2}-\frac{3x^2}{2}.....-\frac{10x^2}{2}\right)\right)$

$= \left(\frac{2}{2}(1+2+3+.....+10)\right) = \frac{10(10+1)}{2} = 55$

28. Let the area of the region enclosed by the curve y = min{sin x, cos x} and the x- axis between x = $-\pi$ to x = π be A. Then A^2 is equal to _____ .

Ans. **NTA (16)**
RESO (16)

Sol.



$$f(x)=\begin{cases}\cos x: & -\pi < x < -\frac{3\pi}{4}\\ \sin x: & -\frac{3\pi}{4} \le x \le \frac{\pi}{4}\\ \cos x: & -\frac{\pi}{4} < \pi \le \pi\end{cases}$$

$$A = \int_{-\pi}^{-3\pi/4} -\cos x\,dx + \int_{\frac{-3\pi}{4}}^{0} -\sin x\,dx + \int_{0}^{\pi/4} \sin x\,dx + \int_{\frac{\pi}{4}}^{\pi/2} \cos x\,dx + \int_{\frac{\pi}{2}}^{\pi} -\cos x\,dx$$

$$A = (-\sin x)_{-\pi}^{-3\pi/4} + (\cos x)_{-3\pi/4}^{0} + (-\cos x)_{0}^{\pi/4} + (\sin x)_{\pi/4}^{\pi/2} + (-\sin x)_{\pi/2}^{\pi}$$

$$= \left(\frac{1}{\sqrt{2}}-0\right)+\left(1+\frac{1}{\sqrt{2}}\right)+\left(-\frac{1}{\sqrt{2}}+1\right)+\left(1-\frac{1}{\sqrt{2}}\right)+(-0+1) = 4$$

29. Let $\vec{a}=9\hat{i}-13\hat{j}+25\hat{k}$, $\vec{b}=3\hat{i}+7\hat{j}-13\hat{k}$ and $\vec{c}=17\hat{i}-2\hat{j}+\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{a}=(\vec{b}+\vec{c})\times\vec{a}$ and $\vec{r}\cdot(\vec{b}-\vec{c})=0$, then $\frac{|593\vec{r}+67\vec{a}|^2}{(593)^2}$ is equal to_____.

Ans. **NTA (569)**
RESO (569)

Sol. $(\bar{r}-\bar{b}-\bar{c})\times\bar{a}=\bar{0}\Rightarrow\bar{r}=\bar{b}+\bar{c}+\lambda\bar{a}$

$\bar{b}-\bar{c}=-14\hat{i}+9\hat{j}-14\hat{k}$

$\bar{r}.(\bar{b}-\bar{c})=0\quad\Rightarrow 0=\bar{b}^2-\bar{c}^2+\lambda\bar{a}.(\bar{b}-\bar{c})$

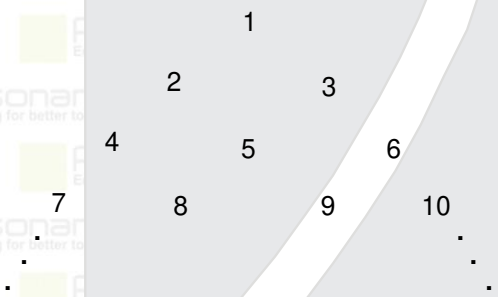
$\Rightarrow 0=227-294+\lambda(-126-117-350)$

$\frac{-67}{593}=\lambda\Rightarrow\bar{r}=20\hat{i}+5\hat{j}-12\hat{k}-\frac{67}{593}\bar{a}$

$\frac{593\bar{r}+67\bar{a}}{593}=20\hat{i}+5\hat{j}-12\hat{k}$

$\frac{|593\bar{r}+67\bar{a}|^2}{(593)^2}=400+25+144=569$

30. Let the positive integers be written in the form:



If the k^{th} row contains exactly k numbers for every natural number k, then the row in which the number 5310 will be, is_____.

Ans. **NTA (103)**
RESO (103)

Sol. Last term of each row follows sequence 1, 3, 6, 10, 15, ...
Differences 2, 3, 4, 5, ... which are in A.P.
$t_n = an^2 + bn + c$
$1 = a + b + c$, $3 = 4a + 2b + c$, $6 = 9a + 3b + c$

$\Rightarrow 2 = 3a + b$, $3 = 5a + b \Rightarrow 1 = 2a$, $b=\frac{1}{2}$, $c = 0$

Last term of n^{th} group $=\frac{1}{2}n^2+\frac{1}{2}n$

$\Rightarrow 1^{st}$ term of n^{th} group $=\frac{1}{2}(n-1)^2+\frac{1}{2}(n-1)+1$

$\Rightarrow\frac{n^2-n+2}{2}\le 5310\le\frac{n(n+1)}{2}$ By trail, n = 100 $\Rightarrow\frac{n(n+1)}{2}=5050$

$n = 102\Rightarrow\frac{n(n+1)}{2}=5253$, $n = 103\Rightarrow\frac{n(n+1)}{2}=5356$

$\Rightarrow$ 5310 lies in 103^{rd} group.

PART : PHYSICS

31. Paramagnetic substances :

A. align themselves along the directions of external magnetic field.

B. attract strongly towards external magnetic field.

C. has susceptibility little more than zero.

D. move from a region of strong magnetic field to weak magnetic field.

(1) B, D only (2) A, C only (3) A, B, C, D (4) A, B, C only

अनुचुम्बकीय पदार्थ :

A. अपने को बाह्य चुम्बकीय क्षेत्र की दिशा में व्यवस्थित रखते हैं।

B. बाह्य चुम्बकीय क्षेत्र की ओर तेजी से आकर्षित होते हैं।

C. उनकी चुम्बकीय प्रवृत्ति, शून्य से कुछ ही ज्यादा होती हैं।

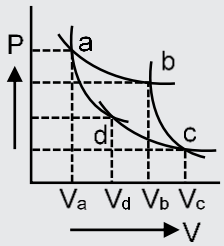
D. किसी शक्तिशाली चुम्बकीय क्षेत्र से कमजोर चुम्बकीय क्षेत्र की ओर गति करते हैं।

(1) केवल B, D (2) केवल A, C (3) A, B, C, D (4) केवल A, B, C

Ans. **(2)**

32. Two different adiabatic paths for the same gas intersect two isothermal curves as shown in P-V diagram. The relation between the ratio $\frac{V_a}{V_d}$ and the ratio $\frac{V_b}{V_c}$ is :

प्रदर्शित P-V आरेख में एक गैस के दो रूद्धोष्म वक्र, दो समतापी वक्रों का प्रतिछेदित करते है। तब $\frac{V_a}{V_d}$ तथा $\frac{V_b}{V_c}$ में सम्बन्ध है :



(1) $\frac{V_a}{V_d}=\left(\frac{V_b}{V_c}\right)^2$ (2) $\frac{V_a}{V_d}=\left(\frac{V_b}{V_c}\right)^{-1}$ (3) $\frac{V_a}{V_d}=\frac{V_b}{V_c}$ (4) $\frac{V_a}{V_d}\neq\frac{V_b}{V_c}$

Ans. **(3)**

Sol. $T_bV_b^{\gamma-1}=T_cV_c^{\gamma-1}$ [adiabatic process] ...(i)

$T_aV_a^{\gamma-1}=T_dV_d^{\gamma-1}$

$$\frac{T_a}{T_b}\left(\frac{v_a}{v_b}\right)^{\gamma-1}=\frac{T_d}{T_c}\left(\frac{v_d}{v_c}\right)^{\gamma-1} \quad ...(2)$$

$T_a = T_b, T_c = T_d$

$$= \frac{v_a}{v_b}=\frac{v_d}{v_c}$$

$$= \frac{v_a}{v_d}=\frac{v_b}{v_c}$$

33. Average force exerted on a non-reflecting surface at normal incidence is 2.4×10^{-4} N. If 360 W/cm^2 is the light energy flux during span of 1 hour 30 minutes. Then the area of the surface is :

एक अपरावर्तित सतह के लम्बवत् औसत लगाया बल 2.4×10^{-4} N है। यदि एक घंटा 30 मिनट की अवधि में 360 W/cm^2 का प्रकाश ऊर्जा फ्लक्स है, तब सतह का क्षेत्रफल है :

(1) 0.02 m^2 (2) 0.2 m^2 (3) 0.1 m^2 (4) 20 m^2

Ans. **(1)**

Sol. $F = \frac{IA}{C}$

$A = \frac{FC}{I} = \frac{2.4\times10^{-4}\times3\times10^{8}\times10^{-4}}{360}$

