PART : MATHEMATICS

1. Let y = y(x) be the solution of the differential equation

$(2x\log_e x)\frac{dy}{dx} + 2y = \frac{3}{x}\log_e x$, x > 0 and $y(e^{-1}) = 0$. Then, y(e) is equal to

(1) $-\frac{3}{e}$ (2) $-\frac{3}{2e}$ (3) $-\frac{2}{e}$ (4) $-\frac{2}{3e}$

Ans. **NTA (1)**
RESO(1)

Sol. $\frac{dy}{dx} + \frac{y}{x\ell nx} = \frac{3}{2x^2}$

I.F. = $e^{\int \frac{1}{x\ell nx}dx}$

$= e^{\ell n(\ell nx)} = \ell nx$

Solution is y. $\ell nx = \int (\ell nx)\frac{3}{2x^2}dx + C$

Put $\ell nx = t$

$\frac{dx}{x} = dt$

$y.\ell nx = \frac{3}{2}\int t..e^{-t} + c$

$= \frac{3}{2}\left[-te^{-t} - \int 1.\left(-e^{-t}\right)dt\right] + C = -\frac{3}{2}te^{-t} + \frac{3}{2}\left(-e^{-t}\right) + c$

$y\ell nx = -\frac{3}{2}\frac{\ell nx}{x} - \frac{3}{2x} + c$

Now $\left(\frac{1}{e},0\right) \Rightarrow c = 0$

when x = e

$y = -\frac{3}{e}$

2. $\int_0^{\pi/4} \frac{\cos^2 x \sin^2 x}{\left(\cos^3 x + \sin^3 x\right)^2}dx$ is equal to

(1) 1/9 (2) 1/12 (3) 1/6 (4) 1/3

Ans. **NTA (3)**
RESO(3)

Sol. Divide by $\cos^6 x$ in Nr and Dr.

$= \int_0^{\pi/4} \frac{\tan^2 x \sec^2 x\, dx}{\left(1+\tan^3 x\right)^2}$ by

$1 + \tan^3 x = t$

$3\tan^2 x \sec^2 x\, dx = dt$

$= \frac{1}{3}\int_1^2 \frac{dt}{t^2}$

$= \frac{1}{3}\left(\frac{-1}{t}\right)_1^2 = \frac{1}{3}\left[\frac{-1}{2}+1\right] = \frac{1}{6}$

3. The function $f(x) = \frac{x^2+2x-15}{x^2-4x+9}$, $x \in R$ is

(1) neither one-one not onto (2) both one-one and onto.

(3) onto but not one-one (4) one-one but not onto

Ans. **NTA (1)**
RESO(1)

Sol. $f'(x) = \frac{(x^2-4x+9)(2x+2)-(x^2+2x-15)(2x-4)}{(x^2-4x+9)^2}$

$= \frac{2x^3-6x^2+10x+18-[2x^3-38x+60]}{(x^2-4x+9)^2} = \frac{-6x^2+48x-42}{(x^2-4x+19)^2} = \frac{-6(x-7)(x-1)}{(x^2-4x+9)^2}$

So f'(x) =positive as well as negative so many one and into.
Since codomain is not given so will be considered it R

4. Let α, β be the distinct roots of the equation $x^2 - (t^2 - 5t + 6)x + 1 = 0$, $t \in R$ and $a_n = \alpha^n + \beta^n$. Then minimum value of $\frac{a_{2023}+a_{2025}}{a_{2024}}$ is

(1) –1/2 (2) 1/4 (3) –1/4 (4) 1/2

Ans. **NTA (3)**
RESO(3)

Sol. By newton's formula
$a_{2025} - (t^2 - 5t + 6)a_{2024} + a_{2023} = 0$

$\Rightarrow \frac{a_{2023}+a_{2025}}{a_{2024}} = t^2 - 5t + 6$

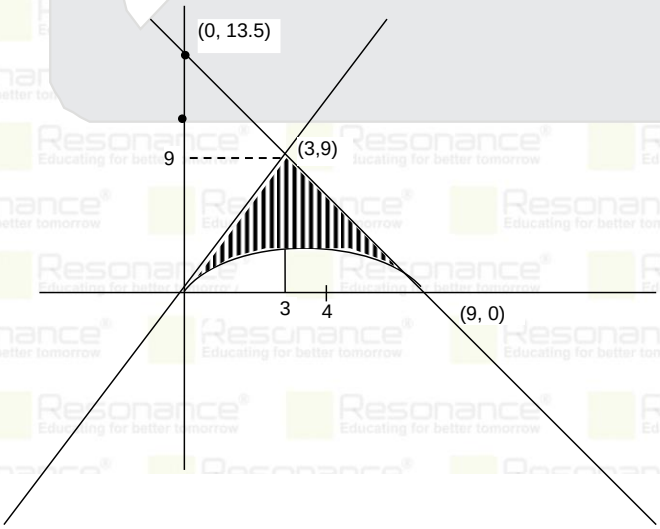
min value = $-\frac{D}{4a} = -\left(\frac{25-24}{4}\right) = -\frac{1}{4}$

5. Let the area of the region enclosed by the curves $y = 3x$, $2y = 27 - 3x$ and $y = 3x - x\sqrt{x}$ be A. Then 10A is equal to

(1) 154 (2) 184 (3) 172 (4) 162

Ans. **NTA (4)**
RESO(4)

Sol.



$y = 3x - x^{\frac{3}{2}}$ $\qquad y' = 3 - \frac{3}{2}\sqrt{x}$

$y' = 3\left(1 - \frac{\sqrt{x}}{2}\right)$

f '(x) : + (0) − (4)

Area of shaded region

$$= \int_0^3 \left(3x - \left(3x - x^{\frac{3}{2}}\right)\right)dx + \int_3^9 \frac{27-3x}{2} - \left(3x - x^{\frac{3}{2}}\right)dx$$

$$= \int_0^3 x^{\frac{3}{2}}dx + \frac{1}{2}\int_3^9 \left(27 - 9x + 2x^{\frac{3}{2}}\right)dx = \left(\frac{2}{5}x^{\frac{5}{2}}\right)_0^3 + \frac{1}{2}\left[27x - \frac{9x^2}{2} + 2\times\frac{2}{5}x^{\frac{5}{2}}\right]_3^9$$

$$= \frac{2}{5}\times 9\sqrt{3} + \frac{1}{2}\left[243 - \frac{729}{2} + \frac{4}{5}\times 81\times 3 - 81 + \frac{81}{2} - \frac{4}{5}\times 9\sqrt{3}\right] = \frac{1}{2}\left[\frac{486-729-81}{2} + \frac{972}{5}\right] = \frac{81}{5} \Rightarrow 10A = 162$$

6. Let f : $(-\infty,\infty) - \{0\} \to R$ be a differentiable function such that $f'(1) = \lim_{a\to\infty} a^2 f\left(\frac{1}{a}\right)$.

Then $\lim_{a\to\infty} \frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2\log_e a$ is equal to

(1) $\frac{3}{2} + \frac{\pi}{4}$ (2) $\frac{3}{4} + \frac{\pi}{8}$ (3) $\frac{5}{2} + \frac{\pi}{8}$ (4) $\frac{3}{8} + \frac{\pi}{4}$

Ans. **NTA (3)**

RESO () Should be Bonus

Sol. $f'(1) = \lim_{a\to\infty} a^2 f\left(\frac{1}{a}\right)$

So $f\left(\frac{1}{a}\right) = 0$ should be at $x \to \infty$

Now $\lim_{a\to\infty} a^2\left[\left[\frac{1}{2} + \frac{1}{2a}\right]\tan^{-1}\left(\frac{1}{a}\right) + 1 + \frac{\ln a^{-2}}{a^2}\right]$

$f\left(\frac{1}{a}\right) = \left[\frac{1}{2} + \frac{1}{2a}\right]\tan^{-1}\left(\frac{1}{a}\right) + 1 + \frac{\ln\left(\frac{1}{a^2}\right)}{a^2}$ (according to question $f\left(\frac{1}{a}\right)$ should be zero whenever a tending to infinite but it is 1 so there is a conceptual mistake in the question)

