PART : MATHEMATICS

1. Let $\vec{a}=6\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a is vector such that $|\vec{c}|\geq 6, \vec{a}\cdot\vec{c}=6|\vec{c}|, |\vec{c}-\vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is 60°, then $|(\vec{a}\times\vec{c})\times\vec{c}|$ is equal to

(1) $\frac{3}{2}\sqrt{6}$ (2) $\frac{3}{2}\sqrt{3}$ (3) $\frac{9}{2}\left(6-\sqrt{6}\right)$ (4) $\frac{9}{2}\left(6+\sqrt{6}\right)$

Ans. **NTA (4)**

Reso (4)

Sol. $|(\vec{a}\times\vec{b})\times\vec{c}| = |\vec{a}\times\vec{b}||\vec{c}|\frac{\sqrt{3}}{2}$

$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 6 & 1 & -1\\ 1 & 1 & 0\end{vmatrix} = \hat{i}-\hat{j}+5\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{27}$$

$\therefore$ $|\vec{c}-\vec{a}| = 2\sqrt{2}$

$$|\vec{c}|^2+|\vec{a}|-2\vec{c}.\vec{a}=8$$

$$|\vec{c}|^2+|\vec{a}|-12|\vec{c}|=8$$

$$|\vec{c}|^2-12|\vec{c}|+30=0$$

$$|\vec{c}| = 6+\sqrt{6} \qquad \therefore \qquad |\vec{c}| > 6$$

$$|(\vec{a}\times\vec{b})\times\vec{c}| = \sqrt{27}\times(6+\sqrt{6})\times\frac{\sqrt{3}}{2} = \frac{9}{2}(6+\sqrt{6})$$

2. Let $0 \le r \le n$. If ${}^{n+1}C_{r+1} : {}^{n}C_{r} : {}^{n-1}C_{r-1} = 55 : 35 : 21$, then $2n + 5r$ is equal to:

(1) 60 (2) 62 (3) 55 (4) 50

Ans. **NTA (4)**

Reso (4)

Sol. $$\frac{{}^{n+1}C_{r+1}}{{}^{n}C_{r}} = \frac{55}{35} \Rightarrow \frac{n+1}{r+1} = \frac{11}{7} \quad ....(1)$$

$$\frac{{}^{n}C_{r}}{{}^{n-1}C_{r-1}} = \frac{35}{21} \Rightarrow \frac{n}{r} = \frac{5}{3} \quad ....(2)$$

From (1) and (2)

n = 10, r = 6

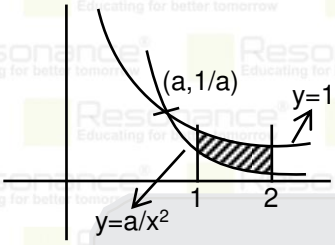
Now, $2n + 5r = 50$.

3. If the area of the region $\left\{(x,y) : \frac{a}{x^2} \le y \le \frac{1}{x}, 1 \le x \le 2, 0 < a < 1\right\}$ is $(\log_e 2) - \frac{1}{7}$ then the value of $7a - 3$ is equal to:

(1) –1 (2) 0 (3) 1 (4) 2

Ans. **NTA (1)**
Reso (1)

Sol. Required Area $= \int_1^2 \left(\frac{1}{x} - \frac{a}{x^2}\right)dx = \left(\ln x + \frac{a}{x}\right)_1^2 = \ln 2 + \frac{a}{2} - a$



Now, $\ln 2 + \frac{a}{2} - a = \ln 2 - \frac{1}{7}$

$\frac{a}{2} = \frac{1}{7} \quad \Rightarrow \quad 7a = 2$ and $7a - 3 = -1$.

4. If three letters can be posted to any one of the 5 different addresses, then the probability that the three letters are posted to exactly two addresses is:

(1) $\frac{18}{25}$ (2) $\frac{4}{25}$ (3) $\frac{6}{25}$ (4) $\frac{12}{25}$

Ans. **NTA (4)**
Reso (4)

Sol. Total out comes $= 5^3$

Favourable outcomes $= {}^5C_2\,(2^3 - 2) = 10 \times 6 = 60$

Required probability $= \frac{60}{125} = \frac{12}{25}$.

5. If z_1, z_2 are two distinct complex number such that $\left|\frac{z_1 - 2z_2}{\frac{1}{2} - z_1\bar{z}_2}\right| = 2$, then

(1) either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$

(2) both z_1 and z_2 lies on the same circle]

(3) either z_1 lies on a circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1.

(4) z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1.

Ans. **NTA (1)**
Reso (1)

Sol. $|z_1 - 2z_2|^2 = 4\left|\frac{1}{2} - z_1\bar{z}_2\right|^2$

$(z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = 4\left(\frac{1}{2} - z_1\bar{z}_2\right)\left(\frac{1}{2} - \bar{z}_1 z_2\right)$

$|z_1|^2 + 4|z_2|^2 - 4|z_1|^2|z_2|^2 = 1$

$(|z_1|^2 - 1)\left(4|z_2|^2 - 1\right) = 0$

$\Rightarrow \quad |z_1| = 1 \quad$ or $\quad |z_2| = \frac{1}{2}$

6. Let ABC be an equilateral triangle. A new triangle is formed by joining the middle points of all sides of the triangle ABC and the same process is repeated infinitely many times. If P is the sum of perimeters and Q is be the sum of area of all the triangles formed in this process, then

(1) $P^2 = 36\sqrt{3}Q$ (2) $P = 36\sqrt{3}Q^2$ (3) $P^2 = 72\sqrt{3}Q$ (4) $P^2 = 6\sqrt{3}Q$

Ans. **NTA (1)**

Reso (1)

Sol. $P = 3a\left(1+\frac{1}{2}+\frac{1}{4}+.....\right)$ a = length of sides of ΔABC

$= 3a\left(\frac{1}{1-\frac{1}{2}}\right) = 6a$

$Q = \frac{\sqrt{3}}{4}a^2\left(1+\frac{1}{4}+\frac{1}{16}+....\right) = \frac{\sqrt{3}}{4}a^2\left(\frac{1}{1-\frac{1}{4}}\right) = \frac{a^2}{\sqrt{3}}$

$\frac{P^2}{Q} = \frac{36a^2\times\sqrt{3}}{a^2} = 36\sqrt{3}$

7. Suppose for a differentiable function h, h(0) = 0, h(1) = 1 and h' (0) = h' (1) = 2. If $g(x) = h(e^x)\, e^{h(x)}$ then g' (0) is equal to:

(1) 8 (2) 4 (3) 3 (4) 5

Ans. **NTA (2)**

Reso (2)

Sol. $g(x) = h(e^x).e^{h(x)}$

$g'(x) = h(e^x).e^{h(x)}.h'(x) + e^{h(x)}.h'(e^x)e^x$

$g'(0) = h(1)\ 2 + 2 = 2 + 2 = 4.$

8. Let A = {1, 2, 3, 4, 5}. Let R be a relation on A defined by xRy if and only if $4x \le 5y$. Let m be the number of elements in R and n be the minimum number of elements from A × A that are required to be added to R to make it a symmetric relation. Then m + n is equal to:

(1) 26 (2) 25 (3) 24 (4) 23

Ans. **NTA (2)**

Reso (2)

Sol. xRy is given by $4x \le 5y$

R = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (2, 2) (2, 3) (2, 4) (2, 5) (3, 3) (3, 4) (3, 5) (4, 4) (4, 5) (5, 4) (5, 5)}

$\Rightarrow$ m = 16

To make symmetric relation add elements:

(2, 1) (3, 1) (4, 1) (5, 1) (3, 2) (4, 2) (5, 2) (4, 3) (5, 3).

$\Rightarrow$ n = 9

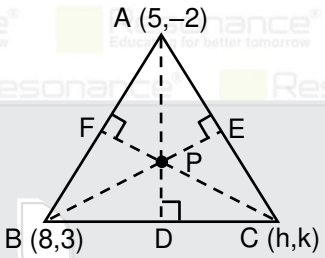
9. If P(6, 1) be the orthocentre of the triangle whose vertices are A(5, –2). B(8, 3) and C(h, k) then the point C lies on the circle:

(1) $x^2 + y - 61 = 0$ (2) $x^2 + y^2 - 65 = 0$ (3) $x^2 + y^2 - 74 = 0$ (4) $x^2 + y^2 - 52 = 0$

Ans. **NTA (2)**
Reso (2)

Sol. Orthocentre P(6,1)

AP $\perp^r$ BC



$$\left(\frac{1+2}{6-5}\right)\left(\frac{K-3}{h-8}\right) = -1$$

$\Rightarrow$ $h + 3k = 17$ ....(1)

BP $\perp^r$ AC

$$\left(\frac{8-6}{3-1}\right)\left(\frac{k+2}{h-5}\right) = -1$$

$\Rightarrow$ $h + k = 3$ ....(2)

From (1) and (2)

$h = -4, k = 7$

Vertex C (–4, 7)

Which lies on $x^2 + y^2 = 65$.

10. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}, \vec{b} = \left(\left(\vec{a} \times \left(\hat{i} + \hat{i}\right)\right) \times \hat{i}\right) \times \hat{i}$. Then the square of the projection of $\vec{a}$ on $\vec{b}$ is:

(1) 2 (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{1}{5}$

Ans. **NTA (1)**
Reso (1)

Sol. $$\vec{a} \times (\hat{i} + \hat{j}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix} = \hat{i} - \hat{j} + \hat{k}$$

$$(\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i} = \hat{k} + \hat{j} \quad \Rightarrow \quad \left(\left(\vec{a} \times \left(\hat{i} + \hat{i}\right)\right) \times \hat{i}\right) \times \hat{i} = \hat{j} - \hat{k} = \vec{b}$$

Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{2}{\sqrt{2}} = \sqrt{2}$

11. If all the word with or without meaning made using all the letters of the word "NAGPUR are arranged as in a dictionary, then the word at 315th position in this arrangement is:

(1) NRAPUG (2) NRAGPU (3) NRAGUP (4) NRAPGU

Ans. **NTA (4)**
Reso (4)

Sol.

NAGPUR		
A		= 5! = 120
G		= 5! = 120
N	A	= 4! = 24
N	G	= 4! = 24
N	P	= 4! = 24
		312 (Total)

NRAGPU $\rightarrow$ 313th word.
NRAGUP $\rightarrow$ 314th word.
NRAPGU $\rightarrow$ 315th word.