$A = \frac{7.2}{360} = 2 \times 10^{-2}$ m^2

$A = 0.02$ m^2

34. A LCR circuit is at resonance for a capacitor C, inductance L and resistance R. Now the value of resistance is halved keeping all other parameters same. The current amplitude at resonance will be now ;

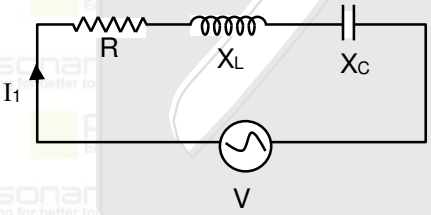
(1) same (2) double (3) zero (4) halved

एक LCR परिपथ धारिता C, प्रेरक L तथा प्रतिरोध R के लिए अनुनादित है। यदि सभी मान स्थिर रखकर केवल प्रतिरोध को आधा कर दें तो अब अनुनाद पर धारा का आयाम होगा :

(1) समान (2) दोगुना (3) शून्य (4) आधा

Ans. **(2)**

Sol. At resonance $X_C = X_L$



$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$

$I_1 = \frac{V}{Z} = \frac{V}{R}$

$Z' = \sqrt{\left(\frac{R}{2}\right)^2 + (X_L - X_C)^2} = \frac{R}{2}$

$I_2 = \frac{V}{Z'} = \frac{2V}{Z}$

$I_2 = 2I_1$

35. Young's modulus is determined by the equation given by $Y = 49000\frac{m}{l}\frac{dyne}{cm^2}$ where M is the mass and l is the extension of wire used in the experiment. Now error in Young modules (Y) is estimated by taking data from M-l plot in graph paper. The smallest scale divisions are 5g and 0.02 cm along load axis and extension axis respectively. If the value of M and l are 500 g and 2 cm respectively then percentage error of Y is :

यंग प्रत्यास्थता गुणांक को समीकरण $Y = 49000\frac{m}{l}\frac{dyne}{cm^2}$ से ज्ञात किया गया है। यहाँ M द्रव्यमान तथा l तार का खिंचाव है। एक ग्राफ पेपर पर M-l ग्राफ से से प्रेक्षण लेकर यंग प्रत्यास्थता गुणांक में त्रुटि ज्ञात की जाती है। खिंचाव–अक्ष तथा द्रव्यमान अक्ष पर क्रमशः 0.2 cm तथा 5 gm सूक्ष्मतम माप के भाग है। यदि M तथा l के मान क्रमशः 500 g एवं 2 cm हैं। तब Y में प्रतिशत त्रुटि है :

(1) 0.2 % (2) 0.5 % (3) 0.02 % (4) 2 %

Ans. **(4)**

Sol. $\left(\frac{dy}{y}\right)\% = \left(\frac{dm}{m} + \frac{d\ell}{\ell}\right)\times 100$

$= \left(\frac{5}{500} + \frac{0.02}{2}\right)\times 100 = 2\%$

36. Three bodies A, B and C have equal kinetic energies and their masses are 400 g, 1.2 kg and 1.6 kg respectively. The ratio of their linear momenta is :

400 ग्राम, 1.2 किग्रा व 1.6 किग्रा द्रव्यमान के क्रमशः तीन पिण्डों की गतिज ऊर्जाएँ समान है। इनके रेखीय संवेगों का अनुपात है :

(1) $\sqrt{3} : \sqrt{2} : 1$ (2) $1 : \sqrt{3} : \sqrt{2}$ (3) $\sqrt{2} : \sqrt{3} : 1$ (4) $1 : \sqrt{3} : 2$

Ans. **(4)**

Sol. $KE = \frac{1}{2}mv^2$ $\quad p = \sqrt{2kE.m}$, $P \propto \sqrt{m}$

$p_1 : p_2 : p_3 = \sqrt{2\times 0.40\times KE} : \sqrt{1.2\times 2\,KE} : \sqrt{1.6\times 2\,KE}$

$= \sqrt{4} : \sqrt{12} : \sqrt{16}$

$1 : \sqrt{3} : 2$ Ans.

37. In an expression $a \times 10^b$:

(1) b is order of magnitude for $5 < a \le 10$

(2) b is order of magnitude for $a \ge 5$

(3) a is order of magnitude for $b \le 5$

(4) b is order of magnitude for $a \le 5$

यहाँ एक व्यंजक $a \times 10^b$ में :

(1) $5 < a \le 10$ के लिए परिमाण की कोटि b है।

(2) $a \ge 5$ के लिए परिमाण की कोटि b है।

(3) $b \le 5$ के लिए परिमाण की कोटि a है।

(4) $a \le 5$ के लिए परिमाण की कोटि b है।

Ans. **(4)**

38. Correct Bernoulli's equation is (simply have their usual meaning) :

(1) $P + mgh + \frac{1}{2}mv^2 = \text{constant}$ (2) $P + \rho gh + \rho v^2 = \text{constant}$

(3) $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$ (4) $P + \frac{1}{2}\rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

सही बरनौली समीकरण है (संकेत सामान्य अर्थ में व्यक्त किय गये हैं)

(1) $P + mgh + \frac{1}{2}mv^2 =$ नियतांक (2) $P + \rho gh + \rho v^2 =$ नियतांक

(3) $P + \rho gh + \frac{1}{2}\rho v^2 =$ नियतांक (4) $P + \frac{1}{2}\rho gh + \frac{1}{2}\rho v^2 =$ नियतांक

Ans. **(3)**

39. Critical angle of incidence for a pair of optical media is 45º. The refractive indices of first and second media are in the ratio :

एक प्रकाशीय माध्यमों के युग्म का आपतन क्रांतिक कोण 45º है। तब प्रथम व द्वितीय माध्यम के अपवर्तनांकों का अनुपात है :

(1) $1:\sqrt{2}$ (2) 1 : 2 (3) 2 : 1 (4) $\sqrt{2}:1$

Ans. **(4)**

Sol. $\sin C = \frac{n_r}{n_d} = \frac{n_2}{n_1}$

$\Rightarrow \frac{n_2}{n_1} = \sin 45° = \frac{1}{\sqrt{2}}$

$\frac{n_1}{n_2} = \sqrt{2}$

40. A player caught a cricket ball of mass 150 g moving at a speed of 20 m/s. If the catching process is completed in 0.1 s, the magnitude of force exerted by the ball on the hand of the player is :

20 मी/से की चाल से गतिमान 150 ग्राम द्रव्यमान की एक गेंद को एक खिलाड़ी पकड़ता है। यदि पकड़ने में 0.1 सेकंड का समय लगा तब खिलाडी के हाथ पर गेंद द्वारा लगाये गये बल का परिमाण है :

(1) 3 N (2) 300 N (3) 150 N (4) 30 N

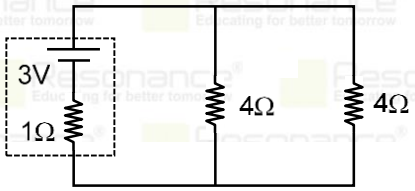
Ans. **(4)**

Sol. $F = m\frac{\Delta v}{\Delta t}$

$= \frac{150}{1000} \times \left(\frac{20-0}{0.1}\right) = 30N$

41. In the given circuit, the terminal potential difference of the cell is :

यदि दिये परितथ में सेल का टर्मिनल विभवान्तर है :



(1) 1.5 V (2) 2 V (3) 3 V (4) 4 V

Ans. **(2)**

Sol. $i = \frac{V}{R_{eq}} = \frac{3}{3} = 1A$

So potential difference across the cell is

$V = \varepsilon - ir = 3 - 1 \times 1 = 3 - 1 = 2V$ Ans.

