PART : MATHEMATICS

1. If the solution y = y(x) of the differential equation $(x^4 + 2x^3 + 3x^2 + 2x + 2)\,dy - (2x^2 + 2x + 3)\,dx = 0$ satisfies $y(-1) = -\frac{\pi}{4}$, then y(0) is equal to :

(1) $-\frac{\pi}{12}$ (2) 0 (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Reso. **(3)**

NTA. **(3)**

Sol. $(x^4 + 2x^3 + 3x^2 + 2x + 2)\,dy = (2x^2 + 2x + 3)\,dx$

$$\Rightarrow \quad dy = \frac{2x^2 + 2x + 3}{x^4 + 2x^3 + 3x^2 + 2x + 2}dx$$

$$\Rightarrow \quad \int dy = \int \frac{\left(2x^2 + 2x + 3\right)}{\left(x^2 + 1\right)\left(x^2 + 2x + 2\right)}\,dx$$

$$\Rightarrow \quad \int dy = \int \left\{\frac{1}{x^2 + 1} + \frac{1}{(x+1)^2 + 1}\right\}dx$$

$$\Rightarrow \quad y(x) = \tan^{-1}x + \tan^{-1}(x+1) + C$$

Now $y = (-1) = -\frac{\pi}{4} \quad \Rightarrow -\frac{\pi}{4} = -\frac{\pi}{4} + 0 + C \Rightarrow C = 0$

Hence $y(x) = \tan^{-1}x + \tan^{-1}(x+1)$

$$y(0) = 0 + \frac{\pi}{4} = \frac{\pi}{4}$$

2. Let the first three term 2, p and q, with $q \neq 2$, of a G.P. be respectively the 7^{th}, 8^{th} and 13^{th} terms of an A.P. If the 5^{th} term of the G.P. is the nth term of the A.P., then n is equal to :

(1) 177 (2) 169 (3) 151 (4) 163

Reso. **(4)**

NTA **(4)**

Sol. 2, p, q are in GP $\Rightarrow p^2 = 2q$ ___(1)

Let first term and common difference of AP are a and d respectively

$\Rightarrow$ $t_7 = a + 6d = 2$ _____ (2)

$t_8 = a + 7d = p$ _____ (3)

$t_{13} = a + 12d = q$ _____ (4)

by (2) & (3) $d = (p - 2)$

and by(2) & (4) $6d = (q - 2)$

$\Rightarrow$ $6(p - 2) = q - 2$

$\Rightarrow$ $6p = q + 10$ _____(5)

by (1) and (5) we have p = 2 or p = 10

but if p = 2 then q = 2 (rejected)

when p = 10 we have q = 50

so GP is 2, 10, 50 , ...

also by (2) and (3) we have

d= 8 and a = –46

Now 5^{th} term of GP = n^{th} term of AP

$\Rightarrow 2(5)^4 = -46 + (n-1)\,8$

$\Rightarrow \frac{1250+46}{8} + 1 = n$

$\Rightarrow n = 163$

3. One of the points of intersection of the curves $y = 1 + 3x - 2x^2$ and $y = \frac{1}{x}$ is $\left(\frac{1}{2}, 2\right)$. Let the area of the region enclosed by these curves be $\frac{1}{24}\left(\ell\sqrt{5}+m\right) - n\log_e\left(1+\sqrt{5}\right)$, where ℓ, m, n $\in$ N. Then ℓ + m + n is equal to

(1) 32 (2) 30 (3) 31 (4) 29

Reso. (2)

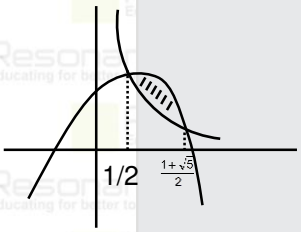
NTA (2)

Sol. Solving curves $y = 1 + 3x - 2x^2$ and $y = \frac{1}{x}$

$2x^3 - 3x^2 - x + 1 = 0$

$(2x-1)(x^2-x-1) = 0$

$x = \frac{1}{2}, \quad x = \frac{1\pm\sqrt{5}}{2}$



$$\text{Area} = \int_{\frac{1}{2}}^{\frac{\sqrt{5}+1}{2}} \left(1+3x-2x^2-\frac{1}{x}\right)dx$$

$$= \left(x + \frac{3x^2}{2} - \frac{2x^3}{3} - \ell n x\right)\Bigg|_{1/2}^{\frac{\sqrt{5}+1}{2}}$$

$$= \frac{\sqrt{5}+1}{2} + \frac{3}{8}\left(\sqrt{5}+1\right)^2 - \frac{1}{12}\left(\sqrt{5}+1\right)^3 - \ell n\left(\frac{\sqrt{5}+1}{2}\right) - \left(\frac{1}{2}+\frac{3}{8}-\frac{1}{12}-\ell n\frac{1}{2}\right)$$

$$= \frac{1}{24}\left[12\left(\sqrt{5}+1\right)+9\left(\sqrt{5}+1\right)^2-2\left(\sqrt{5}+1\right)^3-12-9+2\right] - \ell n\left(\frac{\sqrt{5}+1}{2}\times 2\right)$$

$$= \frac{1}{24}\left[12\left(\sqrt{5}+1\right)+9\left(6+2\sqrt{5}\right)-2\left(5\sqrt{5}+1+3\sqrt{5}\left(\sqrt{5}+1\right)\right)-19\right] - \ell n\left(\sqrt{5}+1\right)$$

$$= \frac{1}{24}\left[14\sqrt{5}+15\right] - \ell n\left(\sqrt{5}+1\right)$$

Now, ℓ = 14, m = 15, n = 1 then ℓ + m + n = 30

4. Let $f(x)=\begin{cases}-2, & -2\le x\le 0\\ x-2, & 0<x\le 2\end{cases}$ and $h(x)=f(|x|)+|f(x)|$. Then $\int_{-2}^{2}h(x)dx$ is equal to :

(1) 2 (2) 6 (3) 4 (4) 1

Reso. **(1)**

NTA **(1)**

Sol. $f(|x|)=\begin{cases}-x-2 & -2\le x<0\\ x-2 & 0\le x\le 2\end{cases}$

And $|f(x)|=\begin{cases}2 & -2\le x<0\\ -(x-2) & 0\le x\le 2\end{cases}$

So $h(x)=\begin{cases}(-x-2)+(2)=-x & -2\le x<0\\ x-2-x+2=0 & 0\le x\le 2\end{cases}$

Hence $\int_{-2}^{2}h(x)dx=\int_{-2}^{0}(-x)dx+\int_{0}^{2}0.dx=-\frac{x^2}{2}\Big|_{-2}^{0}+0=2$

5. The sum of all rational terms in the expansion of $\left(2^{\frac{1}{5}}+5^{\frac{1}{3}}\right)^{15}$ is equal to :

(1) 633 (2) 931 (3) 6131 (4) 3133

Reso. **(4)**

NTA **(4)**

Sol. $T_{r+1}={}^{15}C_r 2^{\frac{15-r}{5}}\times 5^{\frac{r}{3}}$

for rational numbers $\frac{15-r}{5}$ & $\frac{r}{3}$

should be integers

so r = 0, 5, 10, 15 for $\frac{15-r}{5}$

& r = 0, 3, 6, 9, 12, 15 for $\frac{r}{3}$

common values 0, 15

sum of rational terms = $2^3 + 5^5 = 8 + 3125$

= 3133

6. Let a unit vector which makes an angle of 60º with $2\hat{i}+2\hat{j}-\hat{k}$ and an angle of 45º with $\hat{i}-\hat{k}$ be $\vec{C}$.

Then $\vec{C}+\left(-\frac{1}{2}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{\sqrt{2}}{3}\hat{k}\right)$ is :

(1) $\frac{\sqrt{2}}{3}\hat{i}-\frac{1}{2}\hat{k}$

(2) $-\frac{\sqrt{2}}{3}\hat{i}+\frac{\sqrt{2}}{3}\hat{j}+\left(\frac{1}{2}+\frac{2\sqrt{2}}{3}\right)\hat{k}$

(3) $\frac{\sqrt{2}}{3}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{1}{2}\hat{k}$

(4) $\left(\frac{1}{\sqrt{3}}+\frac{1}{2}\right)\hat{i}+\left(\frac{1}{\sqrt{3}}-\frac{1}{3\sqrt{2}}\right)\hat{j}+\left(\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{3}\right)\hat{k}$

Reso. (1)

NTA (1)

Sol. Let $\hat{C} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\hat{C}| = 1 \Rightarrow x^2 + y^2 + z^2 = 1$ _____(1)

as angle between $\hat{C}$ and $2\hat{i} + 2\hat{j} - \hat{k}$ is 60º

$$\Rightarrow \cos 60^\circ = \frac{2x + 2y - z}{3}$$

$\Rightarrow 4x + 4y - 2z = 3$ _____ (2)

and angle between $\hat{C}$ and $(\hat{i} - \hat{k})$ is 45º

$$\Rightarrow \cos 45^\circ = \frac{x - z}{\sqrt{2}}$$

$\Rightarrow x - z = 1$ _____(3)

by (2) and (3)

$4y + 2z + 1 = 0$

$$y = -\left(\frac{2z+1}{4}\right)$$

by (1) $(z+1)^2 + \left(\frac{2z+1}{-4}\right)^2 + z^2 = 1$

$\Rightarrow 16(z^2 + 2z + 1) + (4z^2 + 4z + 1) + 16z^2 = 16$

$\Rightarrow 36z^2 + 36z + 1 = 0$

$$\Rightarrow z = \frac{-36 \pm \sqrt{1296 - 144}}{72}$$

$$\Rightarrow z = \frac{-36 \pm \sqrt{1152}}{72}$$

$$\Rightarrow z = -\frac{1}{2} \pm \frac{24\sqrt{2}}{72}$$

$$\Rightarrow z = -\frac{1}{2} \pm \frac{\sqrt{2}}{3}$$

$$\Rightarrow \hat{C} = (z+1)\hat{i} - \frac{1}{4}(2z+1)\hat{j} + z\hat{k}$$

$$\Rightarrow \vec{C} + \left(-\frac{1}{2}\hat{i} + \frac{1}{3\sqrt{2}}\hat{j} - \frac{\sqrt{2}}{3}\hat{k}\right) = \left(z + \frac{1}{2}\right)\hat{i} - \frac{1}{2}\left(z + \frac{1}{2}\right)\hat{j} + z\hat{k} + \frac{1}{3\sqrt{2}}\hat{j} - \frac{\sqrt{2}}{3}\hat{k}$$

$$= \frac{\sqrt{2}}{3}\hat{i} - \frac{1}{3\sqrt{2}}\hat{j} + \left(-\frac{1}{2} + \frac{\sqrt{2}}{3}\right)\hat{k} + \frac{1}{3\sqrt{2}}\hat{j} - \frac{\sqrt{2}}{3}\hat{k}$$

$= \frac{\sqrt{2}}{3}\hat{i} - \frac{1}{2}\hat{k}$ is Ans. (1)

7. Let $f : R \to R$ be a function given by

$$f(x)=\begin{cases}\dfrac{1-\cos 2x}{x^2}, & x<0\\ \alpha, & x=0\\ \dfrac{\beta\sqrt{1-\cos x}}{x}, & x>0\end{cases}$$

where $\alpha, \beta \in R$. If f is continuous at x = 0, then $\alpha^2 + \beta^2$ is equal to :

(1) 48 (2) 12 (3) 3 (4) 6

Reso. **(2)**

NTA. **(2)**

Sol. f(x) is continuous at x = 0

$\Rightarrow \quad f(0)=\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^+} f(x) \quad \Rightarrow \quad \alpha=\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{x^2}=\lim\limits_{x\to 0}\dfrac{\beta\sqrt{1-\cos x}}{x}$

$\Rightarrow \quad \alpha=\lim\limits_{x\to 0^-}\dfrac{2\sin^2 x}{x^2}=\lim\limits_{x\to 0}\dfrac{\sqrt{2}\,\beta\sin\frac{x}{2}}{x} \quad \Rightarrow \quad \alpha=2=\dfrac{\beta}{\sqrt{2}}$

$\Rightarrow \quad \alpha=2 \ \& \ \beta=2\sqrt{2} \qquad$ So $\quad |\alpha^2+\beta^2| = |4+8| = 12$

8. If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, then the quadratic equation, whose roots are $\dfrac{1}{2a+b}$ and $\dfrac{1}{6a+b}$, is :