12. If the function $f(x) = \left(\frac{1}{x}\right)^{2x}$; $x > 0$ attains the maximum value at $x = \frac{1}{e}$ then :

(1) $e^{2\pi} < (2\pi)^e$ (2) $(2e)^{\pi} > \pi^{(2e)}$ (3) $e^{\pi} > \pi^e$ (4) $e^{\pi} < \pi^e$

Ans. **NTA (3)**

Reso (3)

Sol. $f(x) = \left(\frac{1}{x}\right)^{2x}$

$f\left(\frac{1}{e}\right) > f\left(\frac{1}{\pi}\right)$

$e^{2/e} > \pi^{2/\pi}$

$e^{\pi} > \pi^e$

13. If $\int \frac{1}{a^2\sin^2 x + b^2\cos^2 x}dx = \frac{1}{12}\tan^{-1}(3\tan x)$ + constant, then the maximum value of $a\sin x + b\cos x$, is:

(1) $\sqrt{41}$ (2) $\sqrt{42}$ (3) $\sqrt{39}$ (4) $\sqrt{40}$

Ans. **NTA (4)**

Reso (4)

Sol. Let $I = \int \frac{dx}{a^2\sin^2 x + b^2\cos^2 x}$

divide by $\cos^2 x$

$I = \int \frac{\sec^2 x\,dx}{a^2\tan^2 x + b^2}$

Let $\tan x = t$

$\sec^2 x dx = dt$

$$= \int \frac{dt}{a^2t^2 + b^2} = \frac{1}{a^2}\int \frac{dt}{t^2 + \frac{b^2}{a^2}} = \frac{1}{ab}\tan^{-1}\left(\frac{a}{b}t\right) + C$$

Now compare,

$ab = 12$ and $\frac{a}{b} = 3$

$\Rightarrow$ $a = 6, b = 2$

Now, $6\sin x + 2\cos x$

Its maximum value = $\sqrt{6^2 + 2^2} = \sqrt{40}$

14. $$\lim_{n\to\infty} \frac{(1^2-1)(n-1) + (2^2-2)(n-2) + \ldots\ldots\ldots + ((n-1)^2 - (n-1))1}{(1^3 + 2^3 + \ldots\ldots\ldots + n^3) - (1^2 + 2^2 + \ldots\ldots\ldots + n^2)}$$

(1) $\frac{2}{3}$ (2) $\frac{3}{4}$ (3) $\frac{1}{3}$ (4) $\frac{1}{2}$

Ans. **NTA (3)**

Reso (3)

Sol. $$\lim_{n\to\infty}\frac{n\Sigma r^2-n\Sigma r-\Sigma r^3+\Sigma r^2}{\Sigma r^3-\Sigma r^2}$$

$$\Rightarrow \lim_{n\to\infty}\frac{\frac{(n^2)(n+1)(2n+1)}{6}-\frac{n(n)(n+1)}{2}-\left(\frac{n(n+1)}{2}\right)^2+\frac{n(n+1)(2n+1)}{6}}{\left(\frac{n(n+1)}{2}\right)^2-\frac{n(n+1)(2n+1)}{6}}$$

Required limit is coefficient of n^4

$$=\frac{\left(\frac{2}{6}-\frac{1}{4}\right)}{\frac{1}{4}}=\frac{1}{3}$$ Answer (3).

15. If A is a square matrix of order 3 such that det(A)=3 and det (adj (–4 adj (–3 adj (3adj $(2A)^{-1}$)))) = $2^m 3^n$, then m + 2n is equal to

(1) 4 (2) 6 (3) 3 (4) 2

Ans. **NTA (1)**
Reso (1)

Sol. $|\text{adj}(-4\,\text{adj}(-3\,\text{adj}(3\text{adj}(2A)^{-1})))|$

$= |-4\,\text{adj}(-3\,\text{adj}(3\text{adj}(2A)^{-1}))|^2$

$= (-4)^6\,|\text{adj}(-3\,\text{adj}(3\,\text{adj}(2A)^{-1}))|^2$

$= 4^6.(-3)^{12}\,|\text{adj}(3\,\text{adj}(2A)^{-1})|^4$

$= 4^6.3^{12}.3^{24}\,|\text{adj}(2A)^{-1}|^8$

$= 4^6.3^{36}\,|(2A)^{-1}|^{16}$

$= 4^6.3^{36}.2^{-48}\left(\frac{1}{|A|}\right)^{16}=\frac{4^6.3^{36}.2^{-48}}{3^{16}}=2^{-36}.3^{20}$

$\Rightarrow$ m = – 36, n = 20

m + 2n = – 36 + 40 = 4.

16. Suppose the solution of the differential equation $\frac{dy}{dx}=\frac{(2+\alpha)x-\beta y+2)}{\beta x-2\alpha y-(\beta y-4\alpha)}$ represents a circle passing through origin. Then the radius of this circle is:

(1) 2 (2) $\frac{\sqrt{17}}{2}$ (3) $\sqrt{17}$ (4) $\frac{1}{2}$

Ans. **NTA (2)**
Reso (2)

Sol. $\frac{dy}{dx}=\frac{(2+\alpha)x-\beta y+2)}{\beta x-y(2\alpha+\beta)+4\alpha}$

$\beta(xdy+ydx)-y(2\alpha+\beta)dy+4\alpha dy=(2+\alpha)xdx+2dx$

$\beta\, d(xy)-y(2\alpha+\beta)dy+4\alpha dy=(2+\alpha)xdx+2dx$

Integration on both sides

$\beta xy-(2\alpha+\beta)\frac{y^2}{2}+4\alpha y=(2+\alpha)\frac{x^2}{2}+2x+c$

It represent a circle passing through (0, 0)

$\Rightarrow$ $\beta = 0$, c = 0 and $2+\alpha=2\alpha+\beta \Rightarrow \alpha=2$

Equation of circle is : $2x^2+2y^2+2x-8y=0$ or $x^2+y^2+x-4y=0$

Radius = $\sqrt{\frac{1}{4}+4}=\frac{\sqrt{17}}{2}$.

17. A software company sets up m number of computer systems to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day, 4 more computer systems crashed on the start of the third day and so on, then it took 8 more days to finish the assignment. The value of m is equal to:

(1) 160 (2) 150 (3) 180 (4) 125

Ans. **NTA (2)**
Reso (2)

Sol. $17m = m + (m-4) + (m-8) + (m-12) +$ ---------- 25 terms

$$17m = \frac{25}{2}[2m + (24)(-4)]$$

$17m = 25m - 1200$

$8m = 1200$

$m = 150$

18. Let $f(x) = \frac{1}{7-\sin 5x}$ be a function defined on R. Then the range of the function f(x) is equal to:

(1) $\left[\frac{1}{7},\frac{1}{6}\right]$ (2) $\left[\frac{1}{8},\frac{1}{6}\right]$ (3) $\left[\frac{1}{8},\frac{1}{5}\right]$ (4) $\left[\frac{1}{7},\frac{1}{5}\right]$

Ans. **NTA (2)**
Reso (2)

Sol. $f(x) = \frac{1}{7-\sin x}$, $x \in R$

$-1 \le \sin 5x \le 1$

$-1 \le -\sin 5x \le 1$

$6 \le 7 - \sin 5x \le 8$

$$\frac{1}{8} \le \frac{1}{7-\sin 5x} \le \frac{1}{6}$$

$$\frac{1}{8} \le f(x) \le \frac{1}{6}$$

19. if the locus of the point, whose distance from the point (2, 1) and (1, 3) are in the ratio 5 : 4, is $ax^2 + by^2 + cxy + dx + ey + 170 = 0$, then the value of $a^2 + 2b + 3c + 4d + e$ is equal to

(1) –27 (2) 37 (3) 5 (4) 437

Ans. **NTA (2)**
Reso (2)

Sol. Let variable point is P(h, k)

$$\frac{PA}{PB} = \frac{5}{4}$$

$16PA^2 = 25PB^2$

$16[(h-2)^2 + (k-1)^2] = 25[(h-1)^2 + (k-3)^2]$

$16h^2 + 16k^2 - 64h - 32k + 80 = 25h^2 + 25k^2 - 50h - 150k + 250$

$9h^2 + 9k^2 + 14h - 118k + 170 = 0$

Locus of P(h,k) is

$9x^2 + 9y^2 + 14x - 118y + 170 = 0$

Now $a^2 + 2b + 3c + 4d + e$

$= 81 + 18 + 0 + 56 - 118$

$= 155 - 118 = 37$

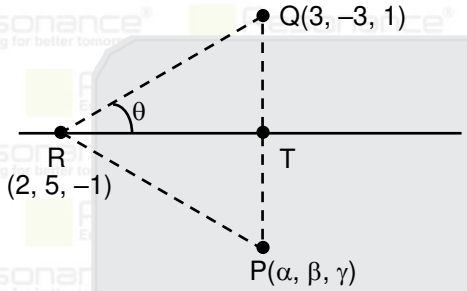
20. Let $P(\alpha, \beta, \gamma)$ be the image of the point Q(3, –3, 1) in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and R be the point (2, 5, –1). If the area of the triangle PQR is λ and $\lambda^2 = 14K$, then K is equal to:

(1) 36 (2) 72 (3) 81 (4) 18

Ans. **NTA (3)**

Reso (3)

Sol.



$QR = \sqrt{(3-2)^2 + (-3-5)^2 + (1+1)^2}$

$= \sqrt{1+64+4} = \sqrt{69}$

$\overrightarrow{RQ} = \hat{i} - 8\hat{j} + 2\hat{k}$

$\overrightarrow{RT} = \hat{i} + \hat{j} - \hat{k}$

$\cos\theta = \frac{1-8-2}{\sqrt{1+64+4}\sqrt{1+1+1}} = \frac{-9}{\sqrt{69}\sqrt{3}} = \frac{-9}{3\sqrt{23}} = \frac{-3}{\sqrt{23}}$

$\sin\theta = \frac{\sqrt{14}}{\sqrt{23}}$

$\sin\theta = \frac{QS}{RQ} = \frac{\sqrt{14}}{\sqrt{23}}$ and $\cos\theta = \frac{RS}{RQ} = \frac{3}{\sqrt{23}}$

$QS = \frac{\sqrt{14}}{\sqrt{23}} \times \sqrt{69} = \sqrt{42}$ $RS = \frac{3}{\sqrt{23}} \times \sqrt{69} = 3\sqrt{3}$

Area of $\Delta PQR = \frac{1}{2} \times (PQ)(RS)$

$= \frac{1}{2} \times 2\sqrt{42} \times 3\sqrt{3}$

$\lambda = 9\sqrt{14}$

$\lambda^2 = 81 \times 14 = 14K$

$\therefore K = 81$

21. Let [t] denote the largest integer less than or equal to t. If $\int_0^3 \left(\left[x^2 \right] + \left[\frac{x^2}{2} \right] \right) dx = a + b\sqrt{2} - \sqrt{3} - \sqrt{5} + c\sqrt{6} - \sqrt{7}$, where a, b, c, $\in$ Z, then a + b + c is equal to_____

Ans. **NTA (23)**
Reso (23)

Sol. $\int_0^3 [x^2]dx + \int_0^3 \left[\frac{x^2}{2} \right] dx$

$= \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^{2} 3\,dx + \int_2^{\sqrt{5}} 4\,dx + \int_{\sqrt{5}}^{\sqrt{6}} 5\,dx + \int_{\sqrt{6}}^{\sqrt{7}} 6\,dx + \int_{\sqrt{7}}^{\sqrt{8}} 7\,dx + \int_{\sqrt{8}}^{3} 8\,dx + \int_0^{\sqrt{2}} 0\,dx + \int_{\sqrt{2}}^{2} 1\,dx + \int_2^{\sqrt{6}} 2\,dx + \int_{\sqrt{6}}^{\sqrt{8}} 3\,dx + \int_{\sqrt{8}}^{3} 4\,dx$

$= 0 + \left(\sqrt{2} - 1\right) + 2\left(\sqrt{3} - \sqrt{2}\right) + 3\left(2 - \sqrt{3}\right) + 4\left(\sqrt{5} - 2\right) + 5\left(\sqrt{6} - \sqrt{5}\right) + 6\left(\sqrt{7} - \sqrt{6}\right) + 7\left(\sqrt{8} - \sqrt{7}\right) + 8\left(3 - \sqrt{8}\right) + 0 + \left(2 - \sqrt{2}\right) + 2\left(\sqrt{6} - 2\right) + 3\left(\sqrt{8} - \sqrt{6}\right) + 4\left(3 - \sqrt{8}\right)$

$= 31 - 6\sqrt{2} - \sqrt{3} - \sqrt{5} - 2\sqrt{6} - \sqrt{7}$

So a = 31, b = –6, c = –2
a + b + c = 23

22. In a triangle ABC, BC = 7, AC = 8, AB = $\alpha \in$ N and $\cos A = \frac{2}{3}$. If $49\cos(3C) + 42 = \frac{m}{n}$, where gcd (m, n) = 1, then m + n is equal to_____.