42. A mixture of one mole of monoatomic gas and one mole of a diatomic gas (rigid) are kept at room temperature (27ºC). The ratio of specific heat of gases at constant volume respectively is :

एक मोल एकल–परमाणु गैस तथा एक मोल द्वि–परमाणु गैस का मिश्रण कमरे के ताप (27ºC) पर लिया गया हैं। स्थिर आयतन पर क्रमशः उनकी विशिष्ट ऊष्माओं का अनुपात है :

(1) $\frac{3}{2}$ (2) $\frac{3}{5}$ (3) $\frac{7}{5}$ (4) $\frac{5}{3}$

Ans. **(2)**

Sol. We know that

$(C_V)_{monatomic} = \frac{3}{2}R$

$(C_V)_{diatonic} = \frac{5}{2}R$ & $\frac{(C_V)_{monatomic}}{(C_V)_{diatonic}} = \frac{3}{5}$

43. A clock has 75 cm, 60 cm long second hand and minute hand respectively. In 30 minutes duration the tip of second hand will travel x distance more than the tip of minute hand. The value of x in meter is nearly (Take $\pi = 3.14$) :

एक घड़ी की सेकंड तथा मिनट वाली सुई क्रमशः 75 cm, 60 cm लम्बी हैं। घड़ी के 30 मिनट के अन्तराल में सेकंड वाली सुई की नोक, मिनट सुई की नोक की अपेक्षा x दूरी अधिक चलती है। यहां x का मान मीटर में लगभग है (यदि $\pi = 3.14$) :

(1) 220.0 (2) 140.5 (3) 118.9 (4) 139.4

Ans. **(4)**

Sol. $X = 2\pi \times 75 \times 30 - 2\pi \times 60 \times 1/2 = 139.4$

44. A stationary particle breaks into two parts of masses m_A and m_B which move with velocities v_A and v_B respectively. The ratio of their kinetic energies ($K_B : K_A$) is :

एक स्थिर कण दो भागों में टूट जाता है जिनका द्रव्यमान m_A एवं m_B है तथा यह क्रमशः v_A तथा v_B की गति से चलते हैं। इनकी गतिज ऊर्जाओं का अनुपात ($K_B : K_A$) है :

(1) $v_B : v_A$ (2) $m_B : m_A$ (3) $m_B\, m_A : m_A\, v_A$ (4) 1 : 1

Ans. **(1)**

Sol. $m_1v_1 = m_2v_2$

$KE = \frac{1}{2}mv^2$

$\frac{KE_A}{KE_B} = \frac{\frac{1}{2}m_Av_A^2}{\frac{1}{2}m_Bv_B^2} = \frac{v_A}{v_B}$ or $\frac{m_B}{m_A}$

$\frac{KE_B}{KE_A} = \frac{v_B}{v_A}$ or $\frac{m_A}{m_B}$

45. Binding energy of a certain nucleus is 18×10^8 J. How much is the difference between total mass of all the nucleons and nuclear mass of the given nucleus :
एक नाभिक की बन्धन ऊर्जा 18×10^8 J है। तब नाभिक के सभी न्युक्लियानों का कुल द्रव्यमान तथा उस नाभिक के द्रव्यमान में कितना अन्तर है :

(1) 10 μg (2) 20 μg (3) 2 μg (4) 0.2 μg

Ans. **(2)**

Sol. B.E. Δmc^2

$$\Rightarrow \Delta m = \frac{B.E.}{C^2} = \frac{18\times10^8}{3\times10^8\times3\times10^8}$$

$$= \frac{2}{10^8}\text{ kg}$$

$$= 2\times 10^{-8}$$

$$= 2 \times 10^{-8} \times 1000 \text{ g} = 2\times 10^{-5} \text{ g} = 20 \text{ μg}$$

46. The diameter of a sphere is measured using a vernier caliper whose 9 divisions of main scale are equal to 10 divisions of vernier scale. The shortest division on the main scale is equal to 1 mm. The main scale reading is 2 cm and second division of vernier scale coincides with a division on main scale. If mass of the sphere is 8.635 g, the density of the sphere is :
वर्नियर पैमाने के उपयोग से एक गोले का व्यास नापा जाता है। यहां मुख्य पैमाने के 9 भाग, वर्नियर पैमाने के 10 भाग के बराबर हैं। मुख्य पैमाने का सबसे छोटा भाग 1 mm है। मुख्य पैमाने का पाठयांक 2 सेमी है। मुख्य पैमाने के किसी चिन्ह के साथ वर्नियर पैमाने का दूसरा खाना मिलता है। यदि गोले का द्रव्यमान 8.635 g है तो गोले का घनत्व है :

(1) 2.0 g/cm^3 (2) 1.7 g/cm^3 (3) 2.5 g/cm^3 (4) 2.2 g/cm^3

Ans. **(1)**

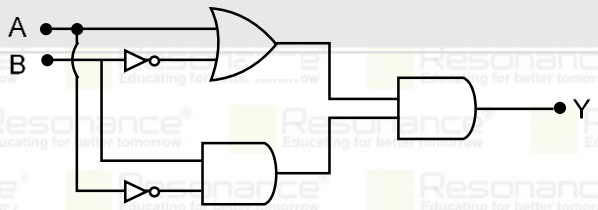
Sol. Reading = M.S.R. + (no. of division coincide × L.C.)

D = 2 cm + 2 × 0.01 = 2.02 cm

$$R = \frac{D}{2} = 1.01\text{cm}$$

$$\rho = \frac{8.635}{\frac{4}{3}\pi(1.01)^3} = 2\text{g/cm}^3$$

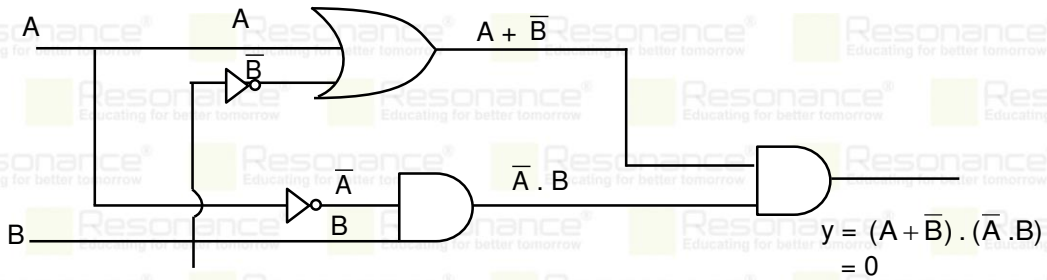
47. The output Y of following circuit for given inputs is :
दिये गये निवेशों के लिए निम्नलिखित परिपथ का निर्गत हैं :



(1) A.B(A + B) (2) A.B (3) $\bar{A}$.B (4) 0

Ans. **(4)**

Sol.



$Y = ((A + \overline{B}).(\overline{A}.B) = 0$

48. Two planets A and B having masses m_1 and m_2 move around the sun in circular orbits of r_1 and r_2 radii respectively. If angular momentum of A is L and that of B is 3L, the ratio of time period $\left(\frac{T_A}{T_B}\right)$ is :

m_1 व m_2 द्रव्यमानों के दो ग्रह सूर्य के परितः क्रमशः r_1 व r_2 त्रिज्याओं की वृत्ताकार कक्षाओं में गति करते हैं। यदि A का कोणीय संवेग L तथा B का कोणीय संवेग 3L है, तब उनके आवर्तकालों का अनुपात $\left(\frac{T_A}{T_B}\right)$ है:

(1) $27\left(\frac{m_1}{m_2}\right)^3$ (2) $\frac{1}{27}\left(\frac{m_2}{m_1}\right)^3$ (3) $\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$ (4) $\left(\frac{r_1}{r_2}\right)^3$

Ans. **(2)**

Sol. $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$

$\frac{L_1}{L_2} = \frac{r_1p_1}{r_2p_2} = \frac{L}{3L} \Rightarrow \frac{r_1m_1v_1}{r_2m_2v_2} = \frac{1}{3}$ .....(i)

For satellite

$\frac{mv^2}{r} = \frac{GMm}{r^2}$

$\Rightarrow v = \sqrt{\frac{GM}{r}}$

$v \propto \frac{1}{\sqrt{r}}$ ....(ii)

by (i) & (ii)

$\frac{r_1m_1 1/\sqrt{r_1}}{r_2m_2 1/\sqrt{r_2}} = \frac{1}{3}$

$\frac{r_1}{r_2} = \left(\frac{m_2}{3m_1}\right)^2$

$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$

$= \left(\left(\frac{m_2}{3m_1}\right)^2\right)^3$

$$\frac{T_A}{T_B} = \left(\frac{m_2}{3m_1}\right)^3$$

$$\frac{T_A}{T_B} = \frac{1}{27}\left(\frac{m_2}{m_1}\right)^3$$

49. A proton and an electron are associated with same de-Broglie wavelength. The ratio of their kinetic energies is :
(Assume $h = 6.63 \times 10^{-34}$ J s, $m_e = 9.0 \times 10^{-31}$ kg and $m_p = 1836$ times m_e)
एक प्रोटॉन तथा एक इलेक्ट्रॉन से संबद्ध डी–ब्रागली तरंगदैर्ध्य समान हैं। तब उनकी गतिज ऊर्जाओं का अनुपात होगा :
(दिया है : $h = 6.63 \times 10^{-34}$ J s, $m_e = 9.0 \times 10^{-31}$ kg तथा $m_p = 1836\ m_e$)

(1) $1 : \frac{1}{\sqrt{1836}}$ (2) $1 : \sqrt{1836}$ (3) 1 : 1836 (4) $1 : \frac{1}{1836}$