(1) $x^2 + 8x + 12 = 0$ (2) $2x^2 + 11x + 12 = 0$

(3) $x^2 + 10x + 16 = 0$ (4) $4x^2 + 14x + 12 = 0$

Reso. **(1)**

NTA. **(1)**

Sol. Sum of roots $-\dfrac{b}{a}=2+6=8 \Rightarrow b=-8a$

Product of roots $\dfrac{1}{a}=12 \Rightarrow a=\dfrac{1}{12} \qquad b=-\dfrac{2}{3}$

Hence new roots $\dfrac{1}{2a+b}=\dfrac{1}{\frac{1}{6}-\frac{2}{3}}=-2$

and $\dfrac{1}{6a+b}=\dfrac{1}{\frac{1}{2}-\frac{2}{3}}=-6$

so other quadratic equation is

$x^2 + 8x + 12 = 0$ Ans. (1)

9. Three urns A, B and C contain 7 red, 5 black; 5 red, 7 black and 6 red, 6 black balls, respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is :

(1) $\frac{4}{17}$ (2) $\frac{7}{18}$ (3) $\frac{5}{18}$ (4) $\frac{5}{16}$

Reso. **(3)**

NTA. **(3)**

Sol. prob. that ball drawn is black

$$= \frac{1}{3}\times\frac{5}{12}+\frac{1}{3}\times\frac{7}{12}+\frac{1}{3}\times\frac{6}{12}$$

Prob. that black ball drawn from bag A is

$$= \frac{\frac{1}{3}\times\frac{5}{12}}{\frac{1}{3}\times\frac{5}{12}+\frac{1}{3}\times\frac{7}{12}+\frac{1}{3}\times\frac{6}{12}}$$

$$= \frac{5}{5+7+6}$$

$$= \frac{5}{18}$$

10. Let $f(x) = x^5 + 2e^{x/4}$ for all $x \in R$. Consider a function g(x) such that (gof) (x) = x for all $x \in R$. Then the value of 8g'(2) is :

(1) 2 (2) 16 (3) 4 (4) 8

Reso. **(2)**

NTA. **(2)**

Sol. $g(f(x)) = x$

$$\Rightarrow \quad g'(f(x)) \cdot f'(x) = 1$$

$$\Rightarrow \quad g'(f(x)) = \frac{1}{f'(x)}$$

Now to find g'(2) let at x = a, f(x) = 2

$$\Rightarrow \quad a^5 + 2e^{a/4} = 2$$

$$\Rightarrow \quad a = 0$$

$$\Rightarrow \quad f(0) = 2$$

Hence $g'(f(0)) = \frac{1}{f'(0)}$

$$\Rightarrow \quad g'(2) = \frac{1}{\left(5x^4+\frac{1}{2}e^{x/4}\right)_{(x=0)}} = \frac{1}{0+\frac{1}{2}} = 2 \qquad \Rightarrow 8g'(2) = 16$$

11. If the system of equations

$x+\left(\sqrt{2}\sin\alpha\right)y+\left(\sqrt{2}\cos\alpha\right)z=0$

$x+(\cos\alpha)\,y+(\sin\alpha)\,z=0$

$x+(\sin\alpha)y-(\cos\alpha)\,z=0$

has a non-trivial solution, then $\alpha\in\left(0,\frac{\pi}{2}\right)$ is equal to :

(1) $\frac{3\pi}{4}$ (2) $\frac{7\pi}{24}$ (3) $\frac{5\pi}{24}$ (4) $\frac{11\pi}{24}$

Reso. **(3)**

NTA. **(3)**

Sol. for non-trivial solution

$$\begin{vmatrix}1 & \sqrt{2}\sin\alpha & \sqrt{2}\cos\alpha\\ 1 & \cos\alpha & \sin\alpha\\ 1 & \sin\alpha & -\cos\alpha\end{vmatrix}=0 \text{ where } \alpha\in\left(0,\frac{\pi}{2}\right)$$

$\Rightarrow$ $1\,(-\cos^2\alpha-\sin^2\alpha)-1\left(-\sqrt{2}\sin\alpha\cos\alpha-\sqrt{2}\sin\alpha\cos\alpha\right)+1\left(\sqrt{2}\sin^2\alpha-\sqrt{2}\cos^2\alpha\right)=0$

$\Rightarrow$ $-1+\sqrt{2}\sin2\alpha-\sqrt{2}\cos2\alpha=0$

$\Rightarrow$ $\sin2\alpha-\cos2\alpha=\frac{1}{\sqrt{2}}$

$\Rightarrow$ $\sin\left(2\alpha-\frac{\pi}{4}\right)=\frac{1}{2}$

$\Rightarrow$ $2\alpha-\frac{\pi}{4}=n\pi+(-1)^n\frac{\pi}{6}\,;\,n\in I$

but $\alpha\in\left(0,\frac{\pi}{2}\right)\Rightarrow\alpha=\frac{5\pi}{24}$

12. The vertices of a triangle are A (–1, 3), B(–2, 2) and C(3, –1). A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the sides of the new triangle nearest to origin is:

(1) $-x+y-\left(2-\sqrt{2}\right)=0$ (2) $x+y-\left(2-\sqrt{2}\right)=0$

(3) $x-y-\left(2+\sqrt{2}\right)=0$ (4) $x+y+\left(2-\sqrt{2}\right)=0$

Reso. **(2)**

NTA. **(2)**

Sol. A (–1, 3); B (–2, 2) and C (3, –1)

$\Rightarrow$ side AB $\equiv$ y = x + 4

side CA $\equiv$ y = –x + 2

and side BC $\equiv$ 5y = –3x + 4

so origin lies out side the triangle ABC

distance of AB from O(0, 0) is $\frac{4}{\sqrt{2}}=2\sqrt{2}$

distance of BC from O(0, 0) is $\frac{4}{\sqrt{34}}$

distance of CA from O(0, 0) is $\frac{2}{\sqrt{2}} = \sqrt{2}$

when each side of Δ ABC shifted by one unit inwards

distance of new sides from origin are $2\sqrt{2}-1$, $\frac{4}{\sqrt{34}}+1$ and $\left(\sqrt{2}-1\right)$ respectively

Clearly new position of CA is nearest to the O(0, 0)

so equation of CA in New position is $x + y - 2 + \sqrt{2} = 0$

13. There are 5 points P_1, P_2, P_3, P_4, P_5 on the side AB, excluding A and B, of a triangle ABC. Similarly there are 6 points P_6, P_7, ... P_{11}, on the side BC and 7 points P_{12}, P_{13}, ... P_{18} on the side CA of the triangle. The number of triangles, that can be formed using the points P_1, P_2, ... P_{18} as vertices, is :

(1) 751 (2) 796 (3) 776 (4) 771

Reso. **(1)**

NTA. **(1)**

Sol. Number of triangles = $^{18}C_3 - (^5C_3 + {}^6C_3 + {}^7C_3) = \frac{18\times17\times16}{6} - (10+20+35) = 816 - 65 = 751$

14. If the domain of the function $\sin^{-1}\left(\frac{3x-22}{2x-19}\right)+\log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is $(\alpha, \beta]$, then $3\alpha + 10\beta$ is equal to :

(1) 97 (2) 100 (3) 98 (4) 95

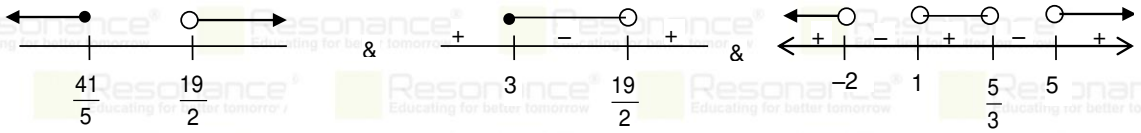
Reso. **(1)**

NTA. **(1)**

Sol. $-1 \le \frac{3x-22}{2x-19} \le 1$ & $\frac{3x^2-8x+5}{x^2-3x-10} > 0$

$\frac{3x-22}{2x-19} \ge -1$ and $\frac{3x-22}{2x-19} \le 1$ & $\frac{(x-1)(3x-5)}{(x-5)(x+2)} > 0$

$\Rightarrow$ $\frac{5x-41}{2x-19} \ge 0$ and $\frac{x-3}{2x-19} \le 0$ and $\frac{(x-1)(3x-5)}{(x-5)(x+2)} > 0$



$\Rightarrow$ $x \in \left(5, \frac{41}{5}\right]$ Hence $3\alpha + 10\beta = 15 + 82 = 97$

15. A square is inscribed in the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. One side of this square is parallel to $y = x + 3$. If (x_i , y_i) are the vertices of the square, then $\sum\left(x_i^2 + y_i^2\right)$ is equal to :

(1) 148 (2) 156 (3) 160 (4) 152

Reso. **(4)**

NTA. **(4)**

Sol. diagonal of square makes 45° angle with side

Let slope of diagonal = m

Then $\tan 45° = \left|\frac{m-1}{1+m}\right|$

So m = not defined and m = 0

So diagonals x = 5 and y = 3 are passing through centre

Now solving diagonals with circle we get vertices of square

x = 5 and $x^2 + y^2 - 10x - 6y + 30 = 0$

$y^2 - 6y + 5 = 0$, y = 1, 5

two vertices (5, 1) & (5, 5)

and with y = 3 and

$x^2 + y^2 - 10x - 6y + 30 = 0$

$x^2 - 10x + 21 = 0 \Rightarrow$ x = 3, 7 $\Rightarrow$ vertices (3, 3) & (7, 3)

So $\sum\left(x_i^2 + y_i^2\right) = 25 + 1+25+25+9+9+49+9 = 152$

16. Let the sum of the maximum and the minimum values of the function $f(x)=\frac{2x^2-3x+8}{2x^2+3x+8}$ be $\frac{m}{n}$, where gcd(m, n) = 1. Then m + n is equal to :

(1) 201 (2) 195 (3) 217 (4) 182

Reso. **(1)**

NTA. **(1)**

Sol. Let $y = \frac{2x^2-3x+8}{2x^2+3x+8}$; $x \in R$

$2(y-1)x^2 + 3(y+1)x + 8(y-1) = 0$; $x \in R$

$\Rightarrow$ $D \geq 0$

$\Rightarrow$ $9(y+1)^2 - 64(y-1)^2 \geq 0$

$\Rightarrow$ $(3y + 3 + 8y - 8)(3y + 3 - 8y + 8) \geq 0$

$\Rightarrow$ $(11y - 5)(5y - 11) \leq 0$

$\Rightarrow$ $\frac{5}{11} \leq y \leq \frac{11}{5}$

So sum of maximum and minimum value of f(x) is equal to $\frac{146}{55}$

$\Rightarrow$ m + n = 201

17. Let the point, on the line passing through the points P(1, –2, 3) and Q(5, –4, 7), farther from the origin and at a distance of 9 units from the points P, be (α, β, γ). Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

(1) 155 (2) 165 (3) 150 (4) 160

Reso. **(1)**

NTA. **(1)**

Sol. Equation of line passes through P(1, –2, 3) and Q (5, –4, 7) is

$$\frac{x-1}{4}=\frac{y+2}{-2}=\frac{z-3}{4}$$

$$\Rightarrow \frac{x-1}{2}=\frac{y+2}{-1}=\frac{z-3}{2}$$

Point on it which are at 9 unit distance from P

are $(1 \pm 6, -2 \mp 3, 3 \pm 6)$

$\equiv$ R (7, –5, 9) or S(–5, 1, –3)

$OR = \sqrt{49+25+81}$

$OS = \sqrt{25+1+9}$ $\Rightarrow$ R is farther from origin

Hence Required point is (7, –5, 9)

$\alpha^2 + \beta^2 + \gamma^2 = 155$

18. Let $\alpha \in (0, \infty)$ and $A=\begin{bmatrix}1 & 2 & \alpha\\ 1 & 0 & 1\\ 0 & 1 & 2\end{bmatrix}$. If $\det(\text{adj}(2A - A^T).\ \text{adj}(A - 2A^T)) = 2^8$, then $(\det(A))^2$ is equal to