$f(x) = \left(\frac{1}{2} + \frac{x}{2}\right)\tan^{-1}x + 1 + x^2\ln(x^2)$

$f'(x) = \frac{x+1}{2}\cdot\frac{1}{1+x^2} + \frac{\tan^{-1}x}{2} + \frac{x^2}{x^2}2x + \ln x^2.2x$

According to NTA $f'(1) = 1 \times \frac{1}{2} + \frac{\pi}{8} + 2 = \frac{5}{2} + \frac{\pi}{8}$

7. The shortest distance between the lines $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ and $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ is

(1) $8\sqrt{3}$ (2) $4\sqrt{3}$ (3) $5\sqrt{3}$ (4) $6\sqrt{3}$

Ans. **NTA (2)**

RESO(2)

Sol. $\vec{a}_1 = 3\hat{i}-15\hat{j}+9\hat{k}$

$\vec{a}_2 = -\hat{i}+\hat{j}+9\hat{k}$

$\vec{b} = 2\hat{i}-7\hat{j}+5\hat{k}$

$\vec{d} = 2\hat{i}+\hat{j}-3\hat{k}$

then

shortest distance = $\left|\frac{(\vec{a}_1-\vec{a}_2).(\vec{b}\times\vec{d})}{(\vec{b}\times\vec{d})}\right|$

$\left|\frac{4(16)+(-16)\times16+0\times16}{\sqrt{16^2+16^2+16^2}}\right|$ (where $\vec{b}\times\vec{d}=16\hat{i}+16\hat{j}+16\hat{k}$)

$= \frac{16\times12}{16\sqrt{3}} = 4\sqrt{3}$

8. Let the relation R_1 and R_2 on the set X = {1, 2, 3, ...20} be given by R_1 = {(x, y): 2x – 3y = 2} and R_2 = {(x, y): –5x + 4y = 0}, if M and N be the minimum number of elements required to be added in R_1 and R_2 respectively in order to make the relation symmetric, then M+N equals

(1) 10 (2) 8 (3) 12 (4) 16

Ans. **NTA (1)**

RESO(1)

Sol. For R_1 : $y=\frac{2x-2}{3}$

R_1 = {(4,2), (7,4), (10,6), (13,8), (16,10), (19,12)}

for R_2 : $y=\frac{5x}{4}$

R_2 = {(4,5), (8,10), (12,15), (16,20)}

For R_1 should be symmetric, we have to add 6 elements.

For R_2 should be symmetric, we have to add 4 elements.

So m + n = 10

9. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation by mistake was taken 8 instead of 12. The correct standard deviation is

(1) $\sqrt{3.86}$ (2) 1.8 (3) 1.94 (4) $\sqrt{3.96}$

Ans. **NTA (4)**

RESO(4)

Sol. $\frac{\sum_{i=1}^{20} x_i}{20} = 10$

$\sum_{i=1}^{20} x_i = 200$ .....(1)

corrected $\sum_{i=1}^{20} x_i = 200 + 12 - 8 = 204$

$\frac{\sum_{i=1}^{20} x_i^2}{20} - (10)^2 = 4 \quad \Rightarrow \quad \sum_{i=1}^{20} x_i^2 = 20 \times 104 = 2080$

corrected $\sum_{i=1}^{20} x_i^2 = 2080 + 12^2 - 8^2 = 2160$

corrected SD = $\sqrt{\frac{2160}{20} - \left(\frac{204}{20}\right)^2} = \sqrt{3.96}$

10. Let y = y(x) be the solution of the differential equation $\left(1+x^2\right)\frac{dy}{dx} + y = e^{\tan^{-1}x}$, y(1) = 0. Then y(0) is

(1) $\frac{1}{2}\left(e^{\pi/2} - 1\right)$ (2) $\frac{1}{4}\left(1 - e^{\pi/2}\right)$ (3) $\frac{1}{4}\left(e^{\pi/2} - 1\right)$ (4) $\frac{1}{2}\left(1 - e^{\pi/2}\right)$

Ans. **NTA (4)**
RESO(4)

Sol. $\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$

I.F. = $e^{\int \frac{1}{1+x^2}dx}$

$= e^{\tan^{-1}x}$

Solution is y. $e^{\tan^{-1}x} = \int \frac{e^{\tan^{-1}x} e^{\tan^{-1}x}}{1+x^2} dx + C$

put $e^{\tan^{-1}x} = t$

$\frac{e^{\tan^{-1}x}}{1+x^2} dx = dt$

$y\ e^{\tan^{-1}x} = \int t dt + C$

$y\ e^{\tan^{-1}x} = \frac{t^2}{2} + C$

$\Rightarrow \quad y\ e^{\tan^{-1}x} = \frac{e^{2\tan^{-1}x}}{2} + C$

$x = 1 \Rightarrow y = 0$

$c = \frac{-e^{\pi/2}}{2}$

now at x = 0

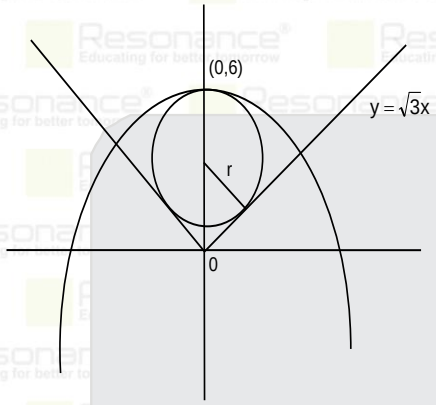
$y \times 1 = \frac{1}{2} - \frac{e^{\pi/2}}{2}$

11. Let C be the circle of minimum area touching the parabola $y = 6 - x^2$ and the lines $y = \sqrt{3}|x|$. Then, which one of the following points lies on the circle C?

(1) (1, 1) (2) (1, 2) (3) (2, 2) (4) (2, 4)

Ans. **NTA (4)**
RESO(4)

Sol.



Centre of circle = (0,6 – r)

Now ⊥' r drop from centre on tangent is equal to radius

$$\left|\frac{\sqrt{3}\times 0-(6-r)}{\sqrt{3+1}}\right| = r$$

$(r - 6)^2 = 4r^2$

$r = 2$

circle is $x^2 + (y - 4)^2 = 4$

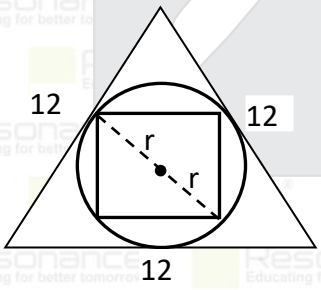
(4) option (2, 4) lies on circle

12. A circle is inscribed in an equilateral triangle of side of length 12. If the area and perimeter of any square inscribed in this circle are m and n, respectively. Then $m + n^2$ is equal to

(1) 312 (2) 396 (3) 408 (4) 414

Ans. **NTA (3)**
RESO(3)

Sol.



Irradius of $\Delta = r = \frac{\Delta}{s} = \frac{\frac{\sqrt{3}}{4}\times 12^2}{18} = \frac{36\sqrt{3}}{18} = 2\sqrt{3}$

side length of square is a , then

$2a^2 = 4r^2$

$a^2 = 2r^2 = 24$

Now m = 24, n = 4. $\sqrt{24}$

$m + n^2 = 24 + 16 \times 24 = 17 \times 24 = 408$

13. If A(3, 1, –1), $B\left(\frac{5}{3},\frac{7}{3},\frac{1}{3}\right)$, C(2, 2, 1) and $D\left(\frac{10}{3},\frac{2}{3},\frac{-1}{3}\right)$ are the vertices of a quadrilateral ABCD, then its area is

(1) $\frac{4\sqrt{2}}{3}$ (2) $\frac{2\sqrt{2}}{3}$ (3) $2\sqrt{2}$ (4) $\frac{5\sqrt{2}}{3}$

Ans. **NTA (1)**
RESO(1)

Sol. $Ar = \frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{BD}\right|$ $\overrightarrow{AC} = -\hat{i}+\hat{j}+2\hat{k}$ $\overrightarrow{BD} = \frac{5\hat{i}}{3}-\frac{5\hat{j}}{3}-\frac{2\hat{k}}{3}$

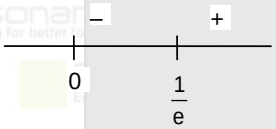
$$\frac{1}{2}\left(\overrightarrow{AC}\times\overrightarrow{BD}\right) = \frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 1 & 2\\ \frac{5}{3} & -\frac{5}{3} & -\frac{2}{3}\end{vmatrix} = = \frac{1}{6}\left(8\hat{i}+8\hat{j}\right) = \left|\frac{1}{2}\left(\overrightarrow{AC}\times\overrightarrow{BD}\right)\right| = \frac{4\sqrt{2}}{3}$$

14. The interval in which the function $f(x) = x^x$, $x > 0$, is strictly increasing is

(1) $\left[\frac{1}{e^2},1\right)$ (2) $\left(0,\frac{1}{e}\right]$ (3) $\left[\frac{1}{e},\infty\right)$ (4) $(0, \infty)$

Ans. **NTA (3)**
RESO(3)

Sol. $f'(x) = (x.\, x^{x-1} + x^x \ell nx)$
$= x^x (1 + \ell nx)$



In $x \in \left[\frac{1}{e},\infty\right)$, f(x) is strictly increasing function.