Ans. **(3)**

Sol. We know that

$$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}$$

λ is same so momentum will also be same

Now kinetic energy $k = \frac{p^2}{2m}$

$$\Rightarrow k \propto \frac{1}{m}$$

$$\Rightarrow \frac{k_p}{k_e} = \frac{m_e}{m_p} = \frac{me}{1836me}$$

= 1 : 1836

50. Two charged conducting spheres of radii a and b are connected to each other by a conducting wire. The ratio of charges of the two spheres respectively is :

a तथा b त्रिज्या के दो आवेशित सुचालक गोले, आपस में एक चालक तार से जोड़े गये हैं। उनके क्रमशः आवेशों का अनुपात है :

(1) $\frac{a}{b}$ (2) $\frac{b}{a}$ (3) ab (4) $\sqrt{ab}$

Ans. **(1)**

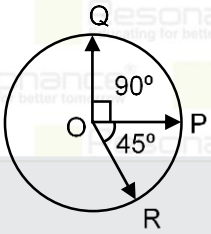
Sol. $v_1 = v_2$

$$\frac{kq_1}{a} = \frac{kq_2}{b}$$

$$\Rightarrow \frac{q_1}{q_2} = \frac{a}{b}$$

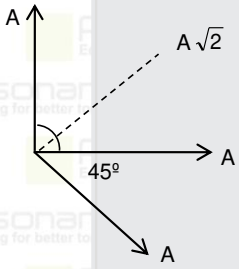
51. Three vectors $\overrightarrow{OP}$, $\overrightarrow{OQ}$ and $\overrightarrow{OR}$ each of magnitude A are acting as shown in figures. The resultant of the three vectors is $A\sqrt{x}$. The value of x is _____.

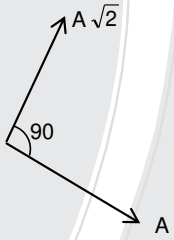
तीन सदिश $\overrightarrow{OP}$, $\overrightarrow{OQ}$ और $\overrightarrow{OR}$ प्रत्येक का परिमाण A है जो चित्र के अनुसार कार्यरत हैं। तीनों सदिशों का परिमाण $A\sqrt{x}$ है। x का मान _____ है।



Ans. **(3)**

Sol.

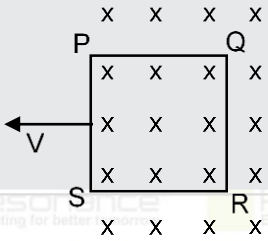




$Art = \sqrt{A^2 + 2A^2} = A\sqrt{3}$ $x = 3$

52. A square loop PQRS having 10 turns, area 3.6×10^{-3} m^2 and resistance 100 Ω is slowly and uniformly being pulled out of a uniform magnetic field of magnitude B = 0.5 T as shown. Work done in pulling the loop out of the field in 1.0 s is _____ $\times 10^{-6}$ J.

10 फेरों, तल क्षेत्रफल 3.6×10^{-3} m^2, प्रतिरोध 100 Ω के एक वर्गाकार लूप PQRS, को B = 0.5 T के एकसमान चुम्बकीय क्षेत्र से धीरे–धीरे बाहर खींचा जाता है जैसा कि चित्र में दर्शाया गया है। इसे 0.1 s में क्षेत्र के बाहर खींचने में किया गया कार्य _____ $\times 10^{-6}$ J है।



Ans. **(3)**

Sol. Side of the square = 6 cm

V = 6 cm/s

$i = \frac{NBvl}{R}$

work done = i^2Rt

$= \frac{(NBvl)^2 \times t}{R}$

$= \frac{(10 \times 0.5 \times 0.06 \times 0.06)^2 \times 1}{100} = 3 \times 10^{-6} J$

53. A closed and an open organ pipe have same lengths. If the ratio of frequencies of their seventh overtones is $\left(\frac{a-1}{a}\right)$ then the value of a is _____.

एक बन्द तथा एक खुले ऑर्गन पाइप की लम्बाई समान है। उनके सातवें गुणवृत्तियों (overtones) की आवृत्तियों का अनुपात $\left(\frac{a-1}{a}\right)$ है तब a का मान _____ है।

Ans. **(16)**

Sol. Close organ pipe

for n^{th} overtone $f_n = \frac{(2n+1)V}{4\ell}$

$n = 7$

$f_c = \frac{15V}{4\ell}$

open organ pipe

for nth overtone, $f_n = \frac{(n+1)V}{2\ell}$, $n = 7$

$f_0 = \frac{8V}{2\ell} = \frac{4V}{\ell}$

$\frac{f_c}{f_0} = \frac{15}{4(4)} = \frac{15}{16} = \left(\frac{a-1}{a}\right)$

$a = 16$

54. In an alpha particle scattering experiment distance of closest approach for the α particle is 4.5×10^{-14} m. If target nucleus has atomic number 80, then maximum velocity of α-particle is _____ $\times 10^5$ m/s approximately.

($\frac{1}{4\pi \in_D} = 9\times10^9$ SI unit, mass of α particle = 6.72×10^{-27} kg)

एक α-कण के प्रकीर्णन प्रयोग में, α-कण की सबसे निकटतम दूरी 4.5×10^{-14} m है। यदि लक्ष्य नाभिक की परमाणु संख्या 80 हो तो α-कण का लगभग अधिकतम वेग _____ $\times 10^5$ m/s है।

(दिया है $\frac{1}{4\pi \in_D} = 9\times10^9$ SI मात्रक, α-कण का द्रव्यमान = 6.72×10^{-27} kg)

Ans. **(156)**

Sol. $KE = \frac{1}{4\pi \in_0}\frac{2Ze^2}{r_0} = \frac{1}{2}mV^2$

$V = \sqrt{\frac{2}{m}\times\frac{1}{4\pi \in_0}\times\frac{2Ze^2}{r_0}}$

$V = \sqrt{\frac{2}{6.72\times10^{-27}}\times9\times10^9\times\frac{2\times80\times\left(1.6\times10^{-19}\right)^2}{4.5\times10^{-14}}}$

$V = 156 \times 10^5$ m/s

55. A liquid column of height 0.04 cm balances excess pressure of a soap bubble of certain radius. If density of liquid is 8×10^3 kg m^{-3} and surface tension of soap solution is 0.28 Nm^{-1}, then diameter of soap bubble is _____ cm.
(if g = 10 m s^{-2})
किसी द्रव के स्तम्भ की ऊंचाई 0.04 cm किसी साबुन के बुलबुले के दाब आधिक्य को संतुलित करती है। यदि द्रव का घनत्व 8×10^3 kg m^{-3} है तथा साबुन का पृष्ठ तनाव 0.28 Nm^{-1} है, तब बुलबुले का व्यास _____ cm है।
(दिया है – g = 10 m s^{-2})

Ans. **(7)**

Sol. $\rho gh = \frac{4T}{R}$

$$R = \frac{4 \times 0.28}{8 \times 10^3 \times 10 \times 0.04 \times 10^{-2}} = 3.5 \times 10^{-2} m$$

R = 3.5 cm
D = 7 cm

56. Resistance of a wire at 0ºC, 100ºC and tºC is found to be 10Ω, 10.2Ω and 10.95Ω respectively. The temperature t in Kelvin scale is _____.
एक तार का 0ºC, 100ºC तथा tºC ताप पर प्रतिरोध क्रमशः 10Ω, 10.2Ω तथा 10.95Ω हैं। तब केल्विन पैमाने पर ताप t का मान _____ है।

Ans. **(748)**

Sol. $R = R_0 (1 + \alpha \Delta T)$
$10.20 = 10 (1 + \alpha\ 100)$ .....(i)
$10.95 = 10(1 + \alpha t)$ .....(ii)
(i) & (ii)

$$\frac{t}{100} = \frac{0.95}{0.20} \Rightarrow t = \frac{19}{4} \times 100 = 475^\circ C \text{ or } 748\ K$$

57. An electric field, $\vec{E} = \frac{2\hat{i} + \hat{j} + 8\hat{k}}{\sqrt{6}}$ passes through the surface of $4m^2$ area having unit vector $\hat{n} = \left(\frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}}\right)$. The electric flux for that surface is _____ V m.