(1) 1 (2) 36 (3) 49 (4) 16

Reso. **(4)**

NTA. **(4)**

Sol. $\left|\text{adj}\left(2A-A^T\right)\text{adj}\left(A-2A^T\right)\right| = \left|\text{adj}\left\{\left(A-2A^T\right)\left(2A-A^T\right)\right\}\right|$

$$= \left|\text{adj}\left\{-\left(2A-A^T\right)^T.\left(2A-A^T\right)\right\}\right|$$

$$= \left|\text{adj}\left(-B^T.B\right)\right| \text{ where } B = 2A - A^T$$

$$= \left|-B^T.B\right|^2$$

$$= \left|-B^T\right|^2|B|^2$$

$$= |B|^4 \quad (\because\ |B| = |B^T|)$$

$2^8 = |2A - A^T|^4$

$\Rightarrow$ $|2A - A^T| = \pm 4$

$$\Rightarrow \begin{vmatrix}1 & 3 & 2\alpha\\ 0 & 0 & 1\\ -\alpha & 1 & 2\end{vmatrix} = \pm 4$$

$\Rightarrow$ $-1\ (1 + 3\alpha) = 4$ or $-1\ (1 + 3\alpha) = -4$

$\Rightarrow$ $\alpha = -\frac{5}{3}$ (Rej) or $\alpha = 1$

Now $|A| = (\alpha - 5) = -4$ $(\det(A))^2 = 16$

19. Let $\alpha, \beta \in R$. Let the mean and the variance of 6 observations –3, 4, 7, –6, α, β be 2 and 23, respectively. Then mean deviation about the mean of these 6 observations is :

(1) $\frac{16}{3}$ (2) $\frac{14}{3}$ (3) $\frac{11}{3}$ (4) $\frac{13}{3}$

Reso. (4)

NTA. (4)

Sol. Mean = $2 = \frac{-3+4+7+(-6)+\alpha+\beta}{6}$

$12 = 2 + \alpha + \beta$

$\alpha + \beta = 10$ _____(1)

again variance

$$23 = \frac{9+16+49+36+\alpha^2+\beta^2}{6} - (2)^2$$

$162 = \alpha^2 + \beta^2 + 110$

$\alpha^2 + \beta^2 = 52$

$\alpha = 6, \quad \beta = 4 \quad$ or $\quad \alpha = 4, \quad \beta = 6$

Now mean deviation about mean.

$$= \frac{|-3-2|+|4-2|+|7-2|+|-6-2|+|6-2|+|4-2|}{6} = \frac{26}{6} = \frac{13}{3}$$

20. Let α and β be the sum and the product of all the non-zero solutions of the equation $(\bar{z})^2 + |z| = 0$, $z \in C$. Then $4(\alpha^2 + \beta^2)$ is equal to :

(1) 2 (2) 6 (3) 4 (4) 8

Reso. (3)

NTA. (3)

Sol. $\bar{z}^2 + |z| = 0$ .....(1)

$z^2 + |\bar{z}| = 0$ .....(2)

By (1) and (2) as $|z| = |\bar{z}|$

$\Rightarrow \bar{z}^2 = z^2$

$\Rightarrow z = \bar{z}$ or $z = -\bar{z}$

$\Rightarrow I(z) = 0$ or $R(z) = 0$

Case-I Let $I(z) = 0 \Rightarrow z = x, x \in R - \{0\}$

By (1) $x^2 + |x| = 0 \Rightarrow x = 0$ only (rejected)

Case-II Let $R(z) = 0 \Rightarrow z = iy, \quad y \in R - \{0\}$

By (1) $-y^2 + |y| = 0$

$y^2 = |y|$

$y = \pm 1$, as $y \neq 0$

Hence $z = \pm i$

$\Rightarrow$ sum $\alpha = +i - i = 0$

And product $\beta = (i) \times (-i) = 1$

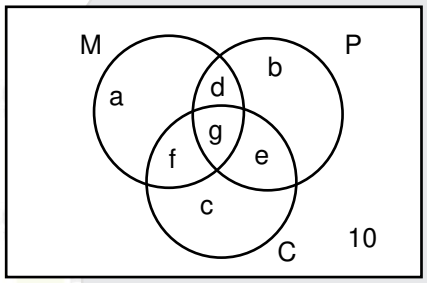
$\Rightarrow 4(\alpha^2 + \beta^2) = 4$

21. In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then m + n is equal to _____ .

Reso. **(45)**

NTA. **(45)**

Sol.



$125 \le n(M) \le 130$

$85 \le n(P) \le 95$

$75 \le n(C) \le 90$

$n(P \cap C) = 30$

$n(C \cap M) = 50$

$n(M \cap P) = 40$

$\Rightarrow \quad (d + e + f) + 3g = 120$

and $\quad a + b + c + d + e + f + g = 210$

$a + b + c = 90 + 2g$

Now $\quad 125 \le a + d + f + g \le 130$

$85 \le b + d + e + g \le 95$

$75 \le c + e + f + g \le 90$

$285 \le (a + b + c) + 2(d + e + f) + 3g \le 315$

$285 \le 90 + 2g + 2(120 - 3g) + 3g \le 315$

$285 \le 330 - g \le 315$

$-45 \le -g \le -15$

$45 \ge g \ge 15$

but $n(P \cap C) = 30 \Rightarrow d + g = 30$

$\Rightarrow g \le 30$

hence $15 \le g \le 30$

so, $m + n = 45$

22. If $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{1+\sin x \cos x}dx = \frac{1}{a}\log_e\left(\frac{a}{3}\right) + \frac{\pi}{b\sqrt{3}}$, where $a, b \in N$, then $a + b$ is equal to _____ .

Reso. **(8)**

NTA. **(8)**

Sol. $I = \int_0^{\frac{\pi}{4}} \frac{Sin^2x}{1+SinxCosx}dx$

$$= \int_0^{\frac{\pi}{4}} \frac{1-\text{Cos}2x}{2+\text{Sin}2x}\,dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{2+\text{Sin}2x}\,dx - \int_0^{\frac{\pi}{4}} \frac{\text{Cos}2x}{2+\text{Sin}2x}\,dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{\text{Sec}^2x}{2+2\tan^2x+2\tan x}\,dx - \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{2\text{Cos}2x}{2+\text{Sin}2x}\,dx$$

$$= \frac{1}{2}\int_0^1 \frac{dt}{t^2+t+1} - \frac{1}{2}\ell n(2+\text{Sin}2x)\Big|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\int_0^1 \frac{dt}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} - \frac{1}{2}\ell n\left(\frac{3}{2}\right)$$

$$= \frac{1}{2}.\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\Big|_0^1 + \frac{1}{2}\ell n\left(\frac{2}{3}\right)$$

$$= \frac{1}{\sqrt{3}}\left(\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}}\right) + \frac{1}{2}\ell n\left(\frac{2}{3}\right)$$

$$= \frac{\pi}{6\sqrt{3}} + \frac{1}{2}\ell n\frac{2}{3}$$

So b = 6 and a = 2

$\Rightarrow$ a + b = 8

Alter divide by $\cos^4x$ in numerator & denominator

$$\int_0^{\frac{\pi}{4}} \frac{\tan^2x\sec^2x\,dx}{(\tan^2x+1+\tan x)(1+\tan^2x)}$$

put t = tanx

dt = sec²xdx

$$= \frac{1}{2}\int_0^1 \frac{2t^2dt}{(t^2+t+1)(t^2+1)} = \frac{1}{2}\int_0^1 \frac{t^2+1}{(t^2+1)(t^2+t+1)} + \frac{t^2-1}{(t^2+1)(t^2+t+1)}dt$$

$$= \frac{1}{2}\int_0^1 \frac{dt}{t^2+t+1} + \frac{1}{2}\int \frac{t^2-1}{(t^2+1)(t^2+t+1)}dt = \frac{1}{2}\int_0^1 \frac{dt}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{1}{2}\int_0^1 \frac{1-\frac{1}{t^2}}{\left(t+\frac{1}{t}\right)\left(t+1+\frac{1}{t}\right)}dt$$

$$= \left(\frac{1}{2}\frac{2}{\sqrt{3}} \tan^{-1}\left| \frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right| \right)_0^1 + I_1$$

$$\frac{1}{\sqrt{3}}\left(\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}} \right) + I_1 = \frac{1}{\sqrt{3}}.\frac{\pi}{6} + I_1$$

Now $I_1 = \frac{1}{2}\int \frac{\left(1-\frac{1}{t^2}\right)dt}{\left(t+\frac{1}{t}\right)\left(t+1+\frac{1}{t}\right)}$

put $t+\frac{1}{t} = u$

$$\left(1-\frac{1}{t^2}\right) dt = du$$

$$= \frac{1}{2}\int_{\infty}^{2} \frac{du}{u(u+1)}$$

$$= \frac{1}{2}\int_{\infty}^{2} \left(\frac{1}{u} - \frac{1}{u+1} \right) du$$

$$= \frac{1}{2}\left(\ell n|u| - \ell n|u+1| \right)_{\infty}^{2}$$

$$= \frac{1}{2}\left(\ell n\left|\frac{u}{u+1}\right| \right)_{\infty}^{2}$$

$$= \frac{1}{2}\left(\ell n\left(\frac{2}{3}\right) - 0 \right)$$

$$= \ell n\sqrt{\frac{2}{3}}$$

So final $= \frac{1}{\sqrt{3}}\left(\frac{\pi}{3} - \frac{\pi}{6}\right) + \ell n\sqrt{\frac{2}{3}}$

$$= \frac{\pi}{6\sqrt{3}} + \frac{1}{2}\ell n\frac{2}{3}$$

So b = 6 and a = 2

$\Rightarrow$ a + b = 8

23. Let A be a square matrix of order 2 such that $|A| = 2$ and the sum of its diagonal elements is –3. If the points (x, y) satisfying $A^2 + xA + yI = O$ lie on a hyperbola, whose transverse axis is parallel to the x-axis, eccentricity is e and the length of the latus rectum is l, then $e^4 + l^4$ is equal to _____ .

Reso. **(Bonus) Data insufficient**

NTA. **(25)**

Sol.

24. Let the solution $y = y(x)$ of the differential equation $\frac{dy}{dx} - y = 1 + 4\sin x$ satisfy $y(\pi) = 1$. Then $y\left(\frac{\pi}{2}\right) + 10$ is equal to _____ .

Reso. **(7)**

NTA. **(7)**

Sol. $\frac{dy}{dx} - y = 1 + 4\sin x$ linear differential equation

I.F. $= e^{\int -dx} = e^{-x}$

So solution of linear differential equation is

$$ye^{-x} = \int e^{-x}(1 + 4\sin x)dx$$

$$ye^{-x} = -e^{-x} + 4\int e^{-x}\sin x\, dx$$

$$ye^{-x} = -e^{-x} + 4\frac{e^{-x}}{2}[-\sin x - \cos x] + c$$

$y = -1 - 2(\sin x + \cos x) + c\,e^{x}$

Now $y(\pi) = 1$

$\Rightarrow 1 = -1 - 2(0 - 1) + ce^{\pi}$

$\Rightarrow c = 0$

Hence $y = -1 - 2(\sin x + \cos x)$

$\Rightarrow y\left(\frac{\pi}{2}\right) + 10 = -1 - 2(1 + 0) + 10$

$= 7$

25. Let A be a 3×3 matrix of non-negative real elements such that $A\begin{bmatrix}1\\1\\1\end{bmatrix} = 3\begin{bmatrix}1\\1\\1\end{bmatrix}$. Then the maximum value of det (A) is _____ .

Reso. **(27)**

NTA. **(27)**

Sol. $A\begin{bmatrix}1\\1\\1\end{bmatrix} = 3\begin{bmatrix}1\\1\\1\end{bmatrix}$

Let $A = \begin{bmatrix}a & b & c\\ p & q & r\\ x & y & z\end{bmatrix}$

$\Rightarrow$ $a + b + c = 3$

$p + q + r = 3$

and $x + y + z = 3$

such that $a, b, c, p, q, r, x, y, z \in [0, 3]$

$|A| = (aqz + brx + cpy) - (cqx + bpz + ary)$

for maximum value of $|A|$, $cqx = 0$, $bpz = 0$ & $ary = 0$

so that at least one of aqz or brx or cpy is non-zero

Let $c = 0$, $p = 0$ & $y = 0$ $\Rightarrow$ $|A| = aqz + brx = aqz + (3 - a)(3 - q)(3 - z)$

$= 27 - 9(a + q + z) + 3(aq + az + qz) \le 27$

for $a + q + z = 0$

26. If the shortest between the lines $\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-5}{4}$ and $\frac{x-3}{1} = \frac{y-2}{-3} = \frac{z+4}{2}$ is $\frac{38}{3\sqrt{5}}k$, and $\int_0^k [x^2]dx = \alpha - \sqrt{\alpha}$, where [x] denotes the greatest integer function, then $6\alpha^3$ is equal to _____.