15. The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is

(1) 56 (2) 48 (3) 16 (4) 24

Ans. **NTA (3)**
RESO(3)

Sol. No. of Δ = $^8C_3 - 8 - 4 \times 8$
= $56 - 8 - 32$
= 16

16. Let $A = \{n \in [100, 700] \cap N: n$ is neither a multiple of 3 nor a multiple of 4$\}$. Then the number of elements in A is

(1) 280 (2) 290 (3) 300 (4) 310

Ans. **NTA (3)**
RESO(3)

Sol. n (divisor by 3) = 233 – 33 = 200
n (divisor by 4) = 175 – 24 = 151
n (divisor by 4 & 3) = 58 – 8 = 50
n (divisor by 4 or 3) = 200 + 151 – 50 = 301
n (which are neither divisible by 3 nor by 4) = 601 – (301) = 300

17. A company has two plants A and B to manufacture motorcycles. 60% motorcycles are manufactured at plant A and the remaining are manufactured at plant B. 80% of the motorcycles manufactured at plant A are rated of the standard quality, while 90% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B, then 126p is

(1) 54 (2) 64 (3) 56 (4) 66

Ans. **NTA (1)**
RESO(1)

Sol. Now P(standard) = $P\left(\frac{St}{A}\right) \times P(A) + +P\left(\frac{St}{B}\right)\times P(B)$

$= \frac{60}{100}\times\frac{80}{100}+\frac{40}{100}\times\frac{90}{100}=\frac{84}{100}$

Now $P\left(\frac{B}{st}\right)=\frac{P\left(\frac{st}{B}\right)\times P(B)}{P(st)} = \frac{\frac{36}{100}}{\frac{84}{100}} = \frac{3}{7}$

So, 126 p = 54

18. If $f(x) = \begin{cases} x^3\sin\left(\frac{1}{x}\right), x\neq 0 \\ 0 \quad\quad\quad , x=0 \end{cases}$, then

(1) f'' (0) = 0 (2) $f''\left(\frac{2}{\pi}\right)=\frac{24-\pi^2}{2\pi}$ (3) f'' (0) = 1 (4) $f''\left(\frac{2}{\pi}\right)=\frac{12-\pi^2}{2\pi}$

Ans. **NTA (2)**
RESO(2)

Sol. $f'(x)=x^3\cos\left(\frac{1}{x}\right).\left(\frac{-1}{x^2}\right)+\left(\sin\frac{1}{x}\right)3x^2$

$= -x\cos\frac{1}{x}+3x^2\sin\frac{1}{x}$

$f''(x)=x\sin\frac{1}{x}\left(-\frac{1}{x^2}\right)-\cos\frac{1}{x}+3x^2\left(\cos\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)+3\sin\left(\frac{1}{x}\right)(2x)$

$f''(x)=-\frac{1}{x}\sin\frac{1}{x}-4\cos\frac{1}{x}+6x\sin\left(\frac{1}{x}\right)$

Here f''(0) is not defined as it is oscillating

$f''\left(\frac{2}{\pi}\right)=-\frac{\pi}{2}\times 1-0+\frac{12}{\pi}=\frac{24-\pi^2}{2\pi}$

19. For $\alpha, \beta \in R$ and a natural number n, let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$. Then $2A_{10} - A_8$ is

(1) 2n (2) $4\alpha + 2\beta$ (3) $2\alpha + 4\beta$ (4) 0

Ans. **NTA (2)**
RESO(2)

Sol. $a_r = 2\begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ r & 1 & \frac{n^2}{2}-\frac{\beta}{2} \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$

$R_1 \to R_1 - R_2$

$= 2\begin{vmatrix} 0 & 0 & \alpha+\frac{\beta}{2} \\ r & 1 & \frac{n^2}{2}-\frac{\beta}{2} \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix} = 2\left(\alpha+\frac{\beta}{2}\right)\{3r - 3r + 2\}$

$a_r = 4\alpha + 2\beta$

Now $2a_{10} - a_8 = 4\alpha + 2\beta$

20. Let a variable line of slope $m > 0$ passing through the point $(4, -9)$ intersect the coordinate axes at the points A and B. The minimum value of the sum of the distances of A and B from the origin is

(1) 10 (2) 30 (3) 15 (4) 25

Ans. **NTA (4)**

RESO(4)

Sol. Let line is $y + 9 = m(x-4)$

$A\left(\frac{9}{m}+4,0\right)$ and B $(0,-9-4m)$

Now (OA + OB)

$= \frac{9}{m}+4+|-9-4m|$

$= 13 + \frac{9}{m} + 4m$

$(OA + OB)_{min} = 25$ By AM $\geq$ GM

21. Let a conic C pass through the point $(4, -2)$ and $P(x, y)$, $x \geq 3$, be any point on C. Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and $(3, -5)$. If the focal distance of the point $(7, -1)$ on C is d, then 12d equal_____.

Ans. **NTA (75)**

RESO(75)

Sol. Given

$\frac{dy}{dx} = \frac{y+5}{2(x-3)}$

$\frac{2dy}{y+5} = \frac{dx}{x-3}$

$2\ln|y + 5| = \ln|x - 3| + \ln c$

P(4,–2) lies on this

$2\ln 3 = \ln 1 + \ln c$

$c = 9$

Now conic is

$(y + 5)^2 = 9(x - 3)$

now focal distance of pt (7,1)

is = d = $|x_1 - 3 + a|$

$= \left|7 - 3 + \frac{9}{4}\right|$

$= \left|\frac{25}{4}\right|$

Now 12d = 75

22. Let $\alpha\beta\gamma = 45$; $\alpha, \beta, \gamma \in R$,. If $x(\alpha, 1, 2) + y(1, \beta, 2) + z(2, 3, \gamma) = (0, 0, 0)$ for some $x, y, z \in R$, $xyz \neq 0$, then $6\alpha + 4\beta + \gamma$ is equal to_____

Ans. **NTA (55)**
RESO(55)

Sol. $\alpha x + y + 2z = 0$
$x + y\beta + 3z = 0$
$2x + 2y + \gamma z = 0$
D = 0

$$\begin{vmatrix} \alpha & 1 & 2 \\ 1 & \beta & 3 \\ 2 & 2 & \gamma \end{vmatrix} = 0$$

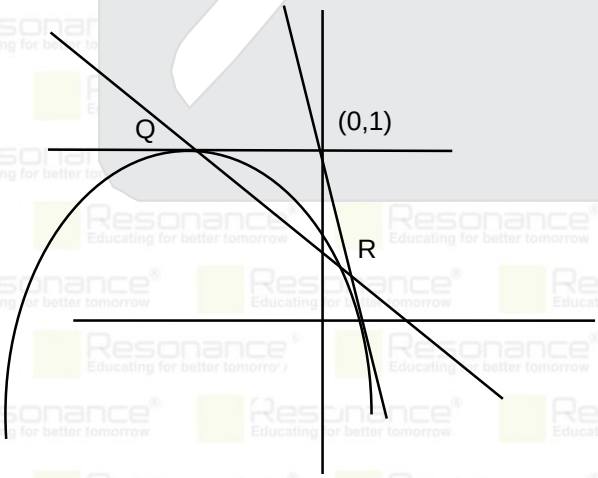
$\alpha(\beta\gamma - 6) - 1(\gamma - 6) + 2(2 - 2\beta) = 0$
$45 - 6\alpha - \gamma + 6 + 4 - 4\beta = 0$
$6\alpha + 4\beta + \gamma = 55$

23. Let L_1, L_2 be the lines passing through the point P(0, 1) and touching the parabola $9x^2 + 12x + 18y - 14 = 0$. Let Q and R be the points on the lines L_1 and L_2 such that the ΔPQR is an isosceles triangle with base QR. If the slopes of the lines QR are m_1 and m_2, then $16\left(m_1^2 + m_2^2\right)$ is equal to____.