एक वैद्युत क्षेत्र $\vec{E} = \frac{2\hat{i} + \hat{j} + 8\hat{k}}{\sqrt{6}}$, किसी $4m^2$ क्षेत्रफल के पृष्ठ जिसका इकाई सदिश $\hat{n} = \left(\frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}}\right)$ है, से गुजरता है।

इस पृष्ठ के लिए वैद्युत फ्लक्स _____ V m है।

Ans. **(12)**

Sol. $\phi = E.A = \frac{\left(2\hat{i} + 6\hat{j} + 8\hat{k}\right) 4\left(2\hat{i} + \hat{j} + \hat{k}\right)}{6}$

$$= \frac{16 + 24 + 32}{6} = \frac{72}{6} = 12$$

58. An electron with kinetic energy 5eV enters a region of uniform magnetic field of 3 μT perpendicular to its direction. An electric field E is applied perpendicular to the direction of velocity and magnetic field. The value of E, so that electron moves along the same path, is _____ NC^{-1}.
(Given, mass of electron = 9×10^{-31} kg, electric charge = 1.6×10^{-19} C)
3μT के एकसमान चुम्बकीय क्षेत्र के लम्बवत् 5eV ऊर्जा का एक इलेक्टॉन प्रवेश करता है। एक वैद्युत क्षेत्र E इसकी गति तथा चुम्बकीय क्षेत्र, दोनों के लम्बवत् लगाया जाता है। वैद्युत क्षेत्र E जिससे इलेक्ट्रॉन उसी दिशा में पथ पर गति कर सके, लगाया जाए तब E = _____ NC^{-1} है।
(दिया है – इलेक्ट्रॉन का द्रव्यमान = 9×10^{-31} kg, इलेक्ट्रॉन आवेश = 1.6×10^{-19} C)

Ans. **(4)**

Sol. $KE = \frac{1}{2}mV^2$

$5 \times 1.6 \times 10^{-19} = \frac{1}{2} \times 9 \times 10^{-31} \times V^2$

$V = \frac{4}{3} \times 10^6 m/s$

$eE = eVB$

$E = VB$

$E = \frac{4}{3} \times 10^6 \times 3 \times 10^{-6}$

$E = 4\ NC^{-1}$

59. A parallel beam of monochromatic light of wavelength 600 nm passes through single slit of 0.4 mm width. Angular divergence corresponding to second order minima would be _____ $\times 10^{-3}$ rad.
एक वर्णीय प्रकाश की समान्तर किरण–पुन्ज की तरंग दैर्ध्य 600 nm है जो एक 0.4 mm चौड़ाई की झिर्री से गुजरती है। इसकी द्वितीय श्रेणी के निम्निष्ठ का कोणीय फैलाव _____ $\times 10^{-3}$ रेडियन होगा।

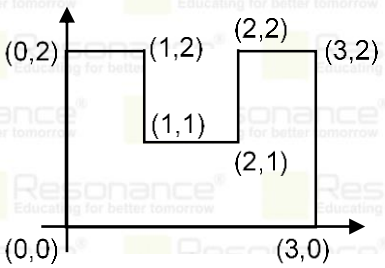
Ans. **(6)**

Sol. $\sin\theta \approx \theta = \frac{n\lambda}{a}$

$\theta = \frac{2 \times 600 \times 10^{-9}}{0.4 \times 10^{-3}} = 3$

$2\theta = 6$

60. A uniform thin metal plate of mass 10 kg with dimensions is shown. The ratio of x and y coordinates of centre of mass of plate is $\frac{n}{9}$. The value of n is _____.
10 किग्रा द्रव्यमान की एक एकसमान पतली धातु की प्लेट दिखायी गयी है। प्लेट के द्रव्यमान केंद्र के x तथा y निर्देशांकों का अनुपात $\frac{n}{9}$ है तब n का मान _____ है।



Ans. **(15)**

Sol. by symmetry x -coordinate of

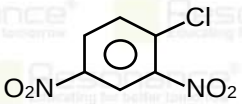
COM, $X_{cm} = \frac{3}{2} = 1.5$ & $Y_{cm} = \frac{3m \times 0.5 + 2m \times 1.5}{5m} = \frac{4.5}{5} = \frac{9}{10}$

so required ratio is : $\frac{3 \times 10}{2 \times 9} = \frac{15}{9}$

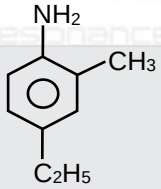
so n = 15

PART : CHEMISTRY

61. Given below are two statements :

Statement I :  IUPAC name of compound A is

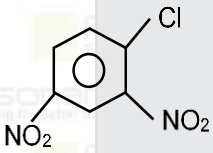
Compound A
4-chloro-1,3-dinitrobenzene

Statement II :  IUPAC name is

Compound B
4-ethyl-2-methylaniline

In the light of above statements, choose the **most appropriate** answer from the options given below :
(1) Statement I is correct but Statement II is incorrect.
(2) Both Statement I & Statement II are correct.
(3) Statement I is incorrect but Statement II is correct.
(4) Both Statement I & Statement II are incorrect.

Ans. **NTA (3)**

Sol. 

lowest locant rule will give 1-chloro.

62. Match List I with List II :

List (i) (Name of the test)		List (ii) (Reaction sequence involved) [M is metal]	
A.	Borax bead test	I.	$MCO_3 \rightarrow MO \xrightarrow[+\Delta]{Co(NO_3)_2} CoO.MO$
B.	Charcoal cavity test	II.	$MCO_3 \rightarrow MCl_2 \longrightarrow M^{2+}$
C.	Cobalt nitrate test	III.	$MSO_4 \xrightarrow[\Delta]{Na_2B_4O_7} M(BO_2)_2 \rightarrow MBO_2 \rightarrow M$
D.	Flame test	IV.	$MSO_4 \xrightarrow[\Delta]{Na_2CO_3} MCO_3 \rightarrow MO \rightarrow M$

Choose the **correct** answer from the options given below :
(1) (A) → (III) ; (B) → (II) ; (C) → (IV) ; (D) → (I)
(2) (A) → (III) ; (B) → (IV) ; (C) → (I) ; (D) → (II)
(3) (A) → (III) ; (B) → (I) ; (C) → (IV) ; (D) → (II)
(4) (A) → (III) ; (B) → (I) ; (C) → (II) ; (D) → (IV)

Ans. **NTA (2)**

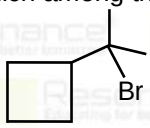
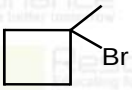
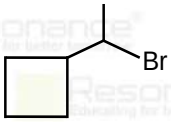
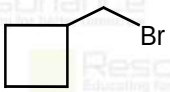
Sol. (A) Borax bead test : (III) $MSO_4 \xrightarrow[\Delta]{Na_2B_4O_7} M(BO_2)_2 \rightarrow MBO_2 \rightarrow M$

(B) Charcoal Cavity test : (IV) $MSO_4 \xrightarrow[\Delta]{Na_2CO_3} MCO_3 \rightarrow MO \rightarrow M$

(C) Cobalt nitrate test : (I) $MCO_3 \rightarrow MO \xrightarrow[+\Delta]{Co(NO_3)_2} CoO.MO$

(D) Flame test : (II) $MCO_3 \rightarrow MCl_2 \longrightarrow M^{2+}$

63. Which among the following compounds will undergo fastest S_N2 reaction.

(1)  (2)  (3)  (4) 

Ans. **NTA (4)**

Sol. $S_N2 \propto \frac{1}{\text{Hinderance}}$

Primary halide give fastest S_N2.

64. Number of complexes with even number of electrons in t_{2g} orbitals is :

$[Fe(H_2O)_6]^{2+}$, $[Co(H_2O)_6]^{2+}$, $[Co(H_2O)_6]^{3+}$, $[Cu(H_2O)_6]^{2+}$, $[Cr(H_2O)_6]^{2+}$

(1) 1 (2) 3 (3) 4 (4) 5

Ans. **NTA (2)**

Sol.

in octahedral complex ion	Electronic configuration (w.r.t (H_2O) WFL)	No. of electron in t_{2g} orbitals
$[Fe(H_2O)_6]^{2+}$	Fe^{+2} : $3d^6$	$t_{2g}^{211}\ eg^{11}$	4
$[Co(H_2O)_6]^{2+}$,	Co^{+3} : $3d^7$	$t_{2g}^{221}\ eg^{11}$	5
$[Co(H_2O)_6]^{3+}$,	Co^{+3} : $3d^6$	$t_{2g}^{211}\ eg^{11}$	4
$[Cu(H_2O)_6]^{2+}$,	Cu^{+2} : $3d^9$	$t_{2g}^{222}\ eg^{21}$	6
$[Cr(H_2O)_6]^{2+}$	Cr^{+2} : $3d^4$	$t_{2g}^{111}\ eg^{10}$	3

Even number of electron present in t_{2g} orbitals in following complexes.