Reso. (48)

NTA. (48)

Sol. $L_1 \equiv \frac{x+2}{2} = \frac{y+3}{3} = \frac{z-5}{4}$ _____(1)

$L_2 \equiv \frac{x-3}{1} = \frac{y-2}{-3} = \frac{z+4}{2}$ _____(2)

Shortest distance b/w L_1 and L_2 is $= \frac{\begin{vmatrix} 5 & 5 & -9 \\ 2 & 3 & 4 \\ 1 & -3 & 2 \end{vmatrix}}{\sqrt{(18)^2 + (0)^2 + (-9)^2}}$

$$\frac{38}{3\sqrt{5}}k = \frac{90 + 0 + 81}{9\sqrt{5}} = \frac{19}{\sqrt{5}}$$

$\Rightarrow$ $k = \frac{3}{2}$

Now $\int_0^k [x^2]dx = \int_0^{\frac{3}{2}} [x^2]dx$

$= \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx + \int_{\sqrt{2}}^{\frac{3}{2}} [x^2]dx$

$= 0 + (\sqrt{2} - 1) + 2\left(\frac{3}{2} - \sqrt{2}\right)$

$= 2 - \sqrt{2}$

$\Rightarrow$ $\alpha = 2$

$\Rightarrow$ $6\alpha^3 = 48$

27. Let $a = 1 + \frac{{}^2C_2}{3!} + \frac{{}^3C_2}{4!} + \frac{{}^4C_2}{5!} + ...,$

$$b = 1 + \frac{{}^1C_0 + {}^1C_1}{1!} + \frac{{}^2C_0 + {}^2C_1 + {}^2C_2}{2!} + \frac{{}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3}{3!} + ...,$$

Then $\frac{2b}{a^2}$ is equal to _____ .

Reso. (8)

NTA. (8)

Sol. $a = 1 + \frac{{}^2C_2}{3!} + \frac{{}^3C_2}{4!} + \frac{{}^4C_2}{5!} + ...$

$$= 1 + \sum_{n=2}^{\infty} \frac{{}^nC_2}{(n+1)!}$$

$$= 1 + \sum_{n=2}^{\infty} \frac{n(n-1)}{2!(n+1)!}$$

$$= 1 + \frac{1}{2}\sum_{n=2}^{\infty} \frac{n(n+1) - 2(n+1) + 2}{(n+1)!}$$

$$= 1 + \frac{1}{2}\sum_{n=2}^{\infty} \left\{ \frac{1}{(n-1)!} - \frac{2}{n!} + \frac{2}{(n+1)!} \right\}$$

$$= 1 + \frac{1}{2}\left\{ (e-1) - 2(e-2) + 2\left(e - \frac{5}{2}\right) \right\}$$

$$= 1 + \frac{1}{2}(e-2) = \frac{1}{2}e$$

and $b = 1 + \frac{{}^1C_0 + {}^1C_1}{1!} + \frac{{}^2C_0 + {}^2C_1 + {}^2C_2}{2!} + ...$

$$= 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + ...$$

$$= e^2$$

$$\frac{2b}{a^2} = \frac{2e^2}{\frac{1}{4}e^2} = 8$$

28. Let the length of the focal chord PQ of the parabola $y^2 = 12x$ be 15 units. If the distance of PQ from the origin is p, then $10p^2$ is equal to _____ .

Reso. (72)

NTA. (72)

Sol. $y^2 = 12x$

Let end points of focal chord are $P(3t^2, 6t)$ & $Q\left(\frac{3}{t^2}, \frac{-6}{t}\right)$

Length of focal chord $3t^2 + 3 + \frac{3}{t^2} + 3 = 15$

$$\Rightarrow \quad \left(t+\frac{1}{t}\right)^2 = 5$$

$$\Rightarrow \quad t+\frac{1}{t} = \pm\sqrt{5} \quad \text{_____(1)}$$

Equation of PQ is $y\left(t-\frac{1}{t}\right) = 2x-6$

distance from origin = $\left|\frac{6}{\sqrt{4+\left(t-\frac{1}{t}\right)^2}}\right|$

$$\Rightarrow \quad P = \left|\frac{6}{\left(t+\frac{1}{t}\right)}\right| = \frac{6}{\sqrt{5}}$$

$$\Rightarrow \quad 10P^2 = 72$$

29. Let ABC be a triangle of area $15\sqrt{2}$ and the vectors $\overrightarrow{AB} = \hat{i}+2\hat{j}-7\hat{k}$, $\overrightarrow{BC} = a\hat{i}+b\hat{j}+c\hat{k}$ and $\overrightarrow{AC} = 6\hat{i}+d\hat{j}-2\hat{k}$, $d > 0$. Then the square of the length of the largest side of the triangle ABC is _____.

Reso. **(54)**

NTA. **(54)**

Sol. $\overrightarrow{AB}+\overrightarrow{BC} = \overrightarrow{AC}$

$\hat{i}+2\hat{j}-7\hat{k}+a\hat{i}+b\hat{j}+c\hat{k} = 6\hat{i}+d\hat{j}-2\hat{k}$

$(a-5)\hat{i}+(b-d+2)\hat{j}+(c-5)\hat{k} = 0$

$\Rightarrow \quad a = 5,\ b-d+2 = 0 \ \&\ c = 5$

Now area of $\Delta ABC = \frac{1}{2}\left|\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\right|$

$$\pm 15\sqrt{2} = \frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -7\\ 6 & d & -2\end{vmatrix}$$

$$\pm 15\sqrt{2} = \frac{1}{2}\left|(7d-4)\hat{i}-40\hat{j}+(d-12)\hat{k}\right|$$

$$\pm 30\sqrt{2} = \sqrt{(7d-4)^2+1600+(d-12)^2}$$

$1800 = 50d^2 - 80d + 1760$

$50d^2 - 80d - 40 = 0$

$5d^2 - 8d - 4 = 0$

$(5d+2)(d-2) = 0$

$d = 2$

$\Rightarrow$ $a = 5, b = 0 \& c = 5$

$\left|\overrightarrow{AB}\right| = \sqrt{54}$

$\left|\overrightarrow{BC}\right| = \sqrt{50}$ & $\left|\overrightarrow{AC}\right| = \sqrt{44}$

$\left|\overrightarrow{AB}\right|^2 = 54$

30. If $\lim\limits_{x\to1}\dfrac{(5x+1)^{\frac{1}{3}}-(x+5)^{\frac{1}{3}}}{(2x+3)^{\frac{1}{2}}-(x+4)^{\frac{1}{2}}} = \dfrac{m\sqrt{5}}{n(2n)^{\frac{2}{3}}}$, where gcd(m, n) = 1, then 8m + 12n is equal to

Reso. **(100)**

NTA. **(100)**

Sol. $\lim\limits_{x\to1}\dfrac{(5x+1)^{\frac{1}{3}}-(x+5)^{\frac{1}{3}}}{(2x+3)^{\frac{1}{2}}-(x+4)^{\frac{1}{2}}}\left(\dfrac{0}{0}\text{ from}\right)$

$$= \lim_{x\to1}\frac{\{(5x+1)-(x+5)\}\left\{\sqrt{2x+3}+\sqrt{x+4}\right\}}{\left\{(5x+1)^{\frac{2}{3}}+(5x+1)^{\frac{1}{3}}(x+5)^{\frac{1}{3}}+(x+5)^{\frac{2}{3}}\right\}\{(2x+3)-(x+4)\}}$$

$$= \lim_{x\to1}\frac{4(x-1)\left\{\sqrt{2x+3}+\sqrt{x+4}\right\}}{\left\{(5x+1)^{\frac{2}{3}}+(5x+1)^{\frac{1}{3}}(x+5)^{\frac{1}{3}}+(x+5)^{\frac{2}{3}}\right\}(x-1)}$$

$$= 4.\frac{\sqrt{5}+\sqrt{5}}{\left(6^{\frac{2}{3}}+6^{\frac{2}{3}}+6^{\frac{2}{3}}\right)} = \frac{8.5^{\frac{1}{2}}}{3.(2.3)^{\frac{2}{3}}}$$

$\Rightarrow m = 8$ and $n = 3$

$\Rightarrow 8m + 12n = 64 + 36 = 100$

PART : PHYSICS

31. The electric field in an electromagnetic wave is given by $\vec{E} = \hat{i}\ 40\cos\omega\ (t - Z/c)\ NC^{-1}$. The magnetic field induction of this wave is (in SI unit) :

एक विद्युत चुंबकीय तरंग में वैद्युत क्षेत्र निम्न प्रकार दिया गया है $\vec{E} = \hat{i}\ 40\cos\omega\ (t - Z/c)\ NC^{-1}$ । इस तरंग का प्रेरित चुम्बकीय क्षेत्र (SI मात्रक) है :

(1) $\vec{B} = \hat{J}\ \frac{40}{c}\cos\omega\ (t - \frac{Z}{C})$

(2) $\vec{B} = \hat{i}\ \frac{40}{c}\cos\omega\ (t - \frac{Z}{C})$

(3) $\vec{B} = \hat{k}\ \frac{40}{c}\cos\omega\ (t - \frac{Z}{C})$

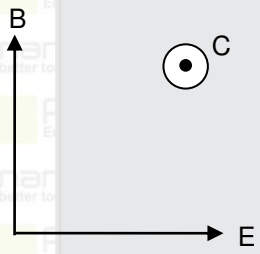
(4) $\vec{B} = \hat{J}\ 40\cos\omega\ (t - \frac{Z}{C})$

Ans. **(1)**

Sol. $E_0 = C\ B_0$

$B_0 = \frac{40}{C}$

$\vec{C} \parallel \vec{E} \times \vec{B} \Rightarrow \vec{B} \| \hat{J}$



32. An electron is projected with uniform velocity along the axis inside a current carrying long solenoid. Thin

(1) The electron will experience a force at 45º to the axis and execute a helical path.

(2) The electron will continue to move with uniform velocity along the axis of the solenoid.

(3) The electron will be accelerated along the axis.

(4) The electron path will be circular about the axis.

एक इलैक्ट्रॉन को एक लम्बी धारावाही परिनालिका में इसकी अक्ष के अनुदिश एकसमान वेग से प्रक्षेपित किया गया है। तब।

(1) इलैक्ट्रॉन अक्ष से 45º के कोण पर एक बल अनुभव करेगा। तथा हैलीकल पथ बनायेगा।

(2) इलैक्ट्रॉन परिनालिका की अक्ष के अनुदिश एकसमान वेग से गति को सुचारू रखेगा।

(3) अक्ष के परितः इलैक्ट्रॉन का पथ वृत्ताकार होगा।

(4) इलैक्ट्रॉन अक्ष के अनुदिश त्वरित होगा

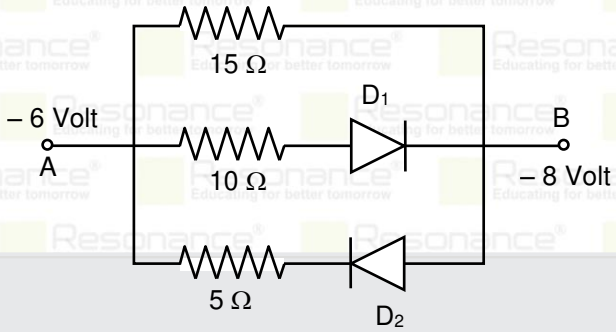
Ans. **(2)**

Sol. $\vec{V} \parallel \vec{B} \Rightarrow F = q\ v\ B \sin\theta$

$F = 0$

Straight line with constant velocity

33. The value of net resistance of the network as shown in the given figure is :
दिये गये चित्र में नेटवर्क के तुल्य प्रतिरोध का मान हैं :



(1) $\frac{15}{4}\Omega$ (2) $\frac{5}{2}\Omega$ (3) 6Ω (4) $\frac{30}{11}\Omega$

Ans. **(3)**

Sol. $-6 > -8$

$V_A > V_B$

$D_1 \Rightarrow$ Forward

$D_2 \Rightarrow$ Revers

$R_{eq} = 15 \| 10$

$R_{eq} = \frac{15 \times 10}{25} = 6$

34. The resistances of the platinum wire of a platinum resistance thermometer at the ice point and steam point are 8Ω and 10Ω respectively. After inserting in a hot bath of temperature 400ºC, the resistance of platinum wire is :
प्लेटिनम प्रतिरोध तापमापी के प्लेटिनम के तार का हिमांक बिन्दु तथा वाष्प बिन्दु पर प्रतिरोध क्रमशः 8Ω व 10Ω है। प्लेटिनाम के तार को 400ºC ताप के एक गर्म वॉथ में धंसाने पर प्लेटिनम के तार का प्रतिरोध है।