Ans. **NTA (39)**
Reso (39)

Sol. $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\frac{2}{3} = \frac{64 + \alpha^2 - 49}{2(8)(\alpha)}$

$\Rightarrow \quad 32\,\alpha = 3\,(15 + \alpha^2)$

$\Rightarrow \quad 3\alpha^2 - 32\alpha + 45 = 0$

$\Rightarrow \quad (3\alpha - 5)\,(\alpha - 9) = 0$

$\Rightarrow \quad \alpha = \frac{5}{3}, 9 \qquad$ But $\alpha \in$ N

So $\alpha = 9$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{49 + 64 - 81}{2(7)(8)} = \frac{2}{7}$

$49\cos 3C + 42 = 49\,(4\cos^3 C - 3\cos C) + 42$

$= 49\left(\frac{32}{49 \times 7} - \frac{6}{7} \right) + 42$

$= \frac{32}{7} - 42 + 42$

$= \frac{32}{7} = \frac{m}{n}$

So m + n = 32 + 7 = 39

23. The length of the latus rectum and directrices of a hyperbola with eccentricity e are 9 and $x = \pm\frac{4}{\sqrt{3}}$, respectively. Let the line $y - \sqrt{3}x + \sqrt{3} = 0$ touch this hyperbola at (x_0, y_0). If m is the product of the focal distances of the point (x_0, y_0), then $4e^2 + m$ is equal to____

Ans. **NTA (61)**

Ans. **Reso (Bonus)**

Sol. Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Length of LR = $\frac{2b^2}{a} = 9$

Directrix $x = \pm\frac{a}{e} = \pm\frac{4}{\sqrt{3}}$

Equation of tangent

$y - \sqrt{3}x + \sqrt{3} = 0$

$y = \sqrt{3}x - \sqrt{3}$

$c^2 = a^2 m^2 - b^2$

$3 = 3a^2 - \frac{9a}{2}$

$\Rightarrow 2a^2 - 3a - 2 = 0$

$\Rightarrow (a - 2)(2a + 1) = 0$

$\Rightarrow a = 2, b^2 = 9$

Equation of hyperbola is $\frac{x^2}{4} - \frac{y^2}{9} = 1$

point of contact will be $\left(4, 3\sqrt{3}\right) = (x_0, y_0)$

$e = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$

Product of focal distances = m = $(ex_0 + a)(ex_0 - a)$

$\Rightarrow m = e^2x_0^2 - a^2$

$\Rightarrow m = 16e^2 - 4$

$\Rightarrow m + 4e^2 = 20e^2 - 4 = 20 \times \frac{13}{4} - 4 = 65 - 4 = 61$

Here equation of directrix should be $x = \pm\frac{4}{\sqrt{13}}$

But in given question it is given $x = \pm\frac{4}{\sqrt{3}}$ which is wrong because eccentricity should be greater than 1

So question is bonus

24. If the system of equations

$2x + 7y + \lambda x = 3$

$3x + 2y + 5z = 4$

$x + \mu y + 32z = -1$

has infinitely many solutions, then $(\lambda - \mu)$ is equal to_____.

Ans. **NTA (38)**

Reso (38)

Sol. For infinitely many solutions $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta_3 = \begin{vmatrix} 2 & 7 & 3 \\ 3 & 2 & 4 \\ 1 & \mu & 1 \end{vmatrix} = 0 \Rightarrow 1(28-6) - \mu(8-9) - 1(4-21) = 0$$

$\Rightarrow \quad 22 + \mu + 17 = 0$

$\Rightarrow \quad \mu = -39$

$$\Delta = \begin{vmatrix} 2 & 7 & \lambda \\ 3 & 2 & 5 \\ 1 & -39 & 32 \end{vmatrix} = 0$$

$\Rightarrow \quad 2(64 + 195) - 7(96 - 5) + \lambda(-119 - 2) = 0$

$\Rightarrow \quad 518 - 637 - 121\lambda = 0$

$\Rightarrow \quad 121\lambda = -121$

$\Rightarrow \quad \lambda = -1$

$\lambda - \mu = -1 + 39 = 38$

25. From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Let items in the sample be drawn one by one without replacement. If variance of X is $\frac{m}{n}$, where gcd(m, n) = 1, then n – m is equal to

Ans. **NTA (71)**
Reso (71)

Sol.

X	0	1	2	3
P(X)	$\frac{{}^9C_5}{{}^{12}C_5}$	$\frac{{}^3C_1 \times {}^9C_4}{{}^{12}C_5}$	$\frac{{}^3C_2 \times {}^9C_3}{{}^{12}C_5}$	$\frac{{}^3C_3 \times {}^9C_2}{{}^{12}C_5}$

$$\text{Mean} = \frac{\Sigma p_1 x_i}{\Sigma p_i} = \frac{{}^3C_1 \times {}^9C_4 + 2.{}^3C_2 \times {}^9C_3 + 3.{}^3C_3 \times {}^9C_2}{{}^{12}C_5} = \frac{5}{4}$$

Variance $\sigma^2 = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2$

$$= \frac{{}^3C_1 \times {}^9C_4 + 4.{}^3C_2 \times {}^9C_3 + 9.{}^3C_3 \times {}^9C_2}{{}^{12}C_5} - \left(\frac{5}{4}\right)^2$$

$$\sigma^2 = \frac{105}{176} = \frac{m}{n}$$

So, n – m = 176 – 105 = 71.

26. Let α, β be roots of $x^2 + \sqrt{2}x - 8 = 0$. If $U_n = \alpha^n + \beta^n$, then $\frac{U_{10} + \sqrt{2}U_9}{2U_8}$ is equal to___.

Ans. **NTA (4)**
Reso (4)

Sol. $x^2 + \sqrt{2}x - 8 = 0$

$U_n = \alpha^n + \beta^n$

By newtons formula

$U_{n+2} + \sqrt{2}U_{n+1} - 8U_n = 0$

Put n = 8

$$U_{10} + \sqrt{2}U_9 - 8U_8 = 0$$

$$\Rightarrow \quad \frac{U_{10} + \sqrt{2}U_9}{2U_8} = 4.$$

27. Let [t] denote the greatest integer less than or equal to t. Let f : [0, ∞) → R be a function defined by $f(x) = \left[\frac{x}{2}+3\right] - \left[\sqrt{x}\right]$. Let S be the set of all points in the interval [0, 8] at which f is not continuous. Then $\sum_{a\in S} a$ is equal to

Ans. **NTA (17)**

Reso (17)

Sol. $f(x) = \left[\frac{x}{2}+3\right] - [\sqrt{x}]$

$S \in [0,8]$

$\left[\frac{x}{2}+3\right]$ is discontinuous at x = 2,4,6,8

$\left[\sqrt{x}\right]$ is discontinuous at x = 1,4

f(4) = 5 – 2 = 3

$f(4^-)$ = 4 – 1 = 3

$f(4^+)$ = 5 – 2 = 3

f(x) is continuous at x = 4

So, f(x) is discontinuous at x = 1,2,6,8

$\sum_{a\in S} a = 1+2+6+8 = 17$

28. If the shortest distance between the lines $\frac{x-\lambda}{3} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $\frac{x+2}{-3} = \frac{y+5}{2} = \frac{z-4}{4}$ is $\frac{44}{\sqrt{30}}$, then the largest possible value of $|\lambda|$ is equal to_____.

Ans. **NTA (43)**

Reso (43)

Sol. $L_1 : \vec{r} = \lambda\hat{i} + 2\hat{j} + \hat{k} + k(-3\hat{i} - \hat{j} + \hat{k})$

$L_2 : \vec{r} = -2\hat{i} - 5\hat{j} + 4\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k})$

Shortest distance between L_1 and L_2

$= \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right| = \frac{44}{\sqrt{30}}$ Here, $\vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} = -6\hat{i} - 15\hat{j} + 3\hat{k}$

$\Rightarrow \left|\frac{((-2-\lambda)\hat{i} - 7\hat{j} + 3\hat{k}).(-6\hat{i} - 15\hat{j} + 3\hat{k})}{\sqrt{36+225+9}}\right| = \frac{44}{\sqrt{30}}$

$\Rightarrow \left|\frac{12 + 6\lambda + 105 + 9}{3\sqrt{30}}\right| = \frac{44}{\sqrt{30}}$

$\Rightarrow |6\lambda + 126| = 132$

$\Rightarrow \lambda = 1, -43$

Largest value of $|\lambda|$ = 43.

29. If the solution y(x) of the given differential equation $(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$ passes through the point $\left(\frac{\pi}{2},0\right)$, then the value of $e^{y\left(\frac{\pi}{6}\right)}$ is equal to_____.