$[Fe(H_2O)_6]^{2+}$, $[Co(H_2O)_6]^{3+}$, $[Cr(H_2O)_6]^{2+}$

65. Given below are two statements :

Statement I : $N(CH_3)_3$ and $P(CH_3)_3$ can act as ligands to form transition metal complexes.

Statement II : As N and P are from same group, the nature of bonding of $N(CH_3)_3$ and $P(CH_3)_3$ is always same with transition metals.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Statement I is correct but Statement II is incorrect.

(2) Both Statement I and Statement II are correct.

(3) Both Statement I and Statement II are incorrect.

(4) Statement I is incorrect but Statement II is correct.

Ans. **NTA (1)**

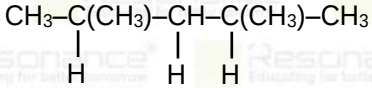
Sol. **S-I :** $N(CH_3)_3$ and $P(CH_3)_3$ both have 1 L.P. of e^- on central atom and also behave as a Lewis base. So both are act as a ligand in formation of transition metal complexes.

S-II : In $N(CH_3)_3$ N-atom has 1 L.P. without vacant d-orbital so behave only as σ donor ligand.

But in : $P(CH_3)_3$ P-atom has 1 L.P. with vacant d-orbital so it behave as a both σ-donor and π-acceptor ligand. There for nature of Bonding of $N(CH_3)_3$ and $P(CH_3)_3$ is different in complexes of transition metal, (as N and P are same group element).

So option (3) S-I is correct and S-II is incorrect.

66. In the given compound, the number of 2º carbon atom/s is_____.



(1) Four (2) Three (3) Two (4) One

Ans. **NTA (4)**

Sol. $-CH_2-$ is 2° carbon present.

67. Match List I with List II :

List (I) (Elements)		List (II) (Properties in their respective groups)	
A.	Cl, S	I.	Elements with highest electronegativity
B.	Ge, As	II.	Elements with largest atomic size
C.	Fr, Ra	III.	Elements which show properties of both metals and non-metal
D.	F, O	IV.	Elements with highest negative electron gain enthalpy

Choose the **correct** answer from the options given below :

(1) (A) → (II) ; (B) → (I) ; (C) → (IV) ; (D) → (III)

(2) (A) → (III) ; (B) → (II) ; (C) → (I) ; (D) → (IV)

(3) (A) → (IV) ; (B) → (III) ; (C) → (II) ; (D) → (I)

(4) (A) → (II) ; (B) → (III) ; (C) → (IV) ; (D) → (I)

Ans. **NTA (3)**

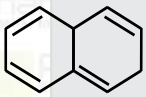
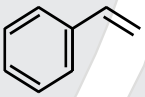
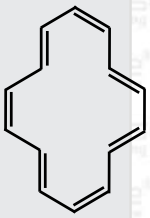
Sol. (A) Cl, S : Elements with highest negative electron gain enthalpy ($\Delta_{eg}H$ of Cl = –349 kJ/mol, $\Delta_{eg}H$ of S = –200 kJ/mol)

(B) Ge, As : Elements which show properties of both metals and non-metal (Metalloid)

(C) Fr, Ra : Elements with largest atomic size (Group number = 7 (n = 7))

(D) F, O : Elements with highest electronegativity (Element number of F = 4.0, O = 3.4)

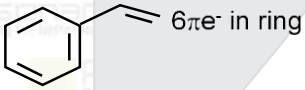
68. Which of the following are aromatic ?

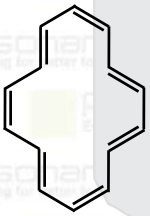
(A)  (B)  (C)  (D) 

(1) A and B only (2) A and C only (3) B and D only (4) C and D only

Ans. **NTA (3)**

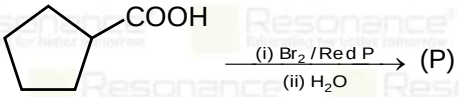
Sol. Huckel rule (4n + 2)

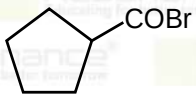
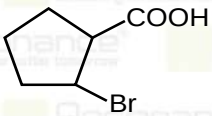
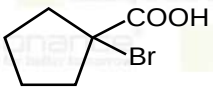
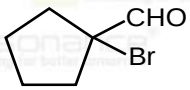




14 annulene is aromatic character of about 42%.

69. Identify the product (P) in the following reaction :

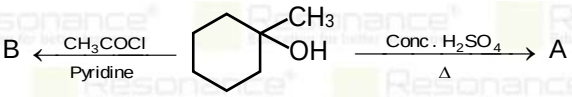


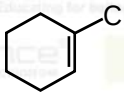
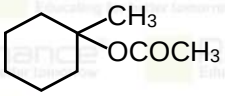
(1)  (2)  (3)  (4) 

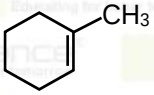
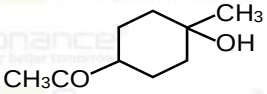
Ans. **NTA (3)**

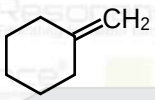
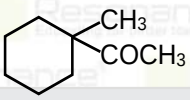
Sol. HVZ reaction is α-halogenation of carboxylic acid.

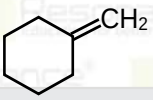
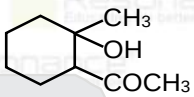
70. Identify the major products A and B respectively in the following set of reactions.



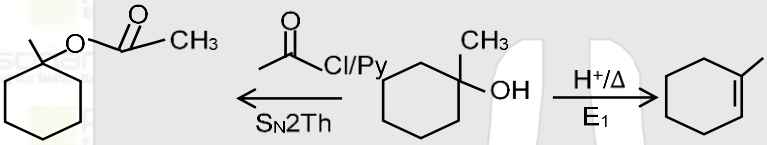
(1) A =  and B = 

(2) A =  and B = 

(3) A =  and B = 

(4) A =  and B = 

Ans. **NTA (1)**

Sol. 

71. Iron(III) catalyses the reaction between iodide and persulphate ions, in which

A. Fe^{3+} oxidises the iodide ion

B. Fe^{3+} oxidises the presulphate ion

C. Fe^{2+} reduces the iodide ion

D. Fe^{2+} reduces the persulphate ion

Choose the **most appropriate** answer from the options given below :

(1) A and D only (2) B and C only (3) B only (4) A only

Ans. **NTA (1)**

Sol. The transition metal ions can change their oxidation states, they become more effective as catalysts. For example, iron(III) catalyses the reaction between iodide and persulphate ions.

$2I^- + S_2O_8^{2-} \rightarrow I_2 + 2SO_4^{2-}$

An explanation of this catalytic action can be given as:

$2Fe^{3+} + 2I^- \rightarrow 2Fe^{2+} + I_2$

$2Fe^{2+} + S_2O_8^{2-} \rightarrow 2Fe^{3+} + 2SO_4^{2-}$

In Step I , Fe^{+3} ion oxidised I^- into I_2

In Step II , Fe^{+2} ion reduced $S_2O_8^{-2}$ into SO_4^{-2}

(Fe^{+3} I^{st} convert into Fe^{+2} then further convert in to Fe^{+3} again)

72. For the given hypothetical reactions, the equilibrium constants are as follows :

$X \rightleftharpoons Y$; $K_1 = 1.0$;

$Y \rightleftharpoons Z$; $K_2 = 2.0$;

$Z \rightleftharpoons W$; $K_3 = 4.0$

The equilibrium constant for the reaction $X \rightleftharpoons W$ is

(1) 6.0 (2) 8.0 (3) 7.0 (4) 12.0

Ans. **NTA (2)**

Sol. $X \rightleftharpoons Y$; $K_1 = 1.0$; $K_1 = \frac{[Y]}{[X]}$

$Y \rightleftharpoons Z$; $K_2 = 2.0$; $K_2 = \frac{[Z]}{[Y]}$

$Z \rightleftharpoons W$; $K_3 = 4.0$; $K_3 = \frac{[W]}{[Z]}$

Net reaction $X \rightleftharpoons W$; $K = K_1 . K_2 . K_3 = \frac{[W]}{[X]}$

$K = 1 \times 2 \times 4 = 8.0$

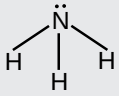
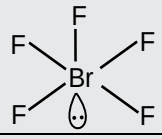
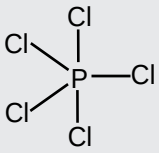
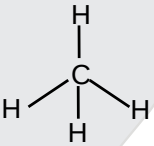
73. Match List I with List II :

List (I) (Molecule)		List (II) (Shape)	
A.	NH_3	I.	Square pyramid
B.	BrF_5	II.	Tetrahedral
C.	PCl_5	III.	Trigonal pyramidal
D.	CH_4	IV.	Trigonal bipyramidal

Choose the **correct** answer from the options given below :
(1) (A) → (III) ; (B) → (I) ; (C) → (IV) ; (D) → (II)
(2) (A) → (III) ; (B) → (IV) ; (C) → (I) ; (D) → (II)
(3) (A) → (II) ; (B) → (IV) ; (C) → (I) ; (D) → (III)
(4) (A) → (IV) ; (B) → (III) ; (C) → (I) ; (D) → (II)

Ans. **NTA (1)**

Sol.