(1) 16Ω (2) 10Ω (3) $2\ \Omega$ (4) 8Ω

Ans. **(1)**

Sol. $R = R_0 (1 + \alpha \Delta\theta)$

$10 = 8 (1 + \alpha \times 100)$

$\left(\frac{10}{8} - 1\right) \times \frac{1}{100} = \alpha = \frac{2}{8 \times 100}$

$\alpha = \frac{1}{400}$

$R_{400} = 8 \left(1 + \frac{1}{400} \times 400\right)$

$= 16$ Ans

35. The equation of stationary wave is :

$y = 2a \sin\left(\frac{2\pi nt}{\lambda}\right) \cos\left(\frac{2\pi x}{\pi}\right)$. Which of the following is NOT correct :

(1) The dimensions of nt is [L]

(2) The dimensions of n is $[LT^{-1}]$

(3) The dimensions of x is [L]

(4) The dimensions of $\frac{n}{\lambda}$ is [T]

अप्रगामी तंरग का समीकरण $y = 2a \sin\left(\frac{2\pi nt}{\lambda}\right) \cos\left(\frac{2\pi x}{\pi}\right)$ है। निम्नलिखित में से कौन –सा विकल्प सही नहीं है।

(1) nt की विमा [L] है

(2) n की विमा $[LT^{-1}]$ है

(3) x की विमा [L] है

(4) $\frac{n}{\lambda}$ की विमा [T] है

Ans. **(4)**

Sol. (i) $nt = \lambda = L \Rightarrow$ True

(ii) $nt = L \Rightarrow n = LT^{-1} \Rightarrow$ True

(iii) $x = \lambda = L \Rightarrow$ True

(iv) $\frac{n}{\lambda} = \frac{L}{L \times T} = T^{-1}$ False

36. On Celsius scale the temperature of body increases by 40ºC. The increase in temperature on Fahrenheit scale is?

सेल्सियस पैमाने पर एक वस्तु का तापमान 40ºC. बढ़ता है। फॉरेनहाइट पैमाने पर ताप वृद्धि है ?

(1) 70ºF (2) 68ºF (3) 72ºF (4) 75ºF

Ans. **(3)**

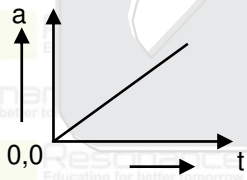
Sol. $\frac{C-0}{100} = \frac{F-32}{180} \Rightarrow \frac{\Delta C}{100} = \frac{\Delta F}{180}$

$\Delta F = \frac{40 \times 180}{100} = 72$ Ans.

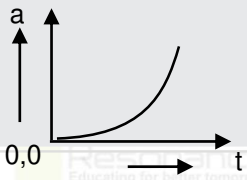
37. A wooden block, initially at rest on the ground, is pushed by a force which increases linearly with time t. Which of the following curve best describes acceleration of the block with time :

एक लकड़ी का गुटका, प्रारंभ में जमीन पर आराम की स्थिति में, एक बल द्वारा धकेला जाता है जो समय t के साथ रैखिक रूप से बढ़ता है। निम्नलिखित में से कौन सा वक्र समय के साथ ब्लॉक के त्वरण का सबसे अच्छा वर्णन करता है।

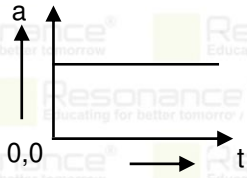
(1)



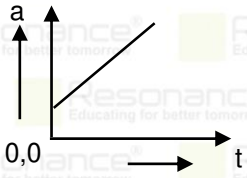
(2)



(3)



(4)



Ans. **NTA (1)**
Reso (1 and 4)

Sol. $F = \alpha$

$F = at + b$ A = acceleration

$A = \frac{at+b}{m}$

38. A body travels 102.5 m in n^{th} second and 115.0 m in $(n + 2)^{th}$ second. The acceleration is :
एक पिण्ड n वें सेकंड में 102.5 m तथा (n + 2) वे सेकंण्ड में 115.0 m दूरी तय करता है। इसका त्वरण है :
(1) 5 m/s² (2) 9m/s² (3) 6.25 m/s² (4) 12.5 m/s²

Ans. **(3)**

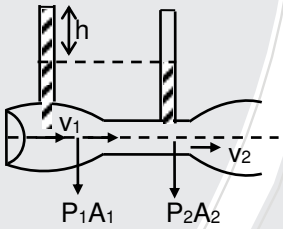
Sol. $102.5 = 4 + \frac{a}{2}(2n - 1)$ $\quad S_n = u + \frac{a}{2}(2n-1)$

$115 = 4 + \frac{a}{2}(2((n+2)^{-1})$

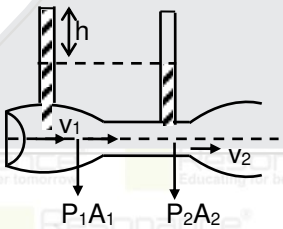
$12.5 = \frac{a}{2}(4)$

$a = \frac{12.5}{2} = 6.25$

39. Given below are two statements :
Statement - I When speed of liquid is zero every where, pressure difference at any two points depends on equation $P_1 - P_2 = \rho g(h_2 - h_1)$.
Statement II : In ventury tube shown $2gh = v_1^2 - v_2^2$



In the light of the above statements, choose the most appropriate answer from the options given below.
(1) Both Statement I and Statement II are incorrect
(2) Both Statement I and Statement II are correct.
(3) Statement I is correct but Statement II is incorrect
(4) Statement I is incorrect but Statement II is correct
नीचे दो कथन दिये गये है :
कथन- I जब प्रत्येक स्थान पर द्रव की चाल शुन्य है तो किन्ही दो बिन्दुओं के बीच दाबान्तर समीकरण $P_1 - P_2 = \rho g(h_2 - h_1)$. पर निर्भर करता है।
कथन II : प्रदर्शित वैन्चुरी नली में $2gh = v_1^2 - v_2^2$



उपरोक्त कथनों के आधार पर, नीचे दिये गये विकल्पों में से सबसे उचित उत्तर चुनिए :
(1) दोनों कथन I व कथन II गलत है।
(2) दोनों कथन I व कथन II सही है।
(3) कथन I सही है। परन्तु कथन II गलत है
(4) कथन I गलत है परन्तु कथन II सही हैं

Ans. **(3)**

Sol. **Statement-1**

$P_1 + 0 + \rho g h_1 = P_2 + \rho g h_2$

$P_1 - P_2 = \rho g (h_2 - h_1)$

Statement-2

$$P_1 + \frac{1}{2} \times v_1^2 = P_2 + \frac{1}{2} \times v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\left(v_2^2 - v_1^2\right) = h \times g$$

$$2gh = v_2^2 - v_1^2$$

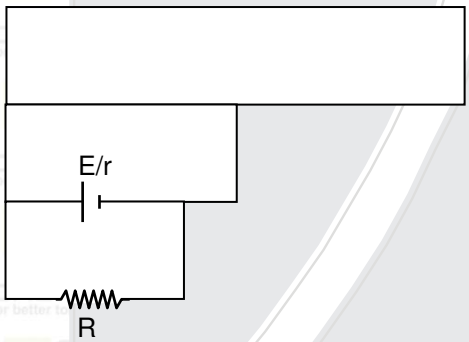
40. To measure the internal resistance of a battery, potentiometer is used. For R = 10Ω , the balance point is observed at ℓ = 500 cm and for R = 1 Ω the balance point is observed at l = 400 cm. The internal resistance of the battery is approximately.

बैटरी के आन्तरिक प्रतिरोध ज्ञात करने के लिए विभवमापी का उपयोग किया गया है। R = 10Ω के लिए सन्तुलन बिन्दु ℓ = 500 cm पर प्राप्त होता है। तथा R = 1 Ω के लिए सन्तुलन बिन्द l = 400 cm. पर प्राप्त होंता है बैटरी का आन्तरिक प्रतिरोध लगभग है।

(1) 0.1Ω (2) 0.3 Ω (3) 0.4Ω (4) 0.2Ω

Ans. **(2)**

Sol.



$V = x \times \ell$

$= E - ir =$

$$iR = \frac{E}{R+r} \times R$$

$$x\ell = R \times \frac{E}{R+r}$$

$$\frac{\ell_1}{\ell_2} = \frac{R_1}{R_1+r} \times \frac{R_2+r}{R_2}$$

$$\frac{500}{400} = \frac{10}{10+r} \times \frac{1+r}{1}$$

$$\frac{5}{4} = \frac{10(1+r)}{(10+r)}$$

$50 + 5r = 40 + 40r$

$10 = 35\ r$

$$r = \frac{2}{7}$$

$= 0.285$

$= 0.3$

41. In an experiment to measure focal length (f) of convex lens, the least counts of the measuring scales for the position of object (u) and for the position of image (v) are Δu and ΔV, respectively. The error in the measurement of the focal length of the convex lens will be :

एक प्रयोग में उत्तल लैंस की फोकस दूरी (f) ज्ञात करने के लिए वस्तु की स्थिति (u) तथा प्रतिबिन्ब की स्थिति (v) के मापन में पैमाने की अल्पतमोक क्रमश Δu व ΔV, है। उत्तल लैंस की फोकस दूरी के मापन में त्रुटि होगी :

(1) $f\left[\frac{\Delta u}{u}+\frac{\Delta v}{v}\right]$ (2) $2f\left[\frac{\Delta u}{u}+\frac{\Delta v}{v}\right]$ (3) $f^2\left[\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2}\right]$ (4) $\frac{\Delta u}{u}+\frac{\Delta v}{v}$

Ans. **(3)**

Sol. $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

$\Rightarrow$ differentiate on both side.

$-\frac{1}{f^2}\times Af=-\frac{1}{v^2}\Delta v+\frac{1}{u^2}\Delta u$

$\Delta f=f^2\left(\frac{\Delta v}{v^2}+\frac{\Delta u}{u^2}\right)$

42. In an ac circuit, the instantaneous current is zero, when the instantaneous voltage is maximum, in this case, the source may be connected to :

A. pure inductor.

B. pure capacitor

C. pure resistor.

D. combination of an inductor and capacitor.

Choose the correct answer form the options given below :

(1) B, C and D only (2) A, B and D only

(2) A, B and C only (4) A and B only

एक प्रत्यावर्ती परिपथ में, जब क्षणिक वोल्टेज अधिकतम है तो क्षणिक धारा शून्य है। इस अवस्था में स्त्रोत किससे जुड़ा हो सकता है।:

A. शुद्ध प्रेरक

B. शुद्ध संधारित्र

C. शुद्ध प्रतिरोध

D. एक प्रेरक व संभारित्र का संयोजन

नीचे दिये गये विकल्पो से सही उत्तर चनिए :

(1) केवल B, C और D (2) केवल A, B और D

(2) केवल A, B और C (4) केवल A और B

Ans. **(2)**

Sol. Phase difference

$\phi=\frac{\pi}{2}$

So A, B and D are correct

43. A metal wire of uniform mass density having length L and mass M is bent to form a semi-circular arc and a particle of mass m is placed at the centre of the arc. The gravitational force on the particle by the wire is

L लम्बाई तथा M द्रव्यमान के एकसमान द्रव्यमान घनत्व के धातु के तार को मोड़कर एक उर्द्धवृत्ताकार चाप बनाया गया है। एवं चाप के केन्द्र पर m द्रव्यमान का एक कण रख दिया गया है। कण पर लगने वाला गुरूत्वीय बल हैं :

(1) $\frac{GmM\pi^2}{L^2}$ (2) $\frac{2GmM\,\pi}{L^2}$ (3) 0 (4) $\frac{GmM\,\pi}{2L^2}$

Ans. **(2)**

Sol. $F = mE$

$$F = m \times \frac{2G\lambda}{R}$$

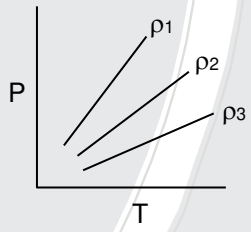
$$\lambda = \frac{M}{L}$$

$$L = \pi R$$

$$F = m \times \frac{2GM \times \pi}{L \times L}$$

44. P-T diagram of an ideal gas having three different densities p_1 p_2, p_3 (in three different cases) is shown in the figure. Which of the following is correct :