Ans. **NTA (3)**
Reso (3)

Sol. $(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$

$\Rightarrow d((e^y+1)\sin x) = 0$

$\Rightarrow (e^y+1)\sin x = c$

it passes through $\left(\frac{\pi}{2},0\right)$

So, $c = 2$

$\therefore (e^y+1)\sin x = 2$

at $x = \frac{\pi}{6}$

$\left(e^y+1\right)\frac{1}{2} = 2$

$(e^y + 1) = 4$

$\Rightarrow e^y = 3$

30. If $S(x) = (1 + x) + 2(1 + x)^2 + 3(1 + x)^3 + \ldots\ldots\ldots + 60(1 + x)^{60}$, $x \neq 0$, and $(60)^2 S(60) = a(b)^b + b$, where $a, b \in N$, then $(a + b)$ equal to_____

Ans. **NTA (3660)**
Reso (3660)

Sol. $S = (1 + x) + 2(1 + x)^2 + (1 + x)^3 + \ldots\ldots\ldots + 60(1 + x)^{60}$ $\quad x \neq 0$

$(1 + x)S = (1 + x)^2 + 2(1 + x)^3 + \ldots\ldots + 59(1 + x)^{60} + 60(1 + x)^{61}$

$\therefore -xS = (1 + x) + (1 + x)^2 + (1 + x)^3 + \ldots\ldots\ldots + (1 + x)^{60} - 60(1 + x)^{61}$

when $x = 60$

$-60S = 61 + (61)^2 + (61)^3 + \ldots\ldots + (61)^{60} - 60(61)^{61}$

$-60S = \frac{61\left((61)^{60} - 1\right)}{60} - 60(61)^{61}$

$\Rightarrow -60^2 S = (61)^{61} - 61 - (60)^2 (61)^{61}$

$\Rightarrow (60)^2 S = (3599)(61)^{61} + 61$

$\therefore a = 3599, b = 61$

$a + b = 3660$

PART : PHYSICS

31. Given below are two statements:

Statement (I): Dimensions of specific heat is $[L^2T^{-2}K^{-1}]$.

Statement (II): Dimensions of gas constant is $[ML^2T^{-1}K^{-1}]$.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both statement (I) and statement (II) are correct.

(2) Statement (I) is incorrect but statement (II) is correct.

(3) Both statement (I) and statement (II) are incorrect

(4) Statement (I) is correct but statement (II) is incorrect

Ans. **(4)**

Sol. $Q = ms\Delta T$

$M^1L^2T^{-2} = [M][S][K] \Rightarrow [S] = M^0L^2T^{-2}K^{-1}$

$PV = nRT$

$$\left(\frac{M^1L^1T^{-2}}{L^2}\right)(L^3) = (mol)[R][K]$$

$[R] = [M^1L^2T^{-2}]\ (mol)^{-1}K^{-1}$

32. In finding out refractive index of glass slab the following observations were made through travelling microscope 50 vernier scale division 49 MSD; 20 divisions on main scale in each cm

For mark on paper

MSR 8.45 cm, VC=26

For mark on paper seen through slab

MSR 7.12 cm, VC=41

For powder particle on the top surface of the glass slab

MSR 4.05 cm, VC=1

(MSR Main Scale Reading, VC = Vernier Coincidence)

Refractive index of the glass slab is:

(1) 1.35 (2) 1.42 (3) 1.52 (4) 1.24

Ans. **(2)**

Sol. 20 MSD = 10mm

$$1\ MSD = \frac{10}{20} = 0.5\ mm$$

50 VSD = 49 MSD = 49 × 0.5 mm

$$1\ VSD = \frac{49}{50} \times 0.5\ mm$$

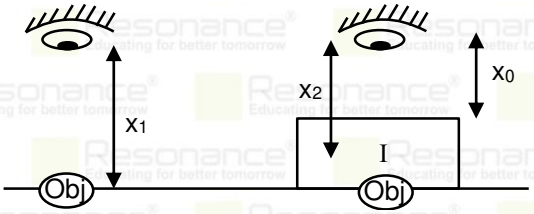
$$L.C. = 1\ MSD - 1\ VSD = (0.5\ mm) - \left(\frac{40}{50} \times 0.5 mm\right)$$

L.C. = 0.01 mm

X_1 = MSR + (VSR)(LC) = 84.5 mm + (0.01 mm) 26 = 84.76 mm

X_2 = MSR + (VSR)(LC) = 71.2 mm + (0.01 mm) 41 = 71.61 mm

X_0 = MSR + (VSR)(LC) = 40.5 mm + (0.01 mm) (1) = 40.51 mm



$$\mu = \frac{\text{actual depth}}{\text{appearent depth}} = \frac{x_1 - x_0}{x_2 - x_0}$$

$$\mu = \frac{84.76 - 40.51}{71.61 - 40.51} = \frac{44.25\ mm}{31.1\,mm}$$

h = 1.42

33. The acceptor level of a p-type semiconductor is 6 eV. The maximum wavelength of light which can create a hole would be Given hc = 1242 eV nm.

(1) 103.5 nm (2) 207 nm (3) 407 nm (4) 414 nm

Ans. **(2)**

Sol. Energy of photon > Energy gap

$$\frac{hc}{\lambda} > 6eV \Rightarrow \lambda < \frac{hc}{6eV} = \frac{1242\ eV.nm}{6eV}$$

$\lambda < 207$ nm $\Rightarrow \lambda_{max} = 207$ nm

34. In the given electromagnetic wave $E_y = 600 \sin(\omega t - kx)\ Vm^{-1}$, intensity of the associated light beam is (in W/m^2: (Given $\varepsilon_0 = 9\times10^{-12}\ C^2N^{-1}m^{-2}$)

(1) 972 (2) 486 (3) 243 (4) 729

Ans. **(2)**

Sol. $E_0 = 600$ V/m

Average electrical energy density = $\frac{1}{2}\varepsilon_0 E_{rms}^2$

$$= \frac{1}{2}\varepsilon_0\left(\frac{E_0}{\sqrt{2}}\right)^2 = \left(\frac{1}{4}\varepsilon_0 E_0^2\right)$$

Average magnetic energy density will also be the same, so average total energy density

$$u_d = \left(\frac{1}{4}\varepsilon_0 E_0^2\right)\times 2 = \frac{1}{2}\varepsilon_0 E_0^2$$

From ICU $\frac{I}{C} = u_d \Rightarrow I = (u_d)(C)$

$$I = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)\times C = \frac{1}{2}\ (9\times10^{-12})(600)^2 \times 3\times10^8$$

$I = 486\ W/m^2$,

35. Assuming the earth to be a sphere of uniform mass density, a body weighed 300 N on the surface of earth. How much it would weigh at R/4 depth under surface of earth ?

(1) 225 N (2) 75 N (3) 375 N (4) 300 N

Ans. **(1)**

Sol. Depth = $\frac{R}{4}$, So the distance from the center $r = R - \frac{R}{4}$

$r = \frac{3R}{4}$ $g_{in} \propto r$

Since $r = \frac{3}{4} \Rightarrow g = \frac{3}{4} \Rightarrow W = \frac{3}{4}$

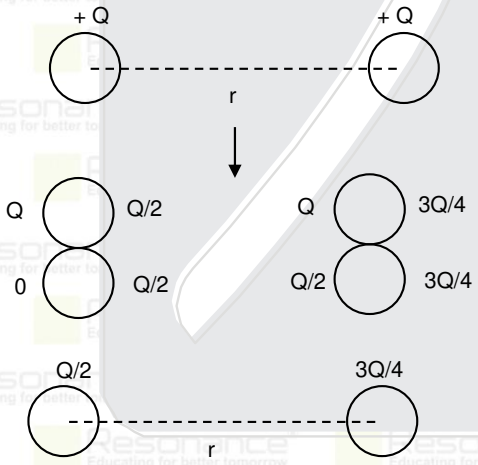
$W_f = 300 \times \frac{3}{4} = 225$ N,

36. Two identical conducting spheres P and S with charge Q on each, repel each other with a force 16 N. A third identical uncharged conducting sphere R is successively brought in contact with the two spheres. The new force of repulsion between P and S is:

(1) 12 N (2) 1 N (3) 4 N (4) 6 N

Ans. **(4)**

Sol.



$F_f = \frac{k\left(\frac{Q}{2}\right)\left(\frac{3Q}{4}\right)}{r^2} = \frac{3}{8}\left(\frac{kQ^2}{r^2}\right)$, $F_f = \left(\frac{3}{8}\right)(16) = 6N$

37. When kinetic energy of a body becomes 36 times of its original value, the percentage increase in the momentum of the body will be:

(1) 500 % (2) 60 % (3) 6% (4) 600%

Ans. **(1)**

Sol. $KE = \frac{p^2}{2m} \Rightarrow P \propto \sqrt{KE}$

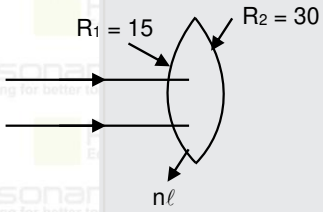
$P \propto \sqrt{36}$ = 6 times $\Rightarrow$ P will increase by 500%

38. For the thin convex lens, the radii of curvature are at 15 cm and 30 cm respectively. The focal length the lens is 20 cm. The refractive index of the material is:

(1) 1.4 (2) 1.8 (3) 1.5 (4) 1.2

Ans. **(3)**

Sol.



$$\frac{1}{f} = \left(\frac{n_\ell}{n_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{+20} = \left(\frac{n}{1} - 1\right)\left(\frac{1}{+15} - \frac{1}{-30}\right)$$

n = 1.5

39. A total of 48 J heat is given to one mole of helium kept in a cylinder. The temperature of helium increases by 2°C. The work done by the gas is: Given, R = 8.3 J K^{-1} mol^{-1},

(1) 23.1 J (2) 24.9 J (3) 72.9 J (4) 48 J

Ans. **(1)**

Sol. $\Delta U = n\left(\frac{f}{2}R\right)\Delta T = (1)\left(\frac{3}{2}\times\frac{25}{3}\right)(2) = 25J$

$Q = W + \Delta U \Rightarrow 48 = W + 25 \Rightarrow W = 23$ J

40. The number of electrons flowing per second in the filament of a 110 W bulb operating at 220 V is: (Given e = 1.6 × 10^{-19} C)

(1) 6.25 x 10^{18} (2) 1.25 × 10^{19} (3) 31.25 × 10^{17} (4) 6.25 × 10^{17}

Ans. **(3)**

Sol. Power consumed = Vi

$110 = (220)(i) \Rightarrow I = \frac{1}{2}A$

$$\mathbf{i} = \frac{dq}{dt} = e\left(\frac{dn}{dt}\right) = \frac{1}{2}A$$

$$\frac{dn}{dt} = \frac{i}{e} = \frac{1/2}{1.6\times10^{-19}} = \frac{10^{19}}{3.2} = 31.25 \times 10^{17} \text{ electron/sec.}$$

41. In a coil, the current changes from – 2 A to +2A in 0.2 s and induces an emf of 0.1 V. The self inductance of the coil is:

(1) 4mH (2) 2.5 mH (3) 1 mH (4) 5 mH

Ans. **(4)**

Sol. $|EMF| = L\frac{\Delta i}{\Delta t}$

$0.1 = (L)\left(\frac{4}{0.2}\right) \Rightarrow L = 5mH$

42. The longest wavelength associated with Paschen series is: (Given $R_H = 1.097\times10^7$ SI unit)

(1) 2.973×10^{-6} m (2) 3.646×10^{-6} m (3) 1.094×10^{-6} m (4) 1.876×10^{-6} m

Ans. **(4)**

Sol. Longest wavelength (smallest energy) of paschen series :

$$\frac{1}{\lambda} = R_e(z)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = (1.097 \times 10^7)(1)^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{1.097\times7}{1.44}\times10^7$$

$$\lambda = \frac{144}{7\times1.097}\times10^{-7}\ m = 1.87 \times 10^{-6}\ m$$

43. In a vernier calliper, when both jaws touch each other, zero of the vernier scale shifts towards left and its 4th division coincides exactly with a certain division on main scale. If 50 vernier scale divisions equal to 49 main scale divisions and zero error in the instrument is 0.04 mm then how many main scale divisions are there in 1 cm?