		B.P.	L.P.	Hyb.	Shape
A.	NH_3 	3	1	sp^3	Trigonal pyramidal
B.	BrF_5 	5	1	sp^3d^2	Square pyramidal
C.	PCl_5 	5	0	sp^3d	Trigonal bipyramidal
D.	CH_4 	4	0	sp^3	Tetrahedral

74. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**:
Assertion A : The stability order of +1 oxidation state of Ga, In and Tl is Ga < In < Tl.
Assertion R : the inert pair effect stabilizes the lower oxidation state down the group.
In the light of the above statements, choose the correct answer from the options given below :
(1) A is true but R is false.
(2) Both A and R are true but R is NOT the correct explanation of A.
(3) Both A and R are true and R is the correct explanation of A.
(4) A is false but R is true.

Ans. **NTA (3)**

Sol. Down the group, due to poor shielding effect of intervening d and f orbitals, the increased effective nuclear charge holds ns electrons tightly (responsible for inter pair effect) and thereby, restricting their participation in bonding. As a result of this, only p-orbital electron may be involved in bonding. In fact in Ga, In and Tl, both +1 and +3 oxidation states are observed. The relative stability of +1 oxidation state progressively increases for heavier elements: Al<Ga<In<Tl.
Therefore option (3) is correct : Both A and R are true and R is the correct explanation of A.

75. Combustion of glucose ($C_6H_{12}O_6$) produces CO_2 and water. The amount of oxygen (in g) required for the complete combustion of 900 g of glucose is : [Molar mass of glucose in g mol^{-1} = 180]
(1) 480 (2) 32 (3) 960 (4) 800

Ans. **NTA (3)**

Sol. $C_6H_{12}O_6 + 6O_2 \longrightarrow 6CO_2 + 6H_2O$

Given w.t. 900 gm

Mole $\frac{900}{180} = 5$ mole

(From mole - mole analysis) : $\frac{\text{Mole of } C_6H_{12}O_6}{1} = \frac{\text{Mole of } O_2 \text{ required}}{6}$

Mole of O_2 required = 6 × 5 = 30 mole

g. wt. of O_2 required = 30 × (MM)O_2 = 30 × 32 = 690 gm

76. Thiosulphate reacts differently with iodine and bromine in the reactions given below :

$2S_2O_3^{2-} + I_2 \rightarrow S_4O_6^{2-} + 2I^-$

$S_2O_3^{2-} + 5Br_2 + 5H_2O \rightarrow 2SO_4^{2-} + 4Br^- + 10H^+$

Which of the following statement justifies the above dual behaviour of thiosulphate ?

(1) Thiosulphate undergoes oxidation by bromine and reduction by iodine in these reactions

(2) Bromine is a stronger oxidant than iodine

(3) Bromine is a weaker oxidant than iodine

(4) Bromine undergoes oxidation and iodine undergoes reduction in these reactions

Ans. **NTA (2)**

Sol. $2S_2O_3^{2-} + I_2 \rightarrow S_4O_6^{2-} + 2I^-$

$S_2O_3^{2-} + 5Br_2 + 5H_2O \rightarrow 2SO_4^{2-} + 4Br^- + 10H^+$

In Reaction $S_2O_3^{2-}$ with I_2, oxidation number of S charge from +2 to 2.5 ($S_2O_3^{2-}$ is oxidised in $S_4O_6^{2-}$) in reaction of $S_2O_3^{2-}$ with Br_2, oxidation number of S charge from +2 to +6 ($S_2O_3^{2-}$ is oxidised in SO_4^{2-}) in

Both reaction I_2 and Br_2 get reduced in I^- and Br^- respectively. Both behave as a oxidizing agent (oxidant)

Br_2 is strong oxidant, as compare to I_2 acc. to SRP values

$E^0_{Br_2|Br^-} = +1.09$ V ; $E^0_{I_2|I^-} = +0.54$ V

i.e. higher SRP species are good oxidant.

77. Match List I with List II :

	List (I) (Compound)		List (II) (Colour)
A.	$Fe_4[Fe(CN)_6.xH_2O$	I.	Violet
B.	$[Fe(CN)_5NOS]^{4-}$	II.	Blood Red
C.	$[Fe(SCN)]^{2+}$	III.	Pressian Blue
D.	$(NH_4)_3PO_4.12MoO_3$	IV.	Yellow

Choose the **correct** answer from the options given below :

(1) (A) → (II) ; (B) → (III) ; (C) → (IV) ; (D) → (I)

(2) (A) → (IV) ; (B) → (I) ; (C) → (II) ; (D) → (III)

(3) (A) → (I) ; (B) → (II) ; (C) → (III) ; (D) → (IV)

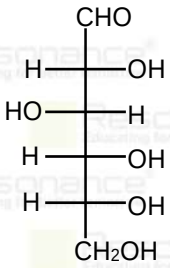
(4) (A) → (III) ; (B) → (I) ; (C) → (II) ; (D) → (IV)

Ans. **NTA (4)**

Sol.

	(Compound)		(Colour)
A.	$Fe_4[Fe(CN)_6.xH_2O$	III	Pressian Blue
B.	$[Fe(CN)_5NOS]^{4-}$	I	Violet
C.	$[Fe(SCN)]^{2+}$	II	Blood Red
D.	$(NH_4)_3PO_4.12MoO_3$	IV	Yellow

78.



The **incorrect** statement regarding the given structure is

(1) has 4 asymmetric carbon atom

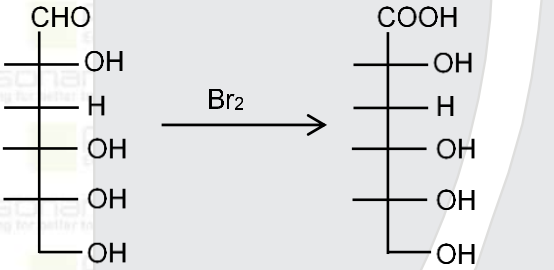
(2) can be oxidized to a dicarboxylic acid with Br_2 water

(3) will coexist in equilibrium with 2 other cyclic structure

(4) despite the presence of –CHO does not give Schiff's test

Ans. **NTA (2)**

Sol.



Monocarboxylic acid

79. An octahedral complex with the formula $CoCl_3.nNH_3$ upon reaction with excess of $AgNO_3$ solution gives 2 moles of AgCl. Consider the oxidation state of Co in the complex is 'x'. The value of "x + n" is_____.

(1) 5 (2) 3 (3) 8 (4) 6

Ans. **NTA (3)**

Sol. $CoCl_3.nNH_3 + AgNO_3(aq.) \longrightarrow AgCl\downarrow$ ppt

(1 mole) (excess) 2 mole

Octahedral complex, C.N. = 6

So complex : $[Co(NH_3)_5Cl]Cl_2$: (n = 5)

Oxidation number of Co = x + 5×(0) + (–1) + 2(–) = 0

x = +3

sum n + x = 5 + 3 = 8

80. Among the following halogens

F_2, Cl_2, Br_2 and I_2

Which can undergo disproportionation reactons ?

(1) F_2, Cl_2 and Br_2 (2) F_2 and Cl_2 (3) Only I_2 (4) Cl_2, Br_2 and I_2

Ans. **NTA (4)**

Sol. For an element of P-block, range of oxidation number = n to (n – 8)

n = Number of valance shell electron

in helogen : n = 7, So range of oxidation number of helogen = (+7) to (–1)

except : (F)

F is highest electron negative element shows only (O) and (–1) oxidation number

So F_2 not get disproportionate other helogens Cl_2, Br_2, I_2 shows –1, 0 and +1, +3, +5 oxidation so they shows disproportionation reaction only.

81. Consider the following reaction

$A + B \rightarrow C$

The time taken for A to become 1/4th of its initial concentration is twice the time taken to become ½ of the same. Also, when the change of concentration of B is plotted against time. The resulting graph gives a straight line with a negative slope and a positive intercept on the concentration axis.

The overall order of the reaction is_____.