Ans. **NTA (68)**
RESO(68)

Sol. $(3x + 2)^2 = -18y + 14 + 4$

$$\left(x + \frac{2}{3}\right)^2 = -2(y - 1)$$



Let tangent be y = mx + 1
then

$\left(x+\frac{2}{3}\right)^2 = -2(mx+1-1)$

$x^2 + x\left(\frac{4}{3}+2m\right)+\frac{4}{9}=0$

tangent D = 0

$\left(\frac{4}{3}+2m\right)^2-\frac{16}{9}=0$

$m = 0 - \frac{4}{3}$

tangent one y = 1

and $y=\frac{-4x}{3}+1$

Now slope of QR

are slope of angle bisectors of tangents

$\frac{4x+3y-3}{5}=\pm\left(\frac{y-1}{1}\right)$

Slopes $m_1 = 2$

and $m_2 = -\frac{1}{2}$

So, $16\left(m_1^2+m_2^2\right)$

$= 16\left(4+\frac{1}{4}\right) = 68$

24. Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}, \vec{b}=3\hat{i}+4\hat{j}-5\hat{k}$ and a vector $\vec{c}$ be such that $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times\vec{c}=\hat{i}+8\hat{j}+13\hat{k}$. If $\vec{a}\times\vec{c}=13$ then $\left(24-\vec{b}.\vec{c}\right)$ is equal to_____

Ans. **NTA (46)**

RESO(46)

Sol. $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times\vec{c}=\hat{i}+8\hat{j}+13\hat{k}$

taking cross product with $\vec{b}$ both side

$\vec{a}\times(\vec{a}\times(\vec{b}+\vec{c}))+\vec{a}\times(\vec{b}\times\vec{c})=\vec{a}\times(\hat{i}+8\hat{j}+13\hat{k})$

$\Rightarrow (\vec{a}.\vec{b}+\vec{a}.\vec{c})\vec{a}-(\vec{a}.\vec{a})(\vec{b}+\vec{c})+(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=\vec{a}\times(\hat{i}+8\hat{j}+13\hat{k})$

$\Rightarrow (-26+13)\ \vec{a}-29(\vec{b}+\vec{c})+13\vec{b}-(-26)\vec{c} = \vec{a}\times(\hat{i}+8\hat{j}+4\hat{k})$

$-13\vec{a}-16\vec{b}-3\vec{c}=(2\hat{i}-3\hat{j}+4\hat{k})\times(\hat{i}+8\hat{j}+13\hat{k})$

$(-26\hat{i}+39\hat{j}-52\hat{k})-(48\hat{i}+64\hat{j}-80\hat{k})-3\vec{c}=-71\hat{i}-22\hat{j}+19\hat{k}$

$-3\hat{i}-3\hat{j}+9\hat{k}=3\vec{c}$

$\vec{c}=-\hat{i}-\hat{j}+3\hat{k}$

Now $24-\vec{b}.\vec{c}$

$= 24 - (-3 - 4 - 5)$

$= 46$

25. Let $r_k = \dfrac{\int_0^1 (1-x^7)^k dx}{\int_0^1 (1-x^7)^{k+1} dx}$, $k \in N$. Then the value of $\sum_{k=7}^{10} \dfrac{1}{7(rk-1)}$ is equal to_____

Ans. **NTA (65)**
RESO(65)

Sol. $r_k = \dfrac{\int_0^1 (1-x^7)^k dx}{\int_0^1 (1-x^7)^{k+1} dx}$

Let $I_{k+1} = \int_0^1 (1-x^7)^{k+1} dx$

$$= \left[(1-x^7)^{k+1} x - \int (k+1)(1-x^7)^k (-7x^6) x\, dx \right] = -7(k+1)\int_0^1 (1-x^7)^k (1-x^7-1) dx$$

$I_{K+1} = -7(x+1)\left[I_{K+1} - I_H\right]$

$7(K+1)I_K = (7K+8)I_{K+1}$

$r^k = \dfrac{I_K}{I_{k+1}} = \dfrac{7K+8}{7(K+1)}$

Now $\sum_{k=1}^{10} \dfrac{1}{7(r_{k-1})}$

$\sum_{k=1}^{10} (k+1) = \dfrac{10\times 11}{2} + 10 = 65$

26. Let the first term of a series be $T_1 = 6$ and its r^{th} term $T_r = 3T_{r-1} + 6^r$, r = 2, 3, ... n. If the sum of the first n terms of this series is $\frac{1}{5}(n^2 - 12n + 39)(4.6^n - 5.3^n + 1)$, then n is equal to _____

Ans. **NTA (6)**
RESO(6)

Sol. $T_1 = 6$

$T_2 = 3 \times 6 + 6^2 = 6^2 \times \left(\frac{1}{2} + 1\right) = \frac{3}{2} \times 6^2$

Similarly $T_3 = 6^3 \times \frac{7}{4}$

Now $S_n = \left(\frac{2-1}{2^0}\right)6 + \left(\frac{2^2-2}{2}\right)6^2 + \left(\frac{2^3-1}{2^2}\right)6^3 + ...$

$S_n = \left(2\times 6 + 2\times 6^2 + 2\times 6^3 + ...n\text{terms}\right) - \left(\frac{6}{2^0} + \frac{6^2}{2^1} + \frac{6^3}{2^2} + ...n\text{terms}\right)$

$= \frac{12}{5}\times 6^n - \frac{12}{5} - \frac{6}{2}\times 3^n + 3 = \frac{3}{5}\left[4\times 6^n - 5(3^n) + 1\right]$

But $Sn = \frac{1}{5}(n^2 - 12n + 39)(4.6^n - 5.3^n + 1)$

So, $n^2 - 12n + 39 = 3$
$(n-6)^2 = 0$
$n = 6$

27. Let P be the point (10, –2, –1) and Q be the foot of the perpendicular drawn from the point R(1, 7, 6) on the line passing through the points (2, –5, 11) and (–6, 7, –5). Then the length of the line segment PQ is equal to_____

Ans. **NTA (13)**
RESO(13)

Sol. Line joining P(2,–5, 11) and T(–6, 7, –5)

$$\frac{x-2}{8}=\frac{y+5}{-12}=\frac{z-11}{16}=\lambda$$

Then, Q $(8\lambda + 2, -12\lambda - 5, 16\lambda + 11)$

Now d.r's of QR

$8\lambda + 1, -12\lambda - 12, 16\lambda + 5$

Now PT$\perp$ QR

$(8\lambda + 1)8 + (-12\lambda - 12)(-12) + (16\lambda + 5)16 = 0$

$\lambda = -\frac{1}{2}$

Q (–2, 1, 3)

distance PQ = $\sqrt{144+9+16} = 13$

28. For $n \in N$, if $\cot^{-1} 3 + \cot^{-1} 4 + \cot^{-1} 5 + \cot^{-1} n = \frac{\pi}{4}$, then n is equal to_____

Ans. **NTA (47)**
RESO(47)

Sol.

$$\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{5}=\frac{\pi}{4}-\tan^{-1}\frac{1}{n}$$

$$\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{4}}{1-\frac{1}{12}}\right)+\tan^{-1}\frac{1}{5}=\frac{\pi}{4}-\tan^{-1}\frac{1}{n}$$

$$\tan^{-1}\frac{7}{11}+\tan^{-1}\frac{1}{5}=\frac{\pi}{4}-\tan^{-1}\frac{1}{n}$$

$$\tan^{-1}\left(\frac{\frac{7}{11}+\frac{1}{5}}{1-\frac{7}{55}}\right)=\frac{\pi}{4}-\tan^{-1}\frac{1}{n}$$

$$\frac{46}{48}=\tan\left(\frac{\pi}{4}-\tan^{-1}\frac{1}{n}\right)$$

$$\frac{46}{48}=\frac{n-1}{n+1}$$

n = 47

29. Let x_1, x_2, x_3, x_4, be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and $\left(4+x_1^2\right)\left(4+x_2^2\right)\left(4+x_3^2\right)\left(4+x_4^2\right)=\frac{125}{16}$ m. Then the value of m is_____