(1) 5 (2) 20 (3) 10 (4) 40

Ans. **(2)**

Sol. There is some error in this question. Actually in the 1st line, it should be written as, " the zero of vernier scale shifts ahead" rather than " the zero of vernier scale shift towards left "

Let 1 MSD = x

50 VSD matches with 49 MSD = 49x

$1VSD = \frac{49}{50}x$

$L.C. = 1\ MSD - 1VSD = x - \frac{49}{50}x = \frac{x}{50}$

zero error = M.S.R. + (V.S.R.)(L.C.)

$0.04\ mm = 0 + (4)\left(\frac{x}{50}\right) \Rightarrow x = 0.5\ mm$

Number of main scale divisions in 10 mm = $\frac{10\ mm}{0.5\ mm} = 20$

44. When UV light of wavelength 300 nm is incident on the metal surface having work function 2.13 eV, electron emission takes place. The stopping potential is: (Given hc 1240 eV nm)

(1) 1.5 V (2) 4 V (3) 4.1 V (4) 2V

Ans. **(4)**

Sol. Energy of photon = $\frac{hc}{\lambda} = \frac{1240}{300}$ = 4.13 eV

$KE_{max} = \frac{hc}{\lambda} - \Psi$ = 4.13 eV – 2.13 eV

KE_{max} = 2 eV = $eV_{st} \Rightarrow V_{st}$ = 2 volt

45. A car of 800 kg is taking turn on a banked road of radius 300 m and angle of banking 30°. If coefficient of static friction is 0.2 then the maximum speed with which car can negotiate the turn safely

(g = 10 m/s^2, $\sqrt{3}$ =1.73)

(1) 51.4 m/s (2) 70.4 m/s (3) 264 m/s (4) 102.8 m/s

Ans. **(1)**

Sol. $V_{max} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{1 - \mu_s \tan\theta}}$

$V_{max} = \sqrt{\frac{300 \times 10(\tan 30° + 0.2)}{1 - (0.2)\tan 30°}}$

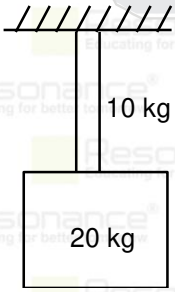
$= 10\sqrt{\frac{23.3}{0.88}}$ = 51.4 m/sec.

46. A body of weight 200 N is suspended from a tree branch through a chain of mass 10 kg. The branch pulls the chain by a force equal to (if g = 10 m/s^2):

(1) 200 N (2) 100 N (3) 150 N (4) 300 N

Ans. **(4)**

Sol. Tension at the highest point of the chain = (20 + 10)g = 300 N

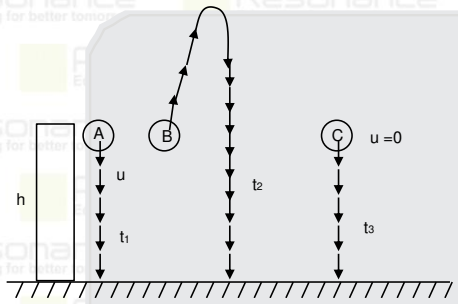


47. A body projected vertically upwards with a certain speed from the top of a tower reaches the ground in t_1. If it is projected vertically downwards from the same point with the same speed, it reaches the ground in t_2. Time required to reach the ground, if it is dropped from the top of the tower, is:

(1) $\sqrt{\frac{t_1}{t_2}}$ (2) $\sqrt{t_1+t_2}$ (3) $\sqrt{t_1t_2}$ (4) $\sqrt{t_1-t_2}$

Ans. **(3)**

Sol.



For ball A : $S = ut + \frac{1}{2}at^2$

$(h = ut_1 + \frac{1}{2}gt_1^2) \times t_2$ ...(1)

For ball B :

$(h = (-u)t_2 + \frac{1}{2}gt_2^2) \times t_1$ ...(2)

Solving $h(t_1 + t_2) = \frac{1}{2}gt_1t_2(t_1+t_2)$

$h = \frac{1}{2}gt_1t_2$

For ball C :

$h = \frac{1}{2}gt_3^2 \Rightarrow gt_1t_2 = \frac{1}{2}gt_3^2$

$\Rightarrow t_3 = \sqrt{t_1t_2}$

48. Energy of 10 non rigid diatomic molecules at temperature T is :

(1) $\frac{7}{2}$ RT (2) 35 RT (3) 35 K_BT (4) 70 K_BT

Ans. **(3)**

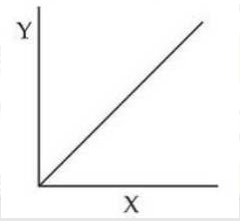
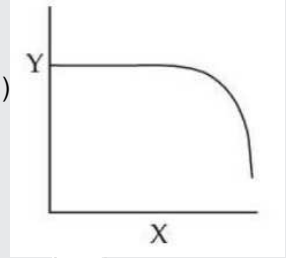
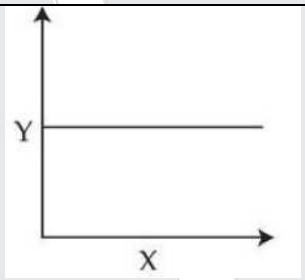
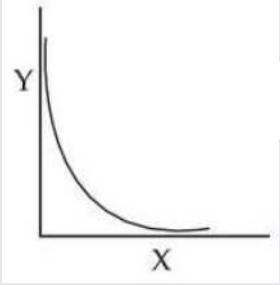
Sol. Since the diatomic molecule is non-rigid so it will also have two vibrational degree of freedom.

So $f = 3 + 2 + 2 = 7$

(KE) of one molecule $= \frac{f}{2}KT = \frac{7}{2}KT$

KE of 10 molecules $= \left(\frac{7}{2}KT\right) \times 10 = 35\ KT$

49. Match List-I with List-II:

List-1 **Y vs X**	**List -II** **Shape of graph**
(A) Y = magnetic susceptibility X = magnetising field	(I) 
(B) Y = magnetic field X = distance from centre of a current carrying wire for x < a (where a radius of wire)	(II) 
(C) Y = magnetic field X = distance from centre of a current carrying wire for x >a (where a radius of wire)	(III) 
(D) Y = magnetic field inside solenoid X = distance from centre	(IV) 

Choose the correct answer from the options given below :

(1) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(2) (A)-(III), (B)-(IV), (C)- (I), (D)-(II)

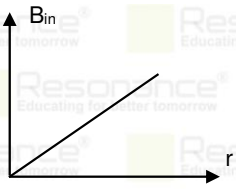
(3) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(4) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

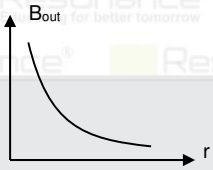
Ans. **(4)**

Sol. (A) Magnetic susceptibility remains constant with applied magnetising field.

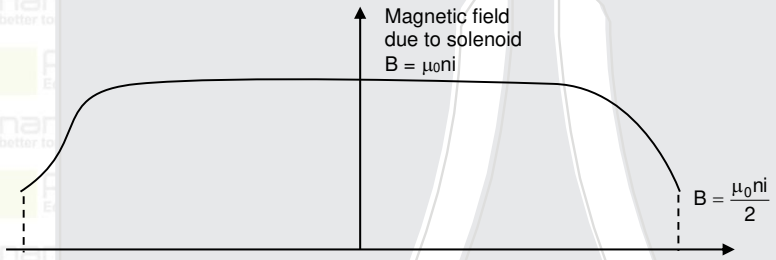
(B) $B_{in} = \frac{\mu_0 i}{2\pi R^2} r \Rightarrow B_{in} \propto r \Rightarrow$



(C) $B_{out} = \frac{\mu_0 i}{2\pi r} \Rightarrow B_{out} \propto \frac{1}{r}$



(D)



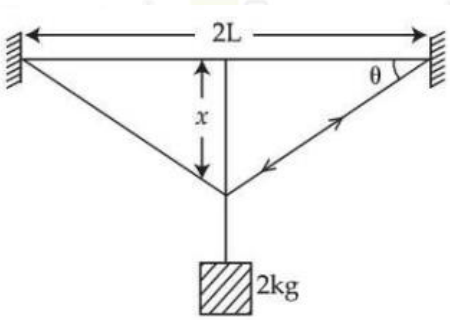
50. Pressure inside a soap bubble is greater than the pressure outside by an amount: (given: R = Radius of bubble, S = Surface tension of bubble)

(1) $\frac{3R}{S}$ (2) $\frac{S}{R}$ (3) $\frac{4S}{R}$ (4) $\frac{2S}{R}$

Ans. **(3)**

Sol. Pressure excess inside a soap bubble = $\frac{4S}{R}$

51. A wire of cross sectional area A, modulus of elasticity 2×10^{11} Nm^{-2} and length 2 m is stretched between two vertical rigid supports. When a mass of 2 kg is suspended at the middle it sags lower from its original position making angle $\theta = \frac{1}{100}$ radian on the points of support. The value of A is ____ $\times 10^{-4}$ m^2 (consider x << L). (given : g = 10 m/s^2)



Ans. **(1)**

Sol. Stretching $= \frac{2L}{\cos\theta} - 2L = 2L\left(\frac{1-\cos\theta}{\cos\theta}\right)$

Stretching $= (2L)\left(\frac{2\sin^2\left(\frac{\theta}{2}\right)}{\cos\theta}\right)$, For small $\theta \to 0$, $\sin\theta \to \theta$, $\cos\theta \to 1$

Stretching $= (2L)\left(\frac{2\times\left(\frac{\theta}{2}\right)^2}{1}\right) = LQ^2 = 2 \times (10^{-2})^2 = 2 \times 10^{-4}$ m

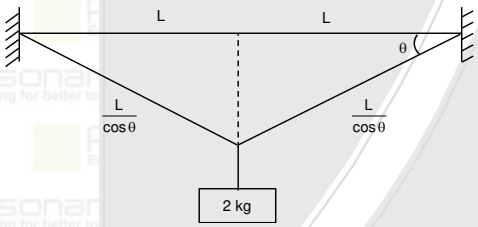
Tension in the wire $T = kx = \left(\frac{YA}{2L}\right)$(stretching)

$T = \frac{(2\times10^{11})(A)}{(2\times2)} \times 2\times10^{-4} = 10^7 A$

For equilibrium, $2T\sin\theta = mg$ where $\sin\theta \approx \theta$

$2(10^7 A)(10^{-2}) = 2 \times 10$

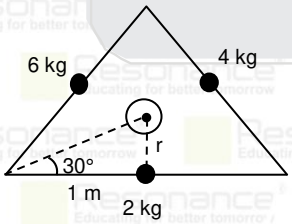
$A = 1 \times 10^{-4}$ m², Answer will be 1



52. Three balls of masses 2kg, 4kg and 6kg respectively are arranged at centre of the edges of an equilateral triangle of side 2 m. The moment of inertia of the system about an axis through the centroid and perpendicular to the plane of triangle, will be kg m².