तीन विभिन्न घनत्वों p_1 p_2, p_3 वाली एक आदर्श गैस का P-T आरेख (तीन स्थितियों में) चित्र में दर्शाया गया है। निम्न में से कौन-सा विकल्प सही है :



(1) $\rho_2 < \rho_3$ (2) $\rho_1 = \rho_2 = \rho_3$ (3) $\rho_1 > \rho_2$ (4) $\rho_1 > \rho_2$

Ans. **(3)**

Sol. Slope α ρ

$\rho_1 > \rho_2 > \rho_3$

45. Which of the following nuclear fragments corresponding to nuclear fission between neutron $\left({}_0^1n\right)$ and uranium isotope $\left({}_{92}^{235}U\right)$ is correct :

न्यूट्रॉन $\left({}_0^1n\right)$ तथा युरेनियम समस्थानिम $\left({}_{92}^{235}U\right)$ के मध्य नाभिकीय विखण्डन के संगत निम्नलिखित कौन से नाभिकीय खण्ड (fragments) सही हैं :

(1) ${}_{56}^{140}Xe + {}_{38}^{94}Sr + 3{}_0^1n$ (2) ${}_{56}^{144}Ba + {}_{36}^{89}Kr + 4{}_0^1n$

(3) ${}_{51}^{153}Sb + {}_{41}^{99}Nb + 3{}_0^1n$ (4) ${}_{56}^{144}Ba + {}_{36}^{89}Kr + 3{}_0^1n$

Ans. **(4)**

Sol. $235 + 1 = 140 + 94 + 3$

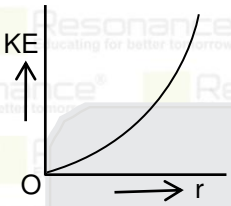
$236 = 237$

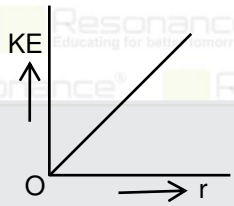
option 1 not correct

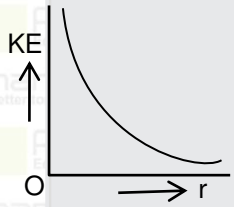
In fission only 2 or 3 neutron emitted so option 2 wrong.

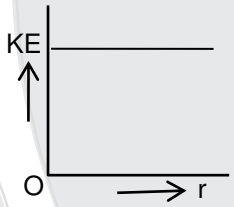
A and z should be same both side of equation so correct ans. 4

46. An infinitely long positively charged straight thread has a linear charge density λ cm^{-1} . An electron revolves along a circular path having axis along the length of the wire. The graph that correctly respects the variation of the kinetic energy of electron as function of radius of circular path form the wire is :
एक अनंत लम्ब धनावेशित धागे का रेखीय आवेश घनत्व λ cm^{-1} है। एक इलैक्ट्रॉप तार की लम्बाई के अनुदिश अक्ष के परितः एक वृत्तीय पथ पर परिक्रमण करता है। वह ग्राफ जो तार से वृत्तीय पथ की त्रिज्या के फलन के रूप में इलैक्ट्रॉन की गतिज ऊर्जा परिवर्तन को प्रदर्शित है।

(1) 

(2) 

(3) 

(4) 

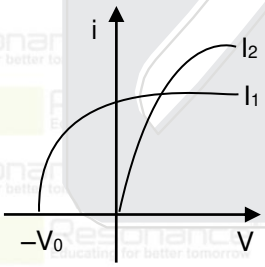
Ans. **(4)**

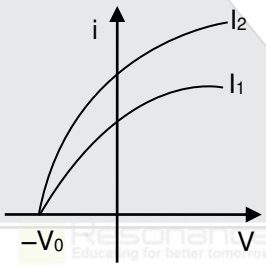
Sol. $\frac{mv^2}{r} = q \times \frac{2k\lambda}{r}$

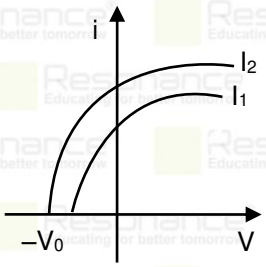
$mv^2/2 = 2kq$

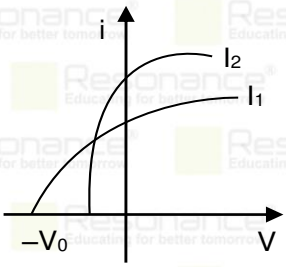
KE = constant

47. Which figure shows the correct variation of applied potential difference (v) with photoelectric current (I) at two different intensities of light ($I_1 < I_2$) of same wavelengths.
प्रकाश की दो तीव्रताओं ($I_1 < I_2$) तथा समान तरंगदैर्ध्यों पर कौन–सा चित्र प्रकाश वैद्युत धारा (I) के साथ आरोपित विभवान्तर (v) के परिवर्तन को प्रदर्शित करता है।

(1) 

(2) 

(3) 

(4) 

Ans. **(2)**

Sol. λ = same = γ
$I \propto N \times \upsilon$
$I \propto N$
i_s = saturation current
$i_s \propto I$
V_s = stopping potential
V_s = same
So Ans. (2)

48. If a rubber ball falls from a height h and rebounds up to the height of h/2. The percentage loss of total energy of the initial system as well as velocity ball before it strikes the ground respectively, are :
यदि रबड़ की एक गेंद h ऊँचाई से गिरती है तथा h/2 ऊँचाई तक उछलती है। प्रारम्भिक निकाय की कुल ऊर्जा में प्रतिशत हानि तथा पृथ्वी तल से टकराने से पर्व गेंद का वेग क्रमशः है :

(1) 40 % $\sqrt{2gh}$ (2) 50% $\sqrt{2gh}$ (3) 50% $\sqrt{\frac{gh}{2}}$ (4) 50% $\sqrt{gh}$

Ans. **(2)**

Sol. $u = \sqrt{2gh}$

$V \propto \sqrt{h}$

$V = \frac{u}{2}$

loss in energy = $\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{mu^2}{4}$

So 50 % loss in energy

49. The co-ordinates of a particle moving in x-y plane are given by :
$x = 2 + 4t$, $y = 3t + 8t^2$
The motion of the particle is :
(1) uniformly accelerated having motion along a parabolic path.
(2) uniformly accelerated having motion along a straight line.
(3) non-uniformly accelerated
(4) uniform motion along a straight line
x-y तल में गतिमान एक कण के निर्देशांक निम्न प्रकार दिये गये है, $x = 2 + 4t$, $y = 3t + 8t^2$ कण की गति है।
(1) परवलयाकार पथ के अनुदिश एकसमान त्वरित गति
(2) सीधी रेखा के अनुदिश एकसमान त्वरित गति
(3) असमान त्वरित
(4) सीधी रेखा के अनुदिश एकसमान त्वरित गति

Ans. **(1)**

Sol. $v_x = 4$
$a_x = 0$
$v_y = 3 + 16t$
$a_y = 16$
accelerating constant, velocity and a are not parallel so motion along parabolic

50. An effective power of a combination of 5 identical convex lenses which are kept in contact along the principal axis is 25D. Focal length of each of the convex lens is :
मुख्य अक्ष के अनुदिश रखे 5 एकसमान उत्तल लैंसों के सयोजन की प्रभावी क्षमता 25D है। प्रत्येक उत्तल लैंस की फोकस दूरी है :

(1) 500 cm (2) 20 cm (3) 25 cm (4) 50 cm

Ans. **(2)**

Sol. $25 = P \times 5$

$P = 5 = \frac{100}{f}$

$f = 20$ cm

51. An elastic spring under tension of 3 N has a length a. Its length is b under tension 2 N. For its length (3a-2b), the value of tension will be ——— N.

3 N तनाव के अन्तर्गत एक प्रत्यास्थ स्प्रिंग की लम्बाई b है। इसकी (3a-2b) लम्बाई के लिए तनाव का मान——— N होगा।

Ans. **05.00**

Sol. $F = Kx$

$3 = K(a - \ell) \Rightarrow \frac{3}{K} + \ell = a$

$2 = K(b - \ell) \Rightarrow \frac{2}{K} + \ell = b$

$F = K\{3a - 2b - \ell\}$

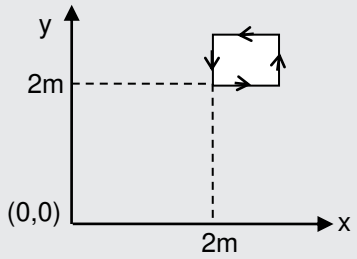
$= K\left[3\left(\frac{3}{K} + \ell\right) - 2\left(\frac{2}{K} + \ell\right) - \ell\right]$

$= \left[\frac{8}{K} + 3\ell - \frac{4}{K} - 2\ell - \ell\right]$

$F = 5$

52. The magnetic field existing in a region is given by $\vec{B} = 0.2(1 + 2x)\,\hat{k}$T. A square loop of edge 50 cm carrying 0.5 A current is placed in x-y plane with its edges parallel to the x-y axes, as shown in figure. The magnitude of the net magnetic force experienced by the loop is ——— mN.

एक परिसर में पाया जाने वाला चुम्बकीय क्षेत्र $\vec{B} = 0.2(1 + 2x)\,\hat{k}$T. है। 50 cm भुजा के एक वर्गाकार लूप जिसमें प्रवाहित धारा 0.5 A है, इसकी भुजाओं को x-y अक्षों के समान्तर रखकर x-y तल में चित्रानुसार रखा गया है। लूप पर लगने वाले परिणामी बल का परिमाण——— mN है।



Ans. **50.00**

Sol. $x = 2.5$

$B = 0.2\,(1 + 2 \times 2.5)$

$= 1.2$

$F_{DC} = B\,i\ell$

$= 1.2 \times 0.5 \times 0.5$

$\pi = 2$

$B = 0.2 \times (1 + 4)$

$= 1$

$F_{AB} = 1 \times 0.5 \times 0.5$

$= 0.25\,(1.2 - 1)$

$= 0.25 \times 0.2$

$= 0.05$

$N = 50$

53. A solid sphere and a hollow cylinder roll up without slipping on same inclined plane with same initial speed υ. The sphere and the cylinder reaches up to maximum heights h_1 and h_2. Respectively, above the initial level. The ratio $h_1 : h_2$ is $\frac{n}{10}$. The value of n is ———.

एक ठोस गोला व एक खोखला बेलन एकसमान आनत तल पर समान प्रारम्भिक चाल υ से बिना फिसले ऊपर की ओर लुढकते है। गोला तथा बेलन प्रारम्भिक स्तर से क्रमशः h_1 व h_2 अधिकतम ऊँचाइयों तक पहुँचते हैं। $h_1 : h_2$ अनुपात $\frac{n}{10}$ है। n का मान ——— है।

Ans. **07.00**

Sol. $\frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right) = mgh. \Rightarrow h \propto \left(1+\frac{k^2}{r^2}\right)$

solid sphere $\frac{k^2}{r^2} = \frac{2}{5}$

Hollow cylin $\frac{k^2}{r^2} = 1$

$$\frac{1+\frac{2}{5}}{1+1} = \frac{h_1}{h_2} = \frac{7}{5\times 2} = \frac{n}{10}$$

n = 7 Ans.

54. A hydrogen atom changes its state from n = 3 to n = 2. due to recoil, the percentage change in the wave length of emitted light is approximately 1×10^{-n}. The value of n is ———.

[Given Rhc = 13.6 eV, hc = 1242 eV nm, h = 66×10^{-34} J s mass of the hydrogenation = 1.6×10^{-27} kg]

एक हाइड्रोजन परमाणु n = 3 सक n = 2 में अवस्था परिवर्तित करता है। स्थानान्तरण के कारण उत्सर्जित प्रकाश की तंरगदैर्ध्यो में परिवर्तन 1×10^{-n} है। n का मान ——— है।

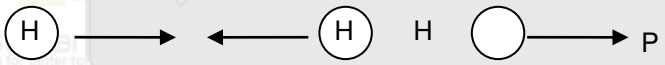
[दिया है Rhc = 13.6 eV, hc = 1242 eV nm, h = 66×10^{-34} J s हाइड्रोजन परमाणु का द्रव्यमान = 1.6×10^{-27} kg]

Ans. **07.00**

Sol. Since H atom falls from n = 3 to h = 2 , 50 energy of emitted photon

E = (– 1.5 lev) – (–3 .4ev) = 1.9 ev

momentum of the emitted photon = $\frac{E}{C} = \frac{1.9\times 1.6\times 10^{-19}}{3\times 10^8} = 10^{-23}$ N.sec



To conserve the momentum, the recoil momentum of the H atom should also be $P = 10^{-27}$ N. sec.