Ans. **NTA (221)**
RESO(221)

Sol. x_1 is root

then

$$4x_1^4+8x_1^3-17x_1^2-12x_1+9=0$$

Now Let $y = x_1^2+4$

$$\pm\sqrt{y-4}=x_1$$

$$4(y-4)^2+8(y-4)\left(\sqrt{y-4}\right)-17(y-4)-12\left(\sqrt{y-4}\right)+9=0$$

$\left|4(y-4)^2-17(y-4)+9\right|^2 = (y-4)(44-8y)^2 = 0$

equation has roots $(4 + x_1^2)$, ……

So product

$= \frac{\text{constant term}}{\text{coefficient } y^4}$

$= \frac{141^2+4\times44^2}{4^2} = \frac{125}{16}m$

$m = 221$

30. If the second, third and fourth terms in the expansion of $(x + y)^n$ are 135, 30 and $\frac{10}{3}$, respectively, then $6(n^3 + x^2 + y)$ is equal to_____

Ans. **NTA (806)**

RESO(806)

Sol. $T_2 = {}^nC_1\, x^{n-1} y^1 = 135$ _____ (1)

$T_3 = {}^nC_2\, x^{n-2} y^2 = 30$ _____ (2)

$T_4 = {}^nC_3\, x^{n-3} y^3 = \frac{10}{3}$ _____ (3)

$\frac{{}^nC_2}{{}^nC_1}\cdot\frac{y}{x} = \frac{30}{135}$ | $\frac{{}^nC_3}{{}^nC_2}\cdot\frac{y}{x} = \frac{1}{9}$

$\frac{n-2+1}{2}\cdot\frac{y}{x} = \frac{2}{9}$ | $\frac{n-3+1}{3}\cdot\frac{y}{x} = \frac{1}{9}$

$9(n-1)\,y = 4x$ ____(4) $3(n-2)\,y = x$ ____(5)

$\Rightarrow 3\left(\frac{n-1}{n-2}\right) = 4$

$\Rightarrow 3n - 3 = 4n - 8$

$\Rightarrow n = 5$

$9y = x$

${}^5C_1.\, x^4 \left(\frac{x}{9}\right) = 135$

$x^5 = 9 \times 27$

$x = 3$

$y = \frac{1}{3}$

Now $6(n^3 + x^2 + y)$

$= 6\left(125+9+\frac{1}{3}\right)$

$= 6\left(134+\frac{1}{3}\right)$

$= 804 + 2$

$= 806$

PART : PHYSICS

31. Four particles A, B, C, D of mass $\frac{m}{2}$, m, 2m, 4m, have same momentum, respectively. The particle with maximum kinetic energy is:

$\frac{m}{2}$, m, 2m, तथा 4m द्रव्यमान के क्रमशः A, B, C, D चार कणों का संवेग एक समान है। अधिकतम गतिज ऊर्जा का कण होगा :

(1) B (2) A (3) D (4) C

Ans. **(2)**

Sol. K.E. = $\frac{p^2}{2m}$

p = same

$\Rightarrow$ K.E. $\propto \frac{1}{\text{mass}}$

$\Rightarrow$ $\frac{m}{2}$ mass will have maximum K.E.

32. Which of the following phenomena does not explain by wave nature of light.

A. reflection
B. diffraction
C. photoelectric effect
D. interference
E. polarization

Choose the most appropriate answer from the options given below:

निम्न में से कौन सी घटना प्रकाश की तरंग प्रकृति द्वारा वर्णित नहीं होती है :

A. परावर्तन
B. विवर्तन
C. प्रकाश विद्युत प्रभाव
D. व्यतिकरण
E. ध्रुवण

सही उत्तर चुनें

(1) C only (2) B, D only (3) E only (4) A, C only
(1) केवल C (2) केवल B, D (3) केवल E (4) केवल A, C

Ans. **(1)**

33. While measuring diameter of wire using screw gauge the following readings were noted. Main scale reading is 1 mm and circular scale reading is equal to 42 divisions. Pitch of screw gauge is 1 mm and it has 100 divisions on circular scale. The diameter of the wire is $\frac{x}{50}$ mm. The value of x is:

पेंचमापी से तार का व्यास नापते समय निम्न पाठ्यांक मिले

मुख्य पैमाने का पाठ्यांक = 1 mm

गोल पैमाने का पाठ्यांक = 42 भाग

स्क्रूगेज का पिंच 1 mm है, गोल पैमाने पर भाग संख्या 100 है। तार का व्यास $\frac{x}{50}$ mm है। तब x का मान है:

(1) 71 (2) 42 (3) 21 (4) 142

Ans. **(1)**

Sol. least count = $\frac{\text{Pich}}{\text{No. of division}}$

$= \frac{1\,mm}{100} = 0.01\,mm$

Final reading = MsR + n × LC

= 1 mm + 42 × 0.01 m

= 1.42 mm

$= \frac{71}{50}\,mm$

x = 71

34. A sample contains mixture of helium and oxygen gas. The ratio of root mean square speed of helium and oxygen in the sample, is:

एक नमूने में हीलियम तथा ऑक्सीजन गैस का मिश्रण है। इसमें हीलियम एवं ऑक्सीजन की वर्ग माध्य मूल चालों का अनुपात है:

(1) $\frac{2\sqrt{2}}{1}$ (2) $\frac{1}{32}$ (3) $\frac{1}{4}$ (4) $\frac{1}{2\sqrt{2}}$

Ans. **(1)**

Sol. $V_{rms} = \sqrt{\frac{3RT}{M_0}}$

$V_{rms} \propto \frac{1}{\sqrt{M_0}}$

$\frac{V_{rmsH_e}}{V_{rmsO_2}} = \sqrt{\frac{M_{0O_2}}{M_{0H_e}}} = \sqrt{\frac{32}{4}} = \frac{2\sqrt{2}}{1}$

35. The ratio of the shortest wavelength of Ballmer series to the shortest wavelength of Lyman series for hydrogen atom is:

हाइड्रोजन परमाणु के लिए बॉमर श्रेणी की निम्नतम तरंगदैर्ध्य तथा लाईमन श्रेणी की निम्नतम तरंगदैर्ध्य का अनुपात है :

(1) 1 : 2 (2) 2 : 1 (3) 1 : 4 (4) 4 : 1

Ans. **(4)**

Sol. For transition $n_2 - n_1$, wavelength is given by

$\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For shortest wavelength in lymen

$n_1 = 1$ and $n_2 \to \infty$

$\frac{1}{\lambda_L} = Rz^2$ .....(i)

For shortest wavelength in balmer series

$n_1 = 2$ and $n_2 \to \infty$

$\frac{1}{\lambda_B} = Rz^2\left(\frac{1}{4}\right) = \frac{Rz^2}{4}$ .....(ii)

(i)/ (ii)

$\frac{\lambda_B}{\lambda_L} = \frac{4}{1}$

36. Electromagnetic waves travel in a medium with speed of 1.5×10^8 m s^{-1}. The relative permeability of the medium is 2.0. The relative permittivity will be:
किसी माध्यम में विद्युतचुम्बकीय तरंगें 1.5×10^8 m s^{-1} की चाल से गति करती है। माध्यम की सापेक्ष चुंबकशीलता 2.0 है, तब सापेक्ष वैद्युतशीलता होगी:
(1) 2 (2) 1 (3) 5 (4) 4

Ans. **(1)**

Sol. $v = \frac{c}{\sqrt{\mu_r \in_r}}$, $\mu_r = 2$

$$\in_r = \frac{c^2}{v^2 \mu_r} = \frac{4}{2} = 2$$

37. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (where $m_2 > m_1$). If the acceleration of the system is $\frac{g}{\sqrt{2}}$ then the ratio of the masses $\frac{m_1}{m_2}$ is :
द्रव्यमान m_1 एवं m_2 (जहाँ $m_2 > m_1$) के दो गुटकों को जोड़ती हुई एक हल्की रस्सी किसी हल्की चिकनी घिरनी से गुजरती है। यदि इस निकाय का त्वरण $\frac{g}{\sqrt{2}}$ हो तब दोनों द्रव्यमानों का अनुपात $\frac{m_1}{m_2}$ है:

(1) $\frac{\sqrt{3}+1}{\sqrt{2}-1}$ (2) $\frac{1+\sqrt{5}}{\sqrt{2}-1}$ (3) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ (4) $\frac{1+\sqrt{5}}{\sqrt{5}-1}$

Ans. **(3)**

Sol. $a = \frac{(m_2 - m_1)g}{m_1 + m_2} = \frac{g}{\sqrt{2}}$

$$\frac{m_2 - m_1}{m_2 + m_1} = \frac{1}{\sqrt{2}}$$

$$\frac{m_1}{m_2} = \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

38. To find the spring constant (k) of a spring experimentally, a student commits 2% positive error in the measurement of time and 1% negative error in measurement of mass. The percentage error in determining value of k is:
प्रयोग द्वारा, स्प्रिंग का स्प्रिंग नियतांक (k) ज्ञात करने में छात्र समय मापन में 2% धनात्मक त्रुटि कर देता है, परन्तु द्रव्यमान ज्ञात करने में 1% की ऋणात्मक त्रुटि कर देता है। k का मान ज्ञात करने में वह कितने प्रतिशत कुल त्रुटि करता है:
(1) 4% (2) 1% (3) 3% (4) 5%

Ans. **(4)**

Sol. $T = 2\pi\sqrt{\frac{k}{m}}$

$$\frac{dT}{T} \times 100 = \frac{1}{2}\frac{dk}{k} \times 100 - \frac{1}{2}\frac{dm}{m} \times 100$$

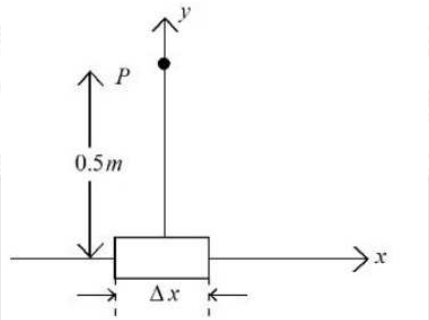
$$2 = \frac{1}{2}\left(\frac{dk}{k} \times 100 - 1\right)$$

$$\frac{dk}{k} \times 100 = 2 \times 2 + 1$$

% error in k = 5%

39. An element $\Delta\ell = \Delta x\,\hat{i}$ is placed at the origin and carries a large current $I = 10A$. The magnetic field on the y-axis at a distance of 0.5m from the elements Δx of 1cm length is:

लम्बाई $\Delta\ell = \Delta x\,\hat{i}$ का एक अवयव मूल बिन्दु पर रखा है जिसमें धारा $I = 10A$ बह रही है। इसमें 0.5m की दूरी पर y-अक्ष पर चुम्बकीय क्षेत्र होगा



(1) 10×10^{-8} T (2) 4×10^{-8} T (3) 8×10^{-8} T (4) 12×10^{-8} T

Ans. **(2)**

Sol. $dB = \dfrac{\mu_0 I d\ell \sin\theta}{4\pi r^2}$

$\theta = 90°$

$dB = \dfrac{10^{-7} \times 10 \times 10^{-2} \times \sin 90°}{25 \times 10^{-2}}$

$dB = 4 \times 10^{-8}$ T

40. Given below are two statements:

Statement I: In an LCR series circuit, current is maximum at resonance.

Statement II: Current in a purely resistive circuit can never be less than that in a series LCR circuit when connected to same voltage source.

In the light of the above statements, choose the correct from the options given below:

नीचे दो कथन दिये गये हैं :

कथन **I:** एक LCR श्रेणीबद्ध परिपथ में अनुनाद पर धारा अधिकतम है।

कथन **II:** किसी शुद्ध प्रतिरोध वाले परिपथ में धारा किसी LCR परिपथ की धारा से कम नहीं हो सकती, यदि दोनों समान वोल्टेज स्रोत से लगे हों।

ऊपर लिखे कथन के अनुसार निम्नमें से सही उत्तर चुनें:

(1) Statement I is false but Statement II is true
(2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are true
(4) Both Statement I and Statement II are false

(1) कथन I असत्य है किन्तु II सत्य है।
(2) कथन I सत्य है किन्तु कथन II असत्य है।
(3) कथन I तथा कथन II दोनों सत्य हैं।
(4) कथन I तथा कथन II दोनों असत्य हैं।

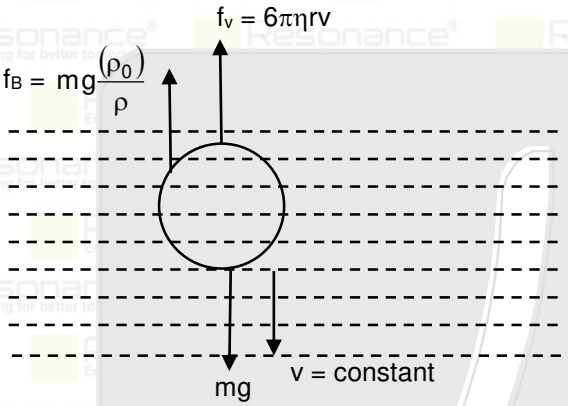
Ans. **(3)**

41. A small ball of mass m and density ρ is dropped in a viscous liquid of density ρ_0. After sometime, the ball falls with constant velocity. The viscous force on the ball is:
m द्रव्यमान व ρ घनत्व की एक छोटी गेंद ρ_0 घनत्व के द्रव में गिरायी जाती है। कुछ समय पश्चात् गेंद स्थिर गति से गिरने लगती है। गेंद पर श्यान बल है :

(1) $mg\left(1-\frac{\rho_0}{\rho}\right)$ (2) $mg(1-\rho\rho_0)$ (3) $mg\left(\frac{\rho_0}{\rho}-1\right)$ (4) $mg\left(1+\frac{\rho}{\rho_0}\right)$

Ans. **(1)**

Sol.



For terminal state

$f_v + f_B = mg$

$6\pi\eta rv + mg\frac{(\rho_0)}{\rho} = mg$

$f_v = mg\left(1-\frac{\rho_0}{\rho}\right)$

42. A train starting from rest first accelerates uniformly up to a speed of 80 km/h for time t, then it moves with a constant speed for time 3t. The average speed of the train for this duration of journey will be (in km/h):
एक रेलगाड़ी विरामावस्था से प्रारम्भ होकर समान त्वरण से 80 km/h गति को समय t में प्राप्त कर लेती है। इसके बाद उसी स्थिर गति से समय 3t तक चलती है। इस अंतराल में गाड़ी की औसत गति (km/h में होगी):

(1) 80 (2) 40 (3) 70 (4) 30

Ans. **(3)**

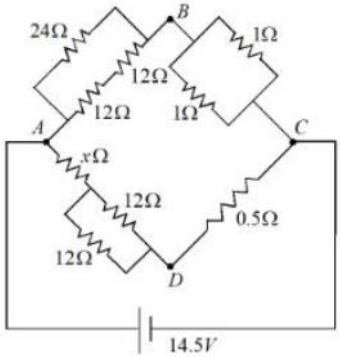
Sol. Avg speed $= \frac{\text{Distance trevelled}}{\text{total time}}$

$= \frac{\text{Area urder speedtime greph}}{\text{total time}}$

$= \frac{\frac{1}{2}\times(4t+3t)(80)}{4t} = \frac{10(7t)}{t}$

Average speed = 70 km/hr

43. The value of unknown resistance (x) for which the potential difference between B and D will be zero in the arrangement shown, is:
दिये गये विन्यास में बिन्दु B तथा D के बीच विभवान्तर शून्य होने पर अज्ञात प्रतिरोध (x) का मान है :