Ans. **(4)**

Sol.



$r = 1 \tan 30° = \frac{1}{\sqrt{3}}$

$I_2 = (2)r^2 + (4)r^2 + (6)r^2$

$I_2 = (12)\left(\frac{1}{\sqrt{3}}\right)^2 = 4$

Answer will be 4

53. In Franck-Hertz experiment, the first dip in the current-voltage graph for hydrogen is observed at 10.2 V. The wavelength of light emitted by hydrogen atom when excited to the first excitation level is_____nm. (Given hc = 1245 eV nm, e = 1.6 × 10^{-19}C).

Ans. **(122)**

Sol. $\Delta E = 10.2 \text{ eV} = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{\Delta E} = \frac{1245}{10.2} = 122 \text{ nm}$$

54. A particle moves in a straight line so that its displacement x at any time t is given by $x^2 = 1 + t^2$. Its acceleration at any time t is x^{-n} where n =

Ans. **(3)**

Sol. $x^2 = 1 + t^2 \Rightarrow x^2 - t^2 = 1$

$$2x\left(\frac{dx}{dt}\right) = 2t \Rightarrow \frac{dx}{dt} = \frac{t}{x} = v$$

$$a = \frac{dv}{dt} = \frac{(x)(1) - (t)\left(\frac{dx}{dt}\right)}{x^2} = \frac{x - (t)\frac{t}{x}}{x}$$

$$a = \frac{x^2 - t^2}{x^3} = \frac{1}{x^3} = x^{-3}$$

so n = 3

55. For a given series LCR circuit it is found that maximum current is drawn when value of variable capacitance is 2.5 nF. If resistance of 2000 and 100 mH inductor is being used in the given circuit. The frequency of ac source is _____ × 10^3 Hz. (given $\pi^2 = 10$)

Ans. **(10)**

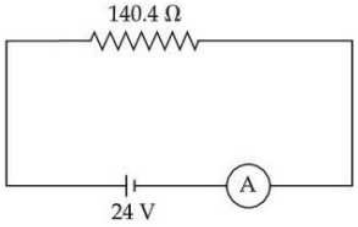
Sol. $\omega_r = \frac{1}{\sqrt{LC}} = 2\pi f_r$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 2.5 \times 10^{-9}}} = \frac{10^5}{2\pi}\sqrt{\frac{10}{25}}$$

$$f_r = \frac{10^5}{2\pi} \times \frac{\pi}{5} = 10 \times 10^3 \text{ Hz}$$

Answer will be 10

56. In the given figure an ammeter A consists of a 240 Ω coil connected in parallel to a 10 Ω shunt. The reading of the ammeter is _____ mA.



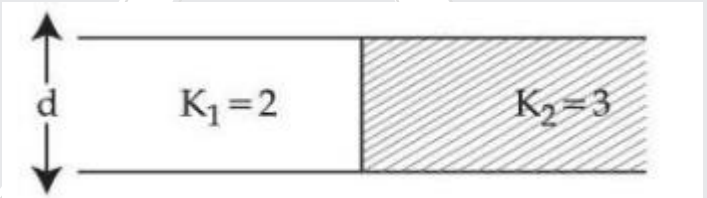
Ans. **(160)**

Sol. $R_A = \frac{(R_g)(5)}{(R_g+5)} = \frac{(240)(10)}{240+10} = 9.6\ \Omega$

$\mathbf{i} = \frac{V}{R_{net}} = \frac{24}{140.4+9.6} = \frac{24}{150} = 160$ mA

Ans. will be 160

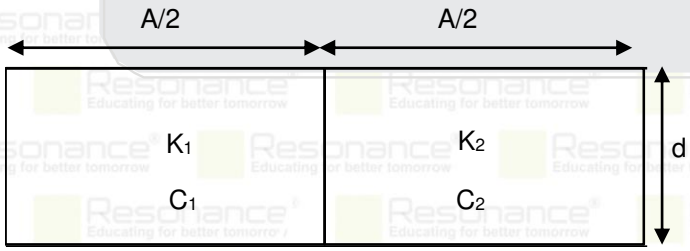
57. A capacitor of 10 μF capacitance whose plates are separated by 10 mm through air and each plate has area 4 cm² is now filled equally with two dielectric media of K_1= 2, K_2= 3 respectively as shown in figure. If new force between the plates is 8 N. The supply voltage is _____V.



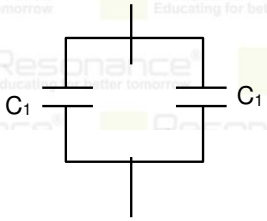
Ans. **(80)**

Sol. Given $C = \frac{\varepsilon_0 A}{d} = 10\ \mu F$

After filled dielectric medium



Both capacitor connected in parallel



$C_{New} = C_1 + C_2$

$$= \frac{\varepsilon_0 A_{1/2} K_1}{d} + \frac{\varepsilon_0 A_{1/2} K_2}{d}$$

$$= \frac{\varepsilon_0 A}{d}\left(\frac{K_1 + K_2}{2}\right)$$

$$= 10\left(\frac{2+3}{2}\right)$$

$C_{new} = 25\ \mu F$

Force between plate of capacitor $F = \frac{CV^2}{2d}$

$$F = 8 = \frac{CV^2}{2\times 10\times 10^{-3}}$$

$$\frac{8\times 2\times 10^{-2}}{25\times 10^{-6}} = V^2$$

$$= \frac{16}{25}\times 10^4 = \frac{16}{25}\times 100\times 100 = V^2$$

$V = 80$

58. A coil having 100 turns, area of $5 \times 10^{-3}\ m^2$, carrying current of 1 mA is placed in uniform magnetic field of 0.20 T such a way that plane of coil is perpendicular to the magnetic field. The work done in turning the coil through 90° is _____ μJ.

Ans. **(100)**

Sol. Work done to turn the coil

$W = \Delta U = MB(1 - \cos\theta)$

$\theta = 90°$

$W = MB$

Magnetic moment of coil = $M = NIA = 100 \times 1 \times 10^{-3} \times 15 \times 10^{-3}$

$= 500 \times 10^{-6}$

$W = 500 \times 10^{-6} \times 20$

$W = 100 \times 10^{-6}\ J$

$W = 100\ \mu J$

59. Two open organ pipes of lengths 60 cm and 90 cm resonate at 6th and 5th harmonics respectively. The difference of frequencies for the given modes is _____Hz. (Velocity of sound in air=333 m/s)

Ans. **(740)**

Sol. For n^{th} harmonics in open organ pipe = fn = $\frac{nv}{2\ell}$

For 1st organ pipe

$\ell_1 = 60$ $\quad$ 6th harmonic $f_{H6} = \frac{6v}{2\ell_1} = \frac{6\times v}{2\times 60}$

For 2nd organ pipe $\ell_2 = 90$

For 5th harmonic $f_{H5} = \frac{nv}{2\ell_2} = \frac{5\times v}{2\times 90}$

$\ell_2 = 90°$

$f_{5th} = \frac{5V}{2\times 90}$

difference of frequencies

$$f_{6th} - f_{5th} = \frac{6v}{120} - \frac{5v}{180} = \frac{18v - 10v}{360}$$

$$= \frac{8v}{360}$$

$$= \frac{v}{45} = \frac{333}{45}$$

$$= 7.4 \times 10^2$$

$$= 740 \text{ Hz}$$

60. Two coherent monochromatic light beams of intensities I and $4I$ are superimposed. The difference between maximum and minimum possible intensities in the resulting beam is x I. The value of x is ____

Ans. **8**

Sol. $I_1 = I$

$I_2 = 4I$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{I} + \sqrt{4\,I}\right)^2 = 9I$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = \left(\sqrt{I} - \sqrt{4\,I}\right)^2 = I$$

$I_{max} - I_{min} = 9I - I = 8I$

$x = 8$

PART : CHEMISTRY

61. Molality (m) of 3M aqueous solution of NaCl is:
(Given : Density of solution = 1.25 g mL^{-1}, Molar mass in g mol^{-1}: Na-23, Cl-35.5)
(1) 3.85 m (2) 2.79 m (3) 2.90 m (4) 1.90 m

Ans. **NTA (2)**

Sol. Given molarity = 3M $\begin{cases} n_{sonate} = 3 \text{ mol} \\ V_{solution} = 1 \text{ L}, d_{soln} = 1.25 \text{ g/mL} \end{cases}$

$$\text{Molality} = \frac{3\text{mol}}{1250\text{g} - 3 \times 58.5\text{ g}} \times 1000 = 2.79 \text{ m}$$

62. The correct IUPAC name of $[PtBr_2(PMe_3)_2]$ is:
(1) bis[bromo(trimethylphosphine)]platinum(II) (2) dibromobis(trimethylp0hosphine)]platinum(II)
(3) bis(trimethylphosphine)dibromoplatinum(II) (4) dibromodi(trimethylphosphine)platinum(II)

Ans. **NTA (2)**

63. During the detection of acidic radical present in a salt, a student gets a pale yellow precipitate soluble with difficulty in NH_4OH solution when sodium carbonate extract was first acidified with dil. HNO_3 and then $AgNO_3$ solution was added. This indicates presence of :
(1) CO_3^{2-} (2) Cl^- (3) I^- (4) Br^-

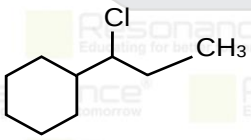
Ans. **NTA (4)**

Sol. $Ag^+ + I^- \rightarrow AgI$ yellow ppt
$Ag^+ + Cl^- \rightarrow Agcl$ white ppt
$Ag^+ + Br^- \rightarrow AgBr$ pale yellow ppt

64. Given below are two statements:
Statement I: PF_5, and BrF_5, both exhibit sp^3d hybridisation.
Statement II: Both SF_6, and $[Co(NH_3)_6]^{3+}$ exhibit sp^3d^2 hybridisation.
In the light of the above statements, choose the correct answer from the options given below:
(1) Statement I is false but Statement II is true (2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are true (4) Both Statement I and Statement II are false

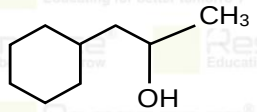
Ans. **NTA (4)**

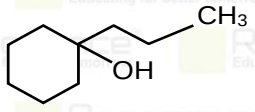
Sol. $PF_5 \xrightarrow[sp^3d]{\text{Hybridisaton}}$
BrF_5 sp^3d^2
SF_6 sp^3d^2
$[Co(NH_3)_6]^{+3}$ d^2sp^3
Both statement (1) and (2) are false

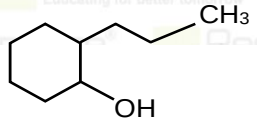
65. 