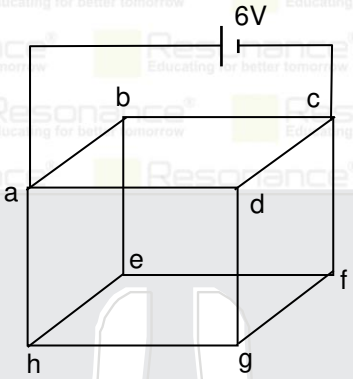
recoil KE of the H-atom = $\frac{P^2}{2m} = \frac{(10-27)^2}{2\times 1.6\times 10^{-27}}$

$= 3.13 \times 10^{-28}$ ev

H atom will take away this much energy, so energy of photo will be this much less % loss in energy of photon = $\frac{\text{lossin energy}}{\text{initialenergy}} \times 100\ \%$

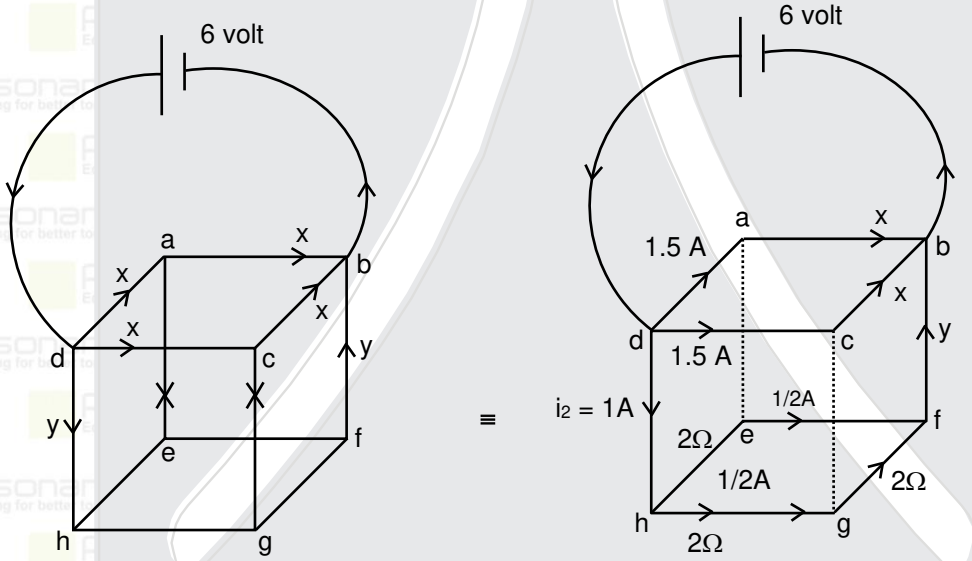
$= \frac{3.3\times 10^{-28}\,\text{ev}}{1.9\times 1.6\times 10^{-19}} \times 100 = 10^{-7}$

55. Twelve wires each having resistance 2Ω are joined to form a cube. A battery of 6 V emf is joined across point a and c. The voltage difference between e and f is ——— v.
बारह एकसमान तारों को जोड़कर एक घन बनाया गया है। जिनमें प्रत्येक तार का प्रतिरोध 2Ω है। बिन्दु a तथा c के बीच एक 6 V वि. वा. बल की बैटरी जोड़ी गई है। e तथा f के बीच विभान्तर ——— v है।



Ans. **01.00**

Sol.



$i_2 = \frac{6}{6} = 1A \Rightarrow \Delta V_{ef} = i_{ef} R_{ef} = \left(\frac{1}{2}\right)(2) = 1$ volt

56. A alternating current at any instant is given by $i = [6 + \sqrt{56}\ \sin(100\pi t + \pi/3)]$ A. The rms value of the current is ———A.
किसी क्षण प्रत्यावर्ती धारा $i = [6 + \sqrt{56}\ \sin(100\pi t + \pi/3)]$ A. दी गई है। धारा का वर्ग माध्य मूल मान———A है।

Ans. **08.00**

Sol. $i = a + b \sin wt$

$$i_{rms} = \sqrt{a^2 + \frac{b^2}{2}}$$

$$= \sqrt{36 + \frac{52}{2}}$$

$$= \sqrt{36+28} = \sqrt{64} = 8 \text{ Ans}$$

57. A soap bubble is blown to diameter of 7 cm 36960 erg of work is done in blowing it further. If surface tension of soap solution is 40 dyne/ cm then the new radius is ——— cm Take $\left(\pi = \frac{22}{7}\right)$.

एक साबुन का बुलबुला 7 cm व्यास तक फूलता है। इसे पुनः फुलाने का कृत्त कार्य 36960 वर्ग है। यदि साबुन के घोल का पृष्ठ तनाव 40 dyne/ cm है तब नई त्रिज्या ——— cm है। दिया है $\left(\pi = \frac{22}{7}\right)$.

Ans. **07.00**

Sol. $d_1 = 7$ cm

$W = 36960$ erg

$W = S \times \Delta A$

$\Delta A = 2\left[4\pi r_2^2 - 4\pi r_1^2\right]$

$5 \times 2 \times (4\pi\ r_2^2 - 4\pi\ r_1^2)$

$S = 40$ dync / cm

$r_2 = ?$

$$36960 = 40 \times 2 \times 4\pi \left[r_2^2 - \left(\frac{7}{2}\right)^2\right]$$

$$36960 = 40 \times 2 \times 4 \times \frac{22}{7}\left[r_2^2 - \frac{49}{4}\right]$$

$$r_2 = \sqrt{36.75 + \frac{49}{4}}$$

$\sqrt{36.75 + 12.25}$

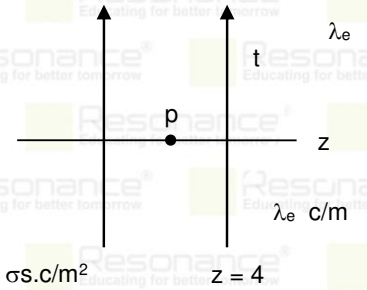
$= 7$ Ans.

58. An infinite plane sheet of charge having uniform surface charge density + σ_s C/m^2 is placed on x-y plane. Another infinitely long line charge having uniform linear charge density + λ_e C/m is placed at z = 4 m plane and parallel to y-axis. If the magnitude values $|\sigma_s| = 2|\lambda_e|$ then at point (0, 0, 2), the ratio of magnitudes of electric field values due to sheet charge to that of line charge is $\pi\sqrt{n}:1$. The value of n is ———

एकसमान पृष्ठ आवेश घनत्व + σ_s C/m^2 की एक अनंत समतल आवेशित चादर x-y तल पर रखी गयी है। दूसरा एकसमान रेखीय आवेश घनत्व + λ_e C/m का अनंत लम्बाई का सीधा रेखीय आवेश y- अक्ष के समान्तर व z = 4 m तल पर रखा गया है। यदि परिमाणिक मान $|\sigma_s| = 2|\lambda_e|$ हो तब बिन्दु (0, 0, 2), पर चादर के आवेश के कारण वैद्युत क्षेत्र के परिमाण तथा रेखीय आवेश के कारण वैद्युत क्षेत्र के परिमाण का अनुपात $\pi\sqrt{n}:1$ है। n का मान ——— है

Ans. **16.00**

Sol.



$$E_s = \frac{\sigma_s}{2E_0}$$

$$\frac{E_s}{E_e} = \frac{\sigma_s}{2E_0} \times \frac{2}{2k\lambda_e}$$

$$= \frac{2\lambda_e}{2E_0} \times \frac{4\pi E_0}{\lambda_e}$$

$\pi \sqrt{n} \cdot 1 = 4\pi$

$n = 16$

59. Two wavelengths λ_1 and λ_2 are used in Young's double slit experiment. $\lambda_1 = 450$ nm and $\lambda_2 = 650$ nm. The minimum order of fringe produced by λ_2 which overlaps with the fringe produced by λ_1 is n. The value of n is ———

यंग द्विझिरी प्रयोग में दो तरंगदैध्यों का उपयोग किया गया है। $\lambda_1 = 450$ nm तथा $\lambda_2 = 650$ nm. । λ_2 द्वारा उत्पन्न फ्रिंज का न्यूनतम क्रम x है जोकि λ_1 द्वारा उत्पन्न फ्रिंज के साथ संपाती है। x का मान ——— है।

Ans. **09.00**

Sol. $y = n_1 \times B_1 = hB_2$

$\Rightarrow h_1 \lambda_1 = n_2 \lambda_2$

$$\frac{h_2}{h_1} = \frac{\lambda_1}{\lambda_2}$$

$$\frac{h_2}{h_1} = \frac{450}{650} = \frac{9}{13}$$

$n_2 = n = 9$ Ans.

60. Two force $\bar{F}_1$ and $\bar{F}_2$ are acting on a body. One force has magnitude thrice that of the other force and the resultant of the two force is equal to the force of larger magnitude. The angle between $\vec{F}_1$ and $\vec{F}_2$ is $\cos^{-1}\left(\frac{1}{n}\right)$. The value of |n| is ———

एक पिण्ड पर दो बल $\bar{F}_1$ व $\bar{F}_2$ कार्यरत है। एक बल का परिमाण दूसरे बल के परिमाण का तीन गुना है। तथा दोनों बलो के परिणामी का परिमाण बड़े बल के परिमाण के बराबर है। $\vec{F}_1$ व $\vec{F}_2$ के बीच का कोण $\cos^{-1}\left(\frac{1}{n}\right)$ है। तब |n| का मान ——— है।

Ans. **06.00**

Sol. $R^2 = F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta$ Given $F_1 = 3F_2 = R$

$9F_2^2 = 9F_2^2 + F_2^2 + 2 \times 3F_2^2 \cos\theta$

$0 = 1 + 6\cos\theta$

$$\cos\theta = -\frac{1}{6}$$

$n = -6$

$|n| = 6$

PART : CHEMISTRY

61. Identify the correct set of reagents or reaction conditions 'X' and 'Y' in the following set of transformation.

$CH_3–CH_2–CH_2–Br \xrightarrow{'X'} Product \xrightarrow{'Y'} CH_3–\underset{\underset{Br}{|}}{CH}–CH_3$

(1) X = conc. alc. NaOH, 80ºC, Y = $Br_2/CHCl_3$
(2) X = conc. alc. NaOH, 80ºC, Y = HBr/acetic acid
(3) X = dil. aq. NaOH, 20ºC, Y = $Br_2/CHCl_3$
(4) X = dil. aq. NaOH, 20ºC, Y = HBr/acetic acid

Ans. **NTA (2)**

Sol.

$CH_3–\underset{\underset{Br}{|}}{CH_2}–CH_2 \xrightarrow[80ºC]{\text{Alc. NaOH}} CH_3–CH=CH_2$

$CH_3–CH=CH_2 \xrightarrow{y = HBr/AcOH} CH_3–\underset{\underset{Br}{|}}{CH}–CH_3$

62. Number of elements from the following that CANNOT form compounds with valencies which match with their respective group valencies is _____.

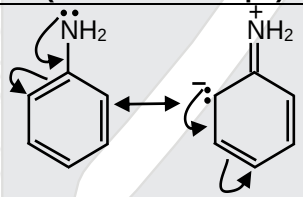
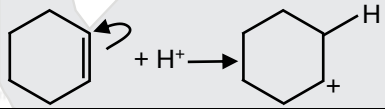
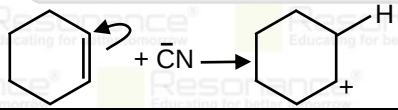
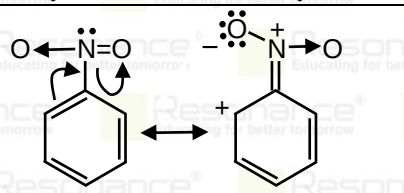
B, C, N, S, O, F, P, Al, Si

(1) 3 (2) 7 (3) 6 (4) 5

Ans. **NTA (1)**

Sol. Following element do not form compound with valencies which match with their respective group valencies is N, O, F.