(1) 3Ω (2) 9Ω (3) 6Ω (4) 42Ω

Ans. **(3)**

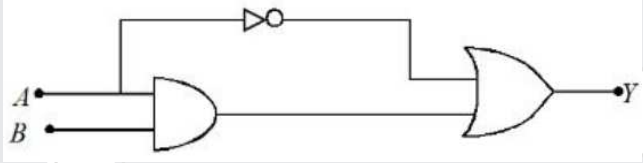
Sol. For $V_B = V_D$. It should be balanced Wheat stone bridge.

$$\frac{L}{K} = \frac{N}{M}$$

$$\frac{6+x}{12} = \frac{0.5}{0.5}$$

$x = 6\ \Omega$

44. The correct truth table for the following logic circuit is :
निम्न तार्किक परिपथ के लिए सही तालिका कौन सी है :



(1)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(3)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1

(4)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

Ans. **(3)**

Sol. $Y = AB + \overline{A}$

45. The specific heat at constant pressure of a real gas obeying $PV^2 = RT$ equation is:
एक वास्तविक गैस जो समीकरण $PV^2 = RT$ का पालन करती है, स्थिर दाब पर इसकी विशिष्ट ऊष्मा है:

(1) $C_V + \frac{R}{2V}$ (2) R (3) $C_V + R$ (4) $\frac{R}{3} + C_V$

Ans. **(1)**

Sol. $PV^2 = nRT$

For P = constant,

$P(2V)dV = nRdT \Rightarrow pdV = \frac{nRdT}{2V}$

$dQ = dW + dU,\ dQ = PdV + nC_VdT$

$dQ = \frac{nRdT}{2V} + nC_VdT \quad \frac{dQ}{ndT} = CP = C_V + \frac{R}{2V}$

$C_P = C_V + \frac{R}{2V}$

46. A bullet of mass 50g is fired with a speed 100 m/s on a plywood and emerges with 40 m/s. The percentage loss of kinetic energy is:

50g द्रव्यमान की एक बन्दूक की गोली 100 m/s से प्लाईवुड पर दागी जाती है एवं यह 40 m/s से पारगतिम होती है। गतिज ऊर्जा में प्रतिशत हानि है:

(1) 16% (2) 44% (3) 32% (4) 84%

Ans. **(4)**

Sol. $\%\ \text{loss} = \frac{\Delta K}{K_i} \times 100\% = \frac{\frac{1}{2}m(100)^2 - \frac{1}{2}m(40)^2}{\frac{1}{2}m(100)^2} \times 100$

$\%\ \text{loss} = \frac{10^4 - 1600}{10^4} \times 100 = \frac{8400}{10^4} \times 100$

% loss = 84%

47. In photoelectric experiment energy of 2.48 eV irradiates a photo sensitive material. The stopping potential was measured to be 0.5 V. Work function of the photo sensitive material is:

किसी प्रकाश–विद्युत प्रयोग में एक प्रकाश सुग्राही पदार्थ पर 2.48 eV ऊर्जा का विकिरण आपतित किया जाता है। इस समय निरोधी विभव 0.5 V नापा गया हो तब पदार्थ का कार्य–फलन होगा :

(1) 2.48 eV (2) 1.68 eV (3) 1.98 eV (4) 0.5 eV

Ans. **(3)**

Sol. $e.V_s = E - \phi$

$0.5\ eV = 2.48\ eV - \phi$

$\phi = 2.48 - 0.5$

$\phi = 1.98\ eV$

48. σ is the uniform surface charge density of a thin spherical shell of radius R. The electric field at any point on the surface of the spherical shell is:

R त्रिज्या के एक पतले गोलीय कोश का एकसमान पृष्ठ आवेश घनत्व σ है। इस गोलीय कोश के पृष्ठ के किसी बिन्दु पर वैद्युत क्षेत्र है:

(1) $\frac{\sigma}{2\epsilon_0}$ (2) $\frac{\sigma}{4\epsilon_0}$ (3) $\frac{\sigma}{\epsilon_0 R}$ (4) $\frac{\sigma}{\epsilon_0}$

Ans. **(4)**

Sol. $E_P = \frac{kQ}{R^2} = \frac{k\sigma 4\pi R^2}{R^2} = \frac{\sigma}{\varepsilon_0}$

49. To project a body of mass m from earth's surface to infinity, the required kinetic energy is (assume, the radius of earth is R_E, g = acceleration due to gravity on the surface of earth):
एक m द्रव्यमान के पिण्ड को पृथ्वी सतह से अनन्त तक फेंकने के लिए आवश्यक गतिज ऊर्जा होगी (यहाँ R_E पृथ्वी की त्रिज्या, g = पृथ्वी सतह पर गुरूत्व त्वरण है):
(1) $2mgR_E$ (2) mgR_E (3) $1/2mgR_E$ (4) $4mgR_E$

Ans. **(2)**

Sol. Escape velocity $V_e = \sqrt{2gR_E}$

so required kinetic energy

$k = \frac{1}{2}mV_e^2$

$= \frac{1}{2}m2gR_E$

$= mgR_E$

50. Match List I with List II

	List I		List II
A.	Torque	I.	$[M^1L^1T^{-2}A^{-2}]$
B.	Magnetic field	II.	$[L^2A^1]$
C.	Magnetic moment	III.	$[M^1T^{-2}A^{-1}]$
D.	Permeability of free space	IV.	$[M^1L^2T^{-2}]$

Choose the correct answer from the options given below:
सूची I का सूची II में मिलान करें

	सूची I		सूची II
A.	बल आघूर्ण	I.	$[M^1L^1T^{-2}A^{-2}]$
B.	चुम्बकीय क्षेत्र	II.	$[L^2A^1]$
C.	चुम्बकीय आघूर्ण	III.	$[M^1T^{-2}A^{-1}]$
D.	मुक्त आकाश की चुंबकशीलता	IV.	$[M^1L^2T^{-2}]$

निम्न में से सही उत्तर चुनें:
(1) A-IV, B-II, C-III, D-I (2) A-III, B-I, C-II, D-IV
(3) A-IV, B-III, C-II, D-I (4) A-I, B-III, C-II, D-IV

Ans. **(3)**

51. A big drop is formed by coalescing 1000 small droplets of water. The ratio of surface energy of 1000 droplets to that of energy of big drop is $\frac{10}{x}$ The value of x is :
पानी की 1000 बूँदें मिलकर एक बड़ी बूँद बनती है। 1000 बूँदों की पृष्ठ ऊर्जा तथा बड़ी एक बूँद की पृष्ठ ऊर्जा का अनुपात $\frac{10}{x}$ हो तो x का मान _____ है:

Ans. **100**

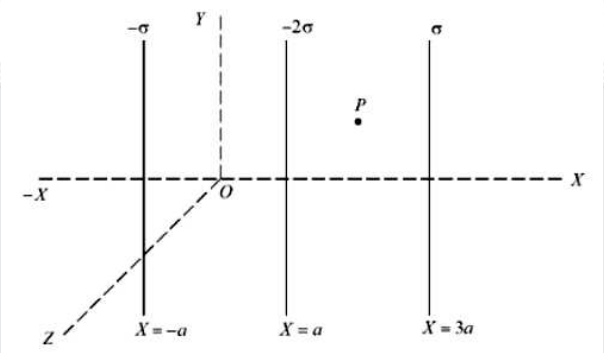
Sol. $V_f = V_i \Rightarrow \frac{4}{3}\pi R^3 = 1000\frac{4}{3}\pi r^3$

$\frac{(4\pi R^2.S)}{1000(4\pi r^2 S)} = \frac{100}{1000} = \frac{10}{100} = \frac{10}{X}$

$X = 100$

52. Three infinitely long charged thin sheets are placed as shown in figure. The magnitude of electric field at the point P is $\frac{x\sigma}{\epsilon_0}$. The value of x is _____(all quantities are measured in SI units).

तीन अनन्त लम्बाई की आवेशित पतली चादरें चित्र के अनुसार रखी हैं। P बिन्दु पर वैद्युत क्षेत्र का मान $\frac{x\sigma}{\epsilon_0}$ है। यहाँ x का मान _____है। (यदि सभी मान SI पद्धति में हैं।).



Ans. **2**

Sol. $E_P = \frac{3\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{2\sigma}{\varepsilon_0}$

53. Radius of a certain orbit of hydrogen atom is 8.48 A. If energy of electron in this orbit is E/x. then x = (Given a_0 = 0.529 Å, E = energy of electron in ground state).