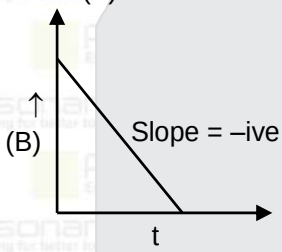
Ans. **NTA (1)**

Sol. For reaction : $A + B \rightarrow C$

w.r.t (A) : $t_{75\%} = 2 \times t_{50\%}$ acc. to question

So order of reaction w.r. to A is 1st order

w.r. to (B) :

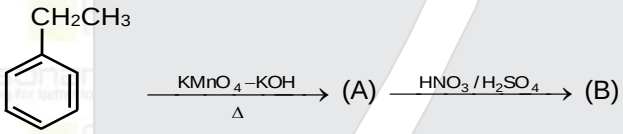


according to Question

So order of reaction w.r. to (B) is zero order

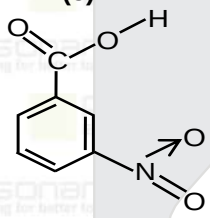
Then over all order of reaction = 1 + 0 = 1.

82. Major product B of the following reaction has_____π-bond.

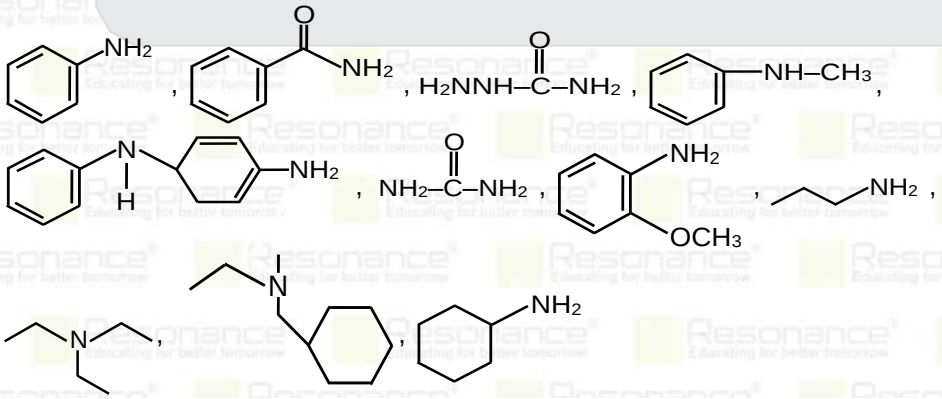


Ans. **NTA (5)**

Sol.

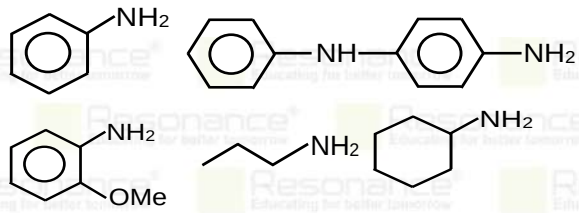


83. Number of amine compounds from the following giving solids which are soluble in NaOH upon reaction with Hinsber's reagent is_____



Ans. **NTA (5)**

Sol.



84. A solution containing 10 g of an electrolyte AB_2 in 100 g of water boils at 100.52ºC. The degree of ionization of the electrolyte (α) is_____$\times 10^{-1}$. (nearest integer)

[Given : Molar mass of AB_2 = 200 g mol^{-1}. K_b (molar boiling point elevation const. of water) = 0.52 K kg mol^{-1}. boiling point of water = 100ºC; AB_2 ionises as $AB_2 \rightarrow A^{2+} + 2B^-$]

Ans. **NTA (5)**

Sol. $AB_2 \longrightarrow A^{+2} + 2B^-$ Number of ion(n) = (3)

MM = 200 g/mole

$i = 1 + (n - 1)(\alpha) = 1 + 2(\alpha)$

K_b = 0.52 k.kg/mole

m = molality = $\frac{W_1}{Mm} \times \frac{1000}{W_2} = \frac{10 \times 1000}{200 \times 100}$ = 0.5 m

T_b = 100.52ºC , T_b^0 = 100 ºC for $H_2O(\ell)$ solvent

for a aquous solution of AB_2 elevation in B.P. =

$T_b - T_b^0 = \Delta T_b = i.Kb.m$

$100.52 - 100.00 = I \times 0.52 \times 0.5$

$i = \frac{0.52}{0.52 \times 0.5} = 2$

and $i = 1 + 2(\alpha)$

$\alpha = \frac{i-1}{2} = \frac{2-1}{2} = (0.5)$

$\alpha = 5 \times 10^{-1}$ Ans. (5)

85. The 'spini only' magnetic moment value of MO_4^{2-} is_____ BM. (Where M is a metal having least metallic radii. among Sc. Ti, V, Cr, Mn and Zn).

(Given atomic number : Sc = 21, Ti = 22, V = 23, Cr = 24, Mn = 25 and Zn = 30)

Ans. **NTA (0)**

Sol.

Element	Sc	Ti	V	Cr	Mn	Zn
Metallic radii (in pm)	164	147	135	129	137	137

Cr has least metallic radii

so given oxiamion is : CrO_4^{-2}, oxidation number of Cr = x + 4(–2) = –2 ; (x = +6)

So Cr^{+6} : $3d^0 4s^0$, No. of unpaired e^- = 0

so spin only magnetic moment $\mu_m = \sqrt{n(n+2)}$ B.M.

μ_m = 0 for CrO_4^{-2} ion Ans. (0)

86. Number of molecules from the following which are exceptions to octet rule is_____

CO_2, NO_2, H_2SO_4, BF_3, CH_4, SiF_4, ClO_2, PCl_5, BeF_2, C_2H_6, $CHCl_3$, CBr_4

Ans. **NTA (6)**

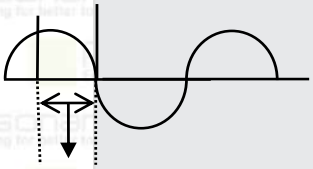
Sol.

	Number valance shell e^- on Central Atom		Number valance shell e^- on Central Atom
CO_2	8	ClO_2	11
NO_2	7	PCl_5	10
H_2SO_4	12	BeF_2	4
BF_3	6	C_2H_6	8
CH_4	8	$CHCl_3$	8
SiF_4	8	CBr_4	8

Exception to octet rule are follows in : NO_2, H_2SO_4, BF_3, ClO_2 PCl_5, BeF_2

so Ans. (6)

87. A hypothetical electromagnetic wave is show below.



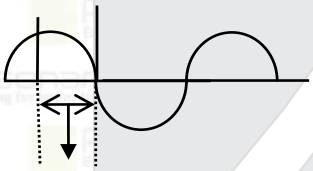
1.5 pm

The frequency of the wave is x × 10^{19} Hz.

x = _____(nearest integer)

Ans. **NTA (5)**

Sol.

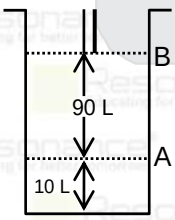


1.5 pm

So $\lambda = 1.5 \times 4 = 6.0$ pm = 6×10^{-12} m

frequency $\nu = \frac{C}{\lambda} = \frac{3\times10^8}{6\times10^{-12}} = 0.5 \times 10^{20} = 5 \times 10^{19}$ Ans. (5)

88.



Consider the figure provided.

1 mol of an ideal gas is kept in a cylinder, Fitted with a piston, at the position A, at 18ºC. If the piston is moved to position B, keeping the temperature unchanged, then 'x' L atm work is done in this reversible process.

x = _____ L atm. (nearest integer)

[Given : Absolute temperature = ºC + 273.15, R = 0.08206 L atm mol^{-1} K^{-1}]

Ans. **NTA (55)**

Sol. According to given figure process is Isothermal reversible expanston

$T = 18^oC = 291.15\ K$

$V_1 = 10\ L$

$V_2 = 100\ L = 10 + 90$

$R = 0.0821\ L.atm/mol.K$

Work done $(w) = -2.303\ RT\log\frac{V_2}{V_1}$

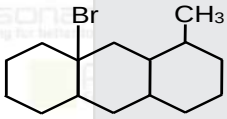
$= -2.303 \times 0.0821 \times 291.15 \times \log\frac{100}{10}$

$w = -55.01281$ atm Ans. (55)

89. If 279 g of aniline is reacted with one equivalent of benzenediazonium chloride, the maximum amount of aniline yellow formed will be_____g. (nearest integer)
(Consider complete conversion).

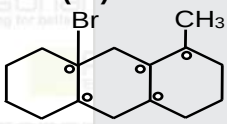
Ans. **NTA (591)**

90. The number of optical isomers in following compound is :_____



Ans. **NTA (32)**

Sol.



o marked carbon are chiral.

$2^5 = 32$