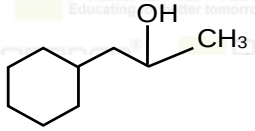
+ NaOH $\xrightarrow{H_2O}$ Major Product "A"

Consider the above chemical reaction. Product "A" is:

(1) 

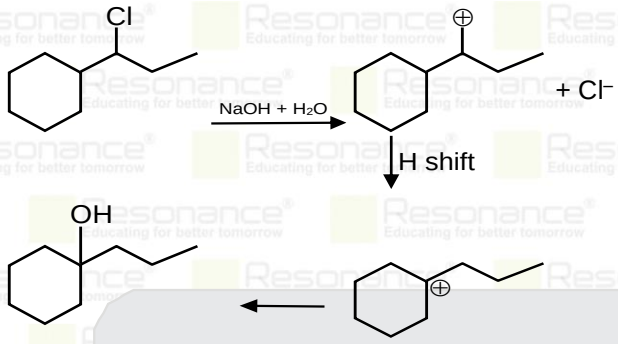
(2) 

(3) 

(4) 

Ans. **NTA (2)**

Sol.



66. The ratio $\frac{K_P}{K_C}$ for the reaction:

$CO_{(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons CO_{2(g)}$ is:

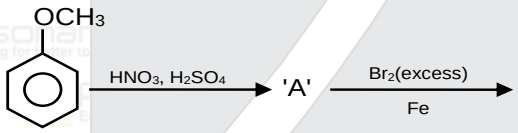
(1) $\frac{1}{\sqrt{RT}}$ (2) 1 (3) $(RT)^{1/2}$ (4) RT

Ans. **NTA (1)**

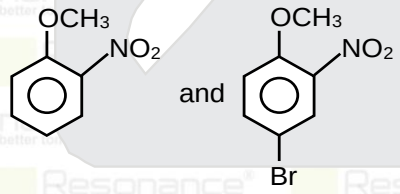
Sol. $Co\,(g) + \frac{1}{2}\,O_2\,(g) \rightleftharpoons CO_2\,(g)$; $\Delta ng = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}$

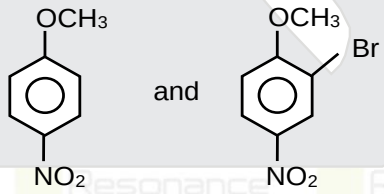
$K_p = K_c\,(RT)^{-1/2}$, $\frac{K_p}{K_C} = \frac{1}{\sqrt{RT}}$

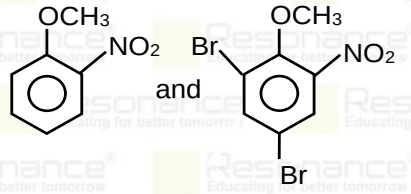
67. The major products formed:

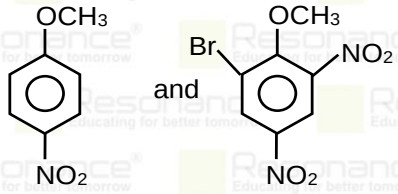


A and B respectively are:

(1) 

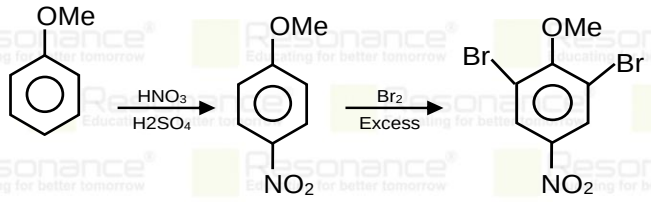
(2) 

(3) 

(4) 

Ans. **NTA (4)**

Sol.



68. Evaluate the following statements related to group 14 elements for their correctness.
(A) Covalent radius decreases down the group from C to Pb in a regular manner.
(B) Electronegativity decreases from C to Pb down the group gradually.
(C) Maximum covalance of C is 4 whereas other elements can expand their covalance due to presence of d orbitals.
(D) Heavier elements do not form $p\pi$-π bonds.
(E) Carbon can exhibit negative oxidation states.
Choose the correct answer from the options given below:
(1) (A), (B) and (C) Only (2) (A) and (B) Only
(3) (C) and (D) Only (4) (C), (D) and (E) Only

Ans. **NTA (4)**

Sol. (A) Radius increases down the group
(B) E.N. does not decrease gradually from C to Pb
(C) Correct
(D) Correct
(E) Oxidation state of carbon can varies from –4 to +4.

69. Match List – I with List – II.

List-I (Tetrahedral Complex)		List-II (Electronic configuration)	
(A)	$TiCl_4$	(I)	e^2, t_2^0
(B)	$[FeO_4]^{2-}$	(II)	e^4, t_2^3
(C)	$[FeCl_4]^-$	(III)	e^0, t_2^0
(D)	$[CoCl_4]^{2-}$	(IV)	e^2, t_2^3

Choose the correct answer from the options given below:
(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II) (2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(3) (A)-(IV), (B)-(III), (C)-(I), (D)-(II) (4) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

Ans. **NTA (1)**

Sol. $TiCl_4 \rightarrow$ $\square\square\square$ f_2^0
$\square\square$ e^0

$FeO_4^{2-} \rightarrow$ $\square\square\square$ t_2^0
$\uparrow\uparrow$ e^2

$[FeCl_4]^- \rightarrow$ $\uparrow\uparrow\uparrow$ t_2^3
$\uparrow\uparrow$ e^2

$[CoCl_4]^{2-} \rightarrow$ $\uparrow\uparrow\uparrow$ t_2^3
$\uparrow\downarrow\uparrow\downarrow$ e^4

70. The number of ions from the following that are expected to behave as oxidising agent is:
Sn^{4+}, Sn^{2+}, Pb^{2+}, Ti^{3+}, Pb^{4+}, Tl^{+}
(1) 1 (2) 4 (3) 3 (4) 2

Ans. **NTA (4)**

Sol. Due to inert pair effect ; Tl^{3+} and Pb^{4+} can behave as oxidising agents.

71. Match List I with List - II.

List – I **Reaction**	**List - II** **Type of redox reaction**
(A) $N_{2(g)}+O_{2(g)} \rightarrow 2NO_{(g)}$	(I) Decomposition
(B) $2Pb(NO_3)_{2(s)} \rightarrow 2PbO_{(s)} +4NO2_{(g)} + O_{2(g)}$	(II) Displacement
(C) $2Na(s) +2H_2O_{(l)} \rightarrow 2NaOH_{(aq)} + H_{2(g)}$	(III) Disproportionation
(D) $2NO_{2(g)} +2OH_{(aq.)} \rightarrow NO_{2(aq)} + NO_{3(aq)} + H_2O_{(l)}$	(IV) Combination

Choose the correct answer from the options given below:
(1) (A)-(III), (B)-(II), (C)-(I), (D)-(IV) (2) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III) (4) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

Ans. **NTA (3)**

Sol. A → IV, B → I, C → II, D → III,

72. The incorrect statements regarding enzymes are:
(A) Enzymes are biocatalysts.
(B) Enzymes are non-specific and can catalyse different kinds of reactions.
(C) Most Enzymes are globular proteins.
(D) Enzyme oxidase catalyses the hydrolysis of maltose into glucose.
Choose the correct answer from the option given below :
(1) (B) and (C) (2) (A), (B) and (C) (3) (B) and (D) (4) (B), (C) and (D)

Ans. **NTA (3)**

Sol. Enzymes are highly specific in their action and follow lock and key mechanism maltase converts maltose to glucose.

73. Arrange the following elements in the increasing order of number of unpaired electrons in it.
(A) Sc
(B) Cr
(C) V
(D) Ti
(E) Mn
Choose the correct answer from the options given below:
(1) (C) < (E) < (B) < (A) < (D) (2) (A) < (D) < (C) < (B) < (E)
(3) (A) < (D) < (C) < (E) < (B) (4) (B) < (C) < (D) < (E) < (A)

Ans. **NTA (3)**

Sol.

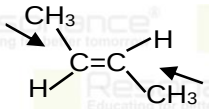
	Number of unpaired
Sc : [Ar] $4s^2$ $3d^1$	1
Cr : [Ar] $4s^1$ $3d^5$	6
V : [Ar] $4s^2$ $3d^3$	3
Ti : [Ar] $4s^2$ $3d^5$	5

Sc < Ti < V < Mn < Cr

74. The incorrect statement regarding the geometrical isomers of 2-butene is:
(1) cis-2-butene has less dipole moment than trans-2-butene.
(2) cis-2-butene and trans-2-butene are not interconvertible at room temperature.
(3) cis-2-butene and trans-2-butene are stereoisomers.
(4) trans-2-butene is more stable than cis-2-butene.

Ans. **NTA (1)**

Sol. trans–2–butene has zero dipole moment.



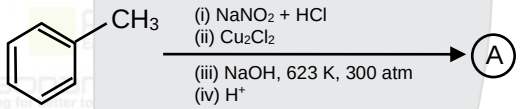
$\ell i = 0$

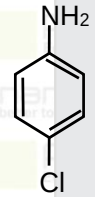
75. How can an electrochemical cell be converted into an electrolytic cell?
(1) Exchanging the electrodes at anode and cathode.
(2) Applying an external opposite potential greater than E^o_{cell}
(3) Applying an external opposite potential lower than E^o_{cell}
(4) Reversing the flow of ions in salt bridge.

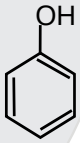
Ans. **NTA (2)**

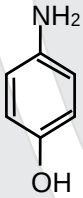
Sol. Applied external potential should be greater than E^o_{cell} in opposite direction

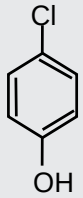
76. Identify the product (A) in the following reaction.



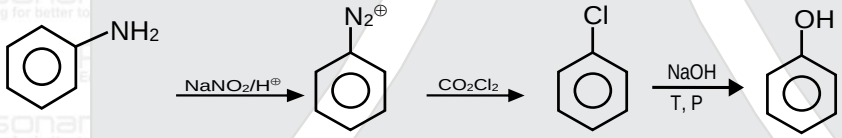
(1) 

(2) 

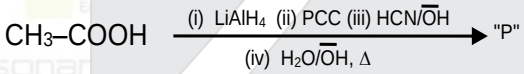
(3) 

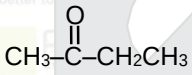
(4) 

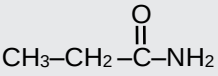
Ans. **NTA (2)**

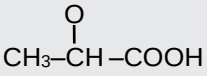
Sol. 