हाइड्रोजन परमाणु की किसी कक्षा की त्रिज्या 8.48 A है। यदि इस कक्षा में इलेक्ट्रॉन की ऊर्जा E/x है तब x = (दिया है a_0 = 0.529 Å, E = इलेक्ट्रॉन की मूल अवस्था में न्यूनतम ऊर्जा)

Ans. **16**

Sol. $r = r_0 \frac{n^2}{Z}$

$8.48 = 0.529\ r_0 \frac{n^2}{Z}$

$n = 4$

$E = E_0 \frac{Z^2}{n^2}$

$E = \frac{E}{4^2} = \frac{E}{16}$

$\therefore X = 16$

54. The refractive index of prism is $\mu = \sqrt{3}$ and the ratio of the angle of minimum deviation to the angle of prism is one. The value of angle of prism is _____°

एक प्रिज्म का अपवर्तनांक $\mu = \sqrt{3}$ है एवं न्यूनतम विचलन कोण तथा प्रिज्म के कोण का अनुपात 1 है। प्रिज्म कोण का मान _____° है।

Ans. **60**

Sol. $A = \delta_m$

$$\frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = n = \sqrt{3}, \quad \frac{2\sin\frac{A}{2}.\cos\frac{A}{2}}{\sin\frac{A}{2}} = \sqrt{3}$$

$$\frac{A}{\delta_m} = 1$$

$$\cos\frac{A}{2} = \frac{\sqrt{3}}{2}$$

$$\frac{A}{2} = 30°$$

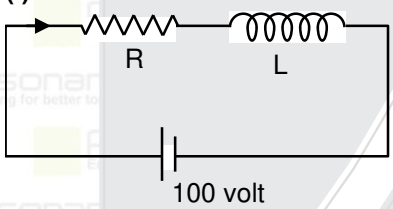
$$A = 60°$$

55. When a dc voltage of 100V is applied to an inductor, a dc current of 5A flows through it. When an ac voltage of 200V peak value is connected to inductor, its inductive reactance is found to be $20\sqrt{3}\,\Omega$. The power dissipated in the circuit is____ W.

एक प्रेरक पर 100V दिष्ट विभव लगाया जाता है तब 5A की दिष्ट धारा बहती है। यदि 200V शिखर मान का प्रत्यावर्ती विभव लगाया जाए तब प्रेरकीय प्रतिघात $20\sqrt{3}\,\Omega$ होता है। परिपथ में शक्ति क्षय ____ W है।

Ans. **250**

Sol. **(i)**

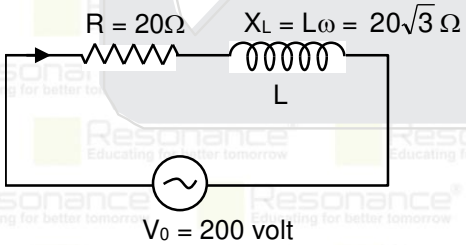


At steady state

$$i = \frac{V}{R} = \frac{100}{R} = 5$$

$$R = 20\Omega$$

(ii)



$$|z| = \sqrt{R^2 + (L\omega)^2}$$

$$= \sqrt{(20)^2 + (20\sqrt{3})^2} = 40\Omega$$

$$i_0 = \frac{v_0}{|z|} = \frac{200}{40} = 5A$$

$$<\text{Power}> = i_{rms}^2 = \left(\frac{i_0}{\sqrt{2}}\right)^2 R = \frac{(5)^2}{2} \times 20 = 250 \text{ watt}$$

56. A circular coil having 200 turns, 2.5×10^{-4} m² area and carrying 100 μA current is 2 placed in a uniform magnetic field of 1T. Initially the magnetic dipole moment ($\vec{M}$) was directed along $\vec{B}$. Amount of work, required to rotate the coil through 90° from its initial orientation such that $\vec{M}$ becomes perpendicular to $\vec{B}$, is_____ μJ.

1T के एकसमान चुम्बकीय क्षेत्र में 200 फेरों, 2.5×10^{-4} m² क्षेत्रफल तथा 100 μA की धारा वाली एक वृत्ताकार कुण्डली रखी गई है। प्रारम्भ में इसका चुम्बकीय द्विध्रुव आघूर्ण ($\vec{M}$) क्षेत्र $\vec{B}$ के अनुदिश है। इसे प्रारम्भिक दिशा में 90° पर घुमाने के लिय, जिससे आघूर्ण $\vec{M}$ क्षेत्र $\vec{B}$ के लम्बवत् हो जाये कृत कार्य का मान _____ μJ है।

Ans. **5**

Sol. $W_{us} = \Delta U = U_f - U_i = (-MB\cos 90°) - (-MB\cos 0°0029$

$= MB = NI\ AB$

$= (200)(10^{-4})(2.5 \times 10^{-4})(1) = 5\mu J$

57. If the radius of earth is reduced to three-fourth of its present value without change in its mass then value of duration of the day of earth will be _____ hours 30 minutes.

पृथ्वी के द्रव्यमान को स्थिर रखकर यदि इसकी त्रिज्या तीन–चौथाई कर दी जाए, तब पृथ्वी का एक दिन _____ घन्टे 30 मिनट का होगा।

Ans. **13**

Sol. According to angular momentum conservation:

$I\omega = \text{constant}$

$\frac{I}{T} = \text{constant}$

$\Rightarrow T \propto I$

$\Rightarrow T \propto R^2$

So $T_{new} = \frac{9}{16} T_{old} = \frac{9}{16} \times 24 = \frac{9}{2} \times 3 = \frac{27}{2} = 13$ hrs. 30 mins

58. A particle is doing simple harmonic motion of amplitude 0.06 m and time period 3.14 s. The maximum velocity of the particle is _____ cm/s.

0.06 m आयाम तथा 3.14 s आवर्तकाल से एक कण सरल आवर्त गति कर रहा है। कण का अधिकतम वेग _____ cm/s है।.

Ans. **12**

Sol. $T = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{T} = 2$

$V_{max} = \omega A$

$= 2 \times 0.6$

$= 12$ cm/sec.

59. For three vectors $\vec{A}=(-x\hat{i}-6\hat{j}-2\hat{k})$, $\vec{B}=(-\hat{i}+4\hat{j}+3\hat{k})$ and $\vec{C}=(-8\hat{i}-\hat{j}+3\hat{k})$, if $\vec{A}.(\vec{B}\times\vec{C})=0$, then value of x is _____

तीन सदिशों $\vec{A}=(-x\hat{i}-6\hat{j}-2\hat{k})$, $\vec{B}=(-\hat{i}+4\hat{j}+3\hat{k})$ तथा $\vec{C}=(-8\hat{i}-\hat{j}+3\hat{k})$ के लिये , यदि $\vec{A}.(\vec{B}\times\vec{C})=0$, हो तो x का मान _____ है।

Ans. **4**

Sol. $\vec{A}.(\vec{B}\times\vec{C})=[\vec{A}\,\vec{B}\,\vec{C}]=\begin{vmatrix}-x & -6 & -2\\ -1 & 4 & 3\\ -8 & -1 & 3\end{vmatrix}=0$

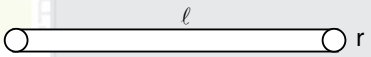
$x=4$

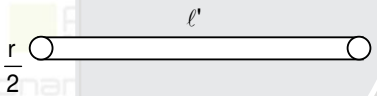
60. A wire of resistance R and radius r is stretched till its radius became r/2. If new resistance of the stretched wire is x R, then value of x is _____

R प्रतिरोध तथा r त्रिज्या का एक तार इस प्रकार खींचा जाता है जिससे इसकी त्रिज्या r/2 हो जाये। तार का नया प्रतिरोध x R हो तब x का मान _____ है।

Ans. **16**

Sol. Resistance $\frac{\rho\ell}{A}$





$\ell(\pi r^2)=\ell'\pi\left(\frac{r}{2}\right)^2$

$\ell'=4\ell$

$R-\frac{\rho\ell}{\pi r^2}$

$R'=\frac{\rho\ell'}{\pi\left(\frac{r}{2}\right)^2}=\frac{\rho(4\ell)(4)}{\pi r^2}=16\left(\frac{\rho\ell}{\pi r^2}\right)$

$R'=16\,R$