77. Consider the given reaction, identify the major product P.



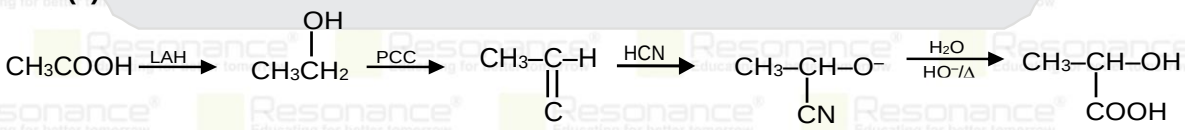
(1) 

(2) 

(3) 

(4) $CH_3 - CH_2 - CH_2 - OH$

Ans. **NTA (3)**

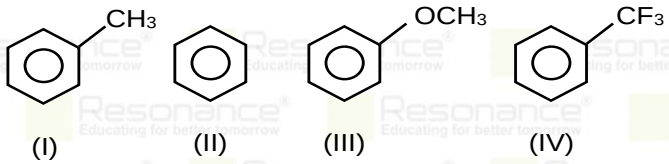
Sol. 

(salt)

78. The correct statement among the following, for a "chromatography" purification method is:
(1) Organic compounds run faster than solvent in the thin layer chromatographic plate.
(2) Non-polar compounds are retained at top and polar compounds come down in column chromatography.
(3) R_f is an integral value.
(4) R_f of a polar compound is smaller than that of a non-polar compound.

Ans. **NTA (4)**

79.



(I) (II) (III) (IV)

The correct arrangement for decreasing order of electrophilic substitution for above compounds is:

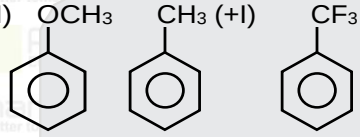
(1) (IV) > (I) > (II) > (III) (2) (III) > (IV) > (II) > (I)

(3) (II) > (IV) > (III) > (I) (4) (III) > (I) > (II) > (IV)

Ans. **NTA (4)**

Sol. Rate of $E^{\oplus}$ sub. in benzene × e^- dense ring

(+M) OCH_3 CH_3 (+I) CF_3 (–I)



80. Match List I with List - II.

List – I Alkali Metal	List - II Emission Wavelength in nm
(A) Li	(I) 589.2
(B) Na	(II) 455.5
(C) Rb	(III) 670.8
(D) Cs	(IV) 780.0

Choose the correct answer from the options given below :

(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II) (2) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(3) (A)-(II), (B)-(IV), (C)-(III), (D)-(I) (4) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

Ans. **NTA (1)**

Sol. Theory & data based

81. Among VO_2^+, MnO_4^-, and $Cr_2O_7^{2-}$, the spin-only magnetic moment value of the species with least oxidising ability is BM (Nearest integer).

(Given atomic member V=23, Mn=25, Cr=24)

Ans. **NTA (0)**

Sol. For 3d series

Oxidising pornet $V^{+5} < Cr^{+6} < Mn^{+7}$

V^{+5} : Ar $4s^0$ $3d^0$

Number of unpaired e^- = 0

$\therefore \quad \mu = 0$

82. For hydrogen atom, energy of an electron in first excited state is –3.4 eV, K.E. of the same electron of hydrogen atom is x eV. Value of x is _____ × 10^{-1} eV. (Nearest integer)

Ans. **NTA (34)**

Sol. T.E. = – (K.E)

–3.4 ev = – (K.E)

K.E = 3.4 ev = 34 × 10^{-1} ev

$\therefore \quad x = 34$

83. Total number of species from the following with central atom utilising sp^2 hybrid orbitals for bonding is_____.

NH_3, SO_2, SiO_2, $BeCl_2$, C_2H_2, C_2H_4, BCl_3, HCHO, C_6H_6, BF_3, $C_2H_4Cl_2$

Ans. **NTA (6)**

Sol. Central atom utilising sp^2 hybrid orbitals

SO_2, C_2H_4, BCl_3, HCHO, C_6H_6, BF_3

84. Consider the two different first order reaction given below
$A + B \rightarrow C$ (Reaction 1)
$P \rightarrow Q$ (Reaction 2)
The ratio of the half life of reaction 1: Reaction 2 is 5 : 2. If t_1 and t_2 represent the time taken to complete $\frac{2}{3}^{rd}$ and $\frac{4}{5}^{th}$ of Reaction 1 and Reaction 2, respectively, then the value of the ratio $t_1 : t_2$ is _____ $\times 10^{-1}$ (nearest integer).
[Given : $\log_{10}(3) = 0.477$ and $\log_{10}(5) = 0.699$]

Ans. **NTA (17)**

Sol. $\frac{\left(t_{\frac{1}{2}}\right)_I}{\left(t_{\frac{1}{2}}\right)_{II}} = \frac{K_2}{K_1} = \frac{5}{2}$

$K_1 t_1 = \ell n \frac{1}{1-\frac{2}{3}} = \ell n\, 3$

$K_2 t_2 = \ell n \frac{1}{1-\frac{4}{5}} = \ell n\, 5$

$\frac{K_1 t_1}{K_2 t_2} = \frac{0.477}{0.699}$

$\therefore \quad \frac{t_1}{t_2} = \frac{0.477}{0.699} \times \frac{5}{2} = 1.7 = 17 \times 10^{-1}$

85. Consider the following reactions

$NiS + HNO_3 + HCl \rightarrow A + NO + S + H_2O$

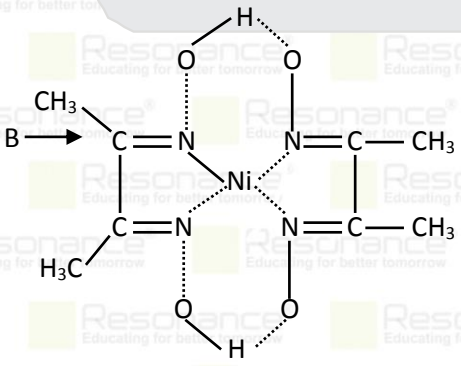
$A + NH_4OH + H_3C{-}\underset{\displaystyle H_3C{-}\overset{\displaystyle |}{C}{=}N{-}OH}{\overset{\displaystyle |}{C}}{=}N{-}OH \longrightarrow B + NH_4Cl + H_2O$

The number of protons that do not involve in hydrogen bonding in the product B is

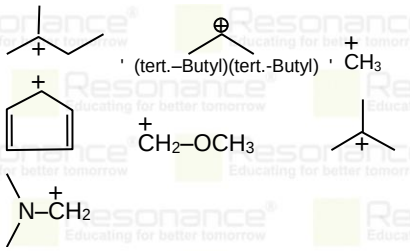
Ans. **NTA (12)**

Sol. $3\,Ni\,s + 2HNO_3 + 6HCl \rightarrow 3NiCl_2 + 2NO + 35 + 4H_2O$

$NiCl_2 + 2NH_4OH + H_3C{-}\underset{\displaystyle H_3C{-}\overset{\displaystyle |}{C}{=}N{-}OH}{\overset{\displaystyle |}{C}}{=}N{-}OH \longrightarrow NH_4Cl + H_2O + B$

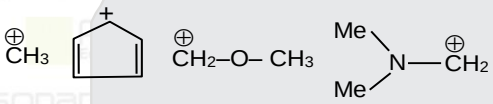


86. Number of carbocation's from the following that are not stabilized by hyper conjugation is _____.



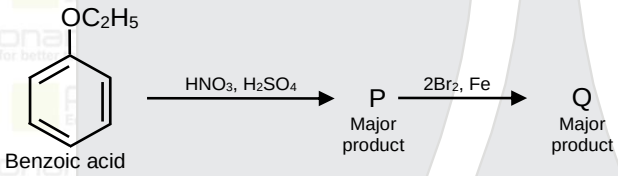
Ans. **NTA (5)**
RESO (4)

Sol.



These are not stable by hyper conjugation.

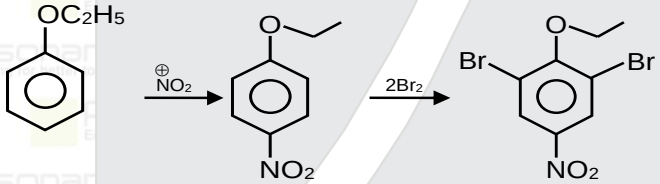
87.



The ratio of number of oxygen atoms to bromine atoms in the product Q is _____ $\times 10^{-1}$.

Ans. **NTA (15)**

Sol.



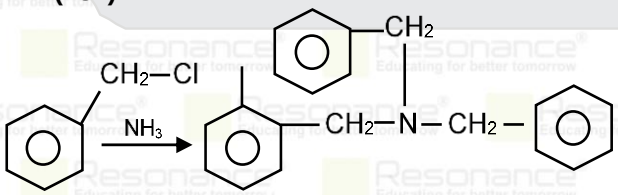
O atom in final pro = 3
Br atom in final pro = 2

$\frac{3}{2} \times 10^{-1} = .15$

88. An amine (X) is prepared by ammonolysis of benzyl chloride. On adding p-toluenesulphonyl chloride to it the solution remains clear. Molar mass of the amine (X) formed is_____ g mol^{-1}.
(Given molar mass in $gmol^{-1}$C : 12, H:1, O : 16, N:14)

Ans. **NTA (287)**

Sol.



3° emine will be formed hims berg reagent will be miscible to this amine ansure (287)

89. For the reaction at 298 K, 2A + B → C. DH = 400 kJ mol^{-1} and ΔS = 0.2 kJ mol^{-1}. The reaction will become spontaneous above _____ K.

Ans. **NTA (2000)**

Sol. For reaction become spontaneous

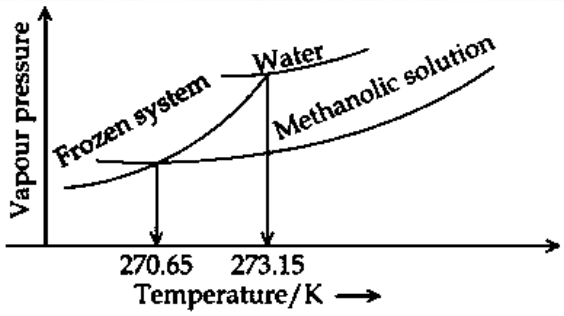
$\Delta G < 0$

$\because$ For $\Delta G = 0$, $T = \frac{\Delta S}{\Delta S} = \frac{400}{0.2} = 2000$ K

$\therefore$ Above 2000 K, $\Delta G < 0$,

$\therefore$ reaction become spontaneous

90. When 'x' × 10^{-2} mL methanol (molar mass = 32 g; density = 0.792 g/cm^3) is added to 100 mL water (density = 1 g/cm^3), the following diagram is obtained.



x = _____ (nearest integer).

[Given : Molal freezing point depression constant of water at 273.15 K is 1.86 K kg mol^{-1}]

Ans. **NTA (543)**

Sol. $\Delta T_f = (T_f)_{solvent} - (T_f)_{solution}$

$= 273.15 - 270.65 = 2.5$ K

$DT_f = K_f\, m$

$2.5 = 1.86 \times \frac{n}{0.1}$

$n = 0.134\ \mu$ mole

$w = 0.1344 \times 32 = 4.3$ g

volume $= \frac{4.3}{0.792} = 5.43$ mL $= 543 \times 10^{-2}$ mL