63. **Match List-I with List-II**

List-I (Mechansim steps)		List-II (Effect)	
(A)		(I)	– E effect
(B)		(II)	– R effect
(C)		(III)	+ E effect
(D)		(IV)	+ R effect

Choose the correct answer from the options given below :

(1) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)
(2) (A) - (I), (B) - (II), (C) - (IV), (D) - (III)
(3) (A) - (II), (B) - (IV), (C) - (III), (D) - (I)
(4) (A) - (III), (B) - (I), (C) - (II), (D) - (IV)

Ans. **NTA (1)**

Sol. $-NH_2$ & $-NO_2$ show +R and –R effect respectively.

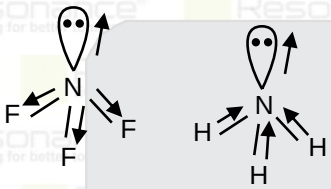
64. Which one of the following molecules has maximum dipole moment ?

(1) NF_3 (2) NH_3 (3) PF_5 (4) CH_4

Ans. **NTA (2)**

Sol. CH_4 & PF_5 have zero dipole moment

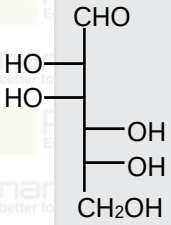
NF_3 NH_3

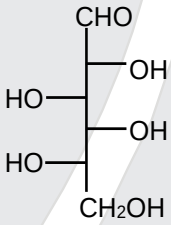


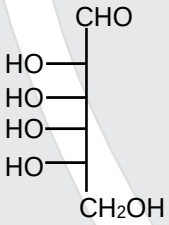
dipole moment of $NH_3 > NF_3$

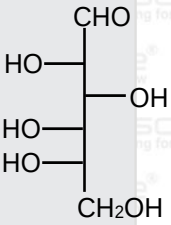
So NH_3 have maximum dipole moment.

65. Which of the following is the correct structure of L-Glucose ?

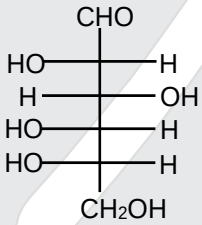
(1) 

(2) 

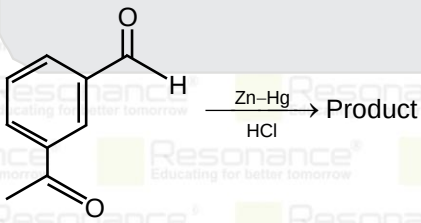
(3) 

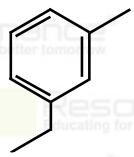
(4) 

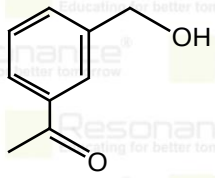
Ans. **NTA (4)**

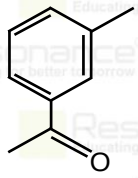
Sol. L-Glucose is  .

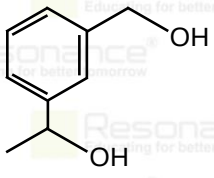
66. Inentify the product in the following reaction :



(1) 

(2) 

(3) 

(4) 

Ans. **NTA (1)**

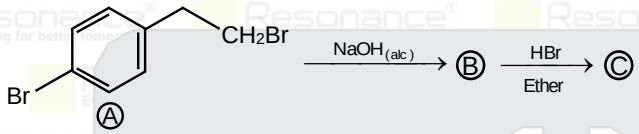
Sol. Zn–Hg/HCl reduces C=O to $-CH_2-$ of carbonyl compounds

67. One of the commonly used electrode is calomel electrode. Under which of the following categorie, calomel electrode comes?

(1) Metal ion - Metal electrodes
(2) Oxidation - Reduction electrodes
(3) Gas - Ion electrodes
(4) Metal - Insoluble Salt - Anion electrodes

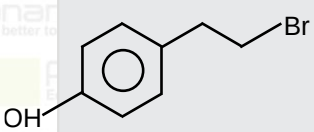
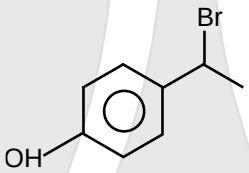
Ans. **NTA (4)**

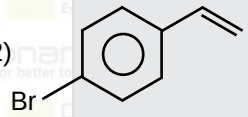
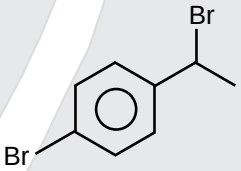
Sol. Calomel electrode comes under.
Metal - insoluble salt - anion electrode.

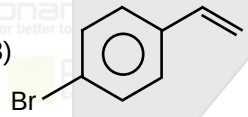
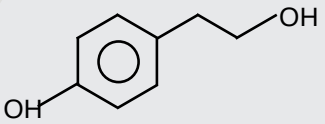
68. 

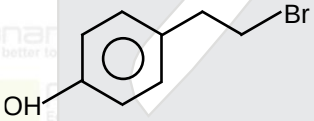
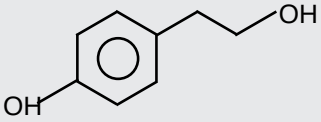
Identify Ⓑ and Ⓒ and how are Ⓑ and Ⓒ related ?

(B) (C)

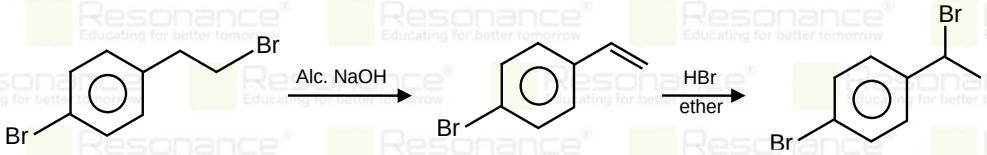
(1)   ; Derivative

(2)   ; Position isomers

(3)   ; Chain isomers

(4)   ; functional group isomers

Ans. **NTA (2)**

Sol. 

69. What pressure (bar) of H_2 would be required to make emf of hydrogen electrode zero in pure water at 25°C ?

(1) 10^{-14} (2) 10^{-7} (3) 1 (4) 0.5

Ans. **NTA (3)**
RESO (1)

Sol. $2H^+(aq) + 2e^- \longrightarrow H_2(g)$

$$E_{N^+/H_2} = E_{H^+/H_2} \frac{-0.059}{2} \log\left(\frac{P_{H_2(g)}}{[H^+]^2}\right)$$

$$0 = 0 \frac{-0.059}{2} \log\frac{P_{H_2}}{[H^+]^2}$$

So $$\left(\frac{P_{H_2}}{[H^+]^2}\right) = 1$$

$$\frac{P_{H_2}}{(10^{-7})^2} = 1$$

$$P_{H_2} = 10^{-14} \text{ bar}$$

70. The Molarity (M) of an aqueous solution containing 5.85 g of NaCl in 500 mL water is :
(Given : Moalr Mass Na : 23 and Cl : 35.5 $gmol^{-1}$)
(1) 0.2 (2) 2 (3) 4 (4) 20

Ans. **NTA (1)**

Sol. $$M = \frac{Ws\,olute \times 1000}{GMM_{Solute} \times V_{M\ell}} = \frac{5.85 \times 1000}{5.85 \times 500} = \frac{1}{10} \times 2 = 0.2 \text{ M}$$

71. Given below are two statements :
Statement-I : Acidity of α-hydrogens of aldehydes and ketones is responsible for Aldol reaction.
Statement-II : Reaction between benzaldehyde and ethanol will NOT give Cross - Aldol product.
In the light of the above statements, choose the **most appropriate** answer from the options given below :
(1) Both **Statement-I** and **Statement-I** are incorrect
(2) Both **Statement-I** and **Statement-I** are correct
(3) **Statement-I** is incorrect but **Statement-II** is correct
(4) **Statement-I** is correct but **Statement-II** is incorrect

Ans. **NTA (4)**

Sol. For aldol condensation $\propto$ H must be present CH_3–CH=O & Ph–CHO gives Ph–CH=CH–CH=O cross aldol product

72. Which of the following nitrogen containing compound does not give Lassaigne's test ?
(1) Glycene (2) Phenyl hydrazine (3) Hydrazine (4) Urea

Ans. **NTA (3)**

Sol. NH_2–NH_2 does not give Lassaigne's test

73. What will be decreasing order of basic strength of the following conjugate bases ?
$^-OH > R\bar{O}, CH_3C\bar{O}O$

(1) $R\bar{O} > {}^-OH > CH_3C\bar{O}O > C\bar{l}$ (2) $C\bar{l} > R\bar{O} > {}^-OH > CH_3C\bar{O}O$

(3) $^-OH > R\bar{O} > CH_3C\bar{O}O > C\bar{l}$ (4) $C\bar{l} > {}^-OH > R\bar{O} > CH_3C\bar{O}O$

Ans. **NTA (1)**

Sol. Conjugate base of weaker acids are stronger.

74. The correct sequence of ligands in the order of decreasing field strength is :
(1) CO > H_2O > F^- > S^{2-} (2) ^-OH > F^- > NH_3 > CN^-
(3) S^{2-} > ^-OH > $EDTA^{4-}$ > CO (4) NCS^- > $EDTA^{4-}$ > CN^- > CO

Ans. **NTA (1)**

Sol. Correct order of decreasing filed strength is $\Rightarrow$ CO > H_2O > F^- > S^{2-}

75. The correct order of first ionization enthalpy values of the following elements is :
(A) O
(B) N
(C) Be
(D) F
(E) B
(1) A < B < D < C < E (2) B < D < C < E < A (3) C < E < A < B < D (4) E < C < A < B < D
Ans. **NTA (4)**
Sol. Correct order 1st IE is $\Rightarrow$ B < Be < O < N < F

76. In the precipitation of the iron group (III) in qualitative analysis, ammonium chloride is added before adding ammonium hydroxide to :
(1) prevent interference by phosphate ions (2) increased concentration of NH_4^+ ion
(3) decrease concentration of ^-OH ions (4) increase concentration of Cl^- ions
Ans. **NTA (3)**
Sol. Decrease concentration of OH^- ions.
So that higher group cation can not precipitated.

77. Which among the following is **incorrect** statement ?
(1) Electromeric effect dominated over inductive effect
(2) The organic compound shows electromeric effect in the presence of the reagent only
(3) Hydrogen ion (H^+) shows negative electromeric effect
(4) The electromeric effect is, temporary effect
Ans. **NTA (3)**
Sol. H^+ show positive electromeric effect

78. The element which show only one oxidation state other than its elemental form is :
(1) Titanium (2) Nickel (3) Scandium (4) Cobalt
Ans. **NTA (3)**
Sol. Sc show only one oxidation state other than elemental state that is +3 oxidation state.

79. Number of complexes from the following with eneven number of unpaired "d" electrons is _____.
$[V(H_2O)_6]^{3+}$, $[Cr(H_2O)_6]^{2+}$, $[Fe(H_2O)_6]^{3+}$, $[Ni(H_2O)_6]^{3+}$, $[Cu(H_2O)_6]^{2+}$
(Given atomic numbers : V = 23, Cr = 24, Fe = 26, Ni = 28, Cu = 29]
(1) 2 (2) 1 (3) 5 (4) 4
Ans. **NTA (1)**
Sol.

	Complex	Electronic configuration	Unpaired Electron
(i)	$[V(H_2O)_6]^{3+}$	$V^{3+} \Rightarrow 3d^2 \Rightarrow t_{2g}^{1,1,0}, e_g^{0,0}$	2
(ii)	$[Cr(H_2O)_6]^{2+}$	$Cr^{2+} \Rightarrow 3d^4 \Rightarrow t_{2g}^{1,1,1}, e_g^{1,0}$	4
(iii)	$[Fe(H_2O)_6]^{3+}$	$Fe^{3+} \Rightarrow 3d^5 \Rightarrow t_{2g}^{1,1,1}, e_g^{2,1}$	5
(iv)	$[Cu(H_2O)_6]^{2+}$	$Cu^{2+} \Rightarrow 3d^9 \Rightarrow t_{2g}^{2,2,2}, e_g^{2,1}$	1

80. Number of molecules/ions from the following in which the central atom is involved in sp3 hybridization is _____.
NO_3^-, BCl_3, ClO_2^-, ClO_3
(1) 3 (2) 1 (3) 2 (4) 4
Ans. **NTA (3)**
Sol.

Molecule/ion	Hybridisation
NO_3^-	sp^2
BCl_3	sp^2
ClO_2^-	sp^3
ClO_3	sp^3

81. Number of molecules/ species from the following having one unpaired electron is _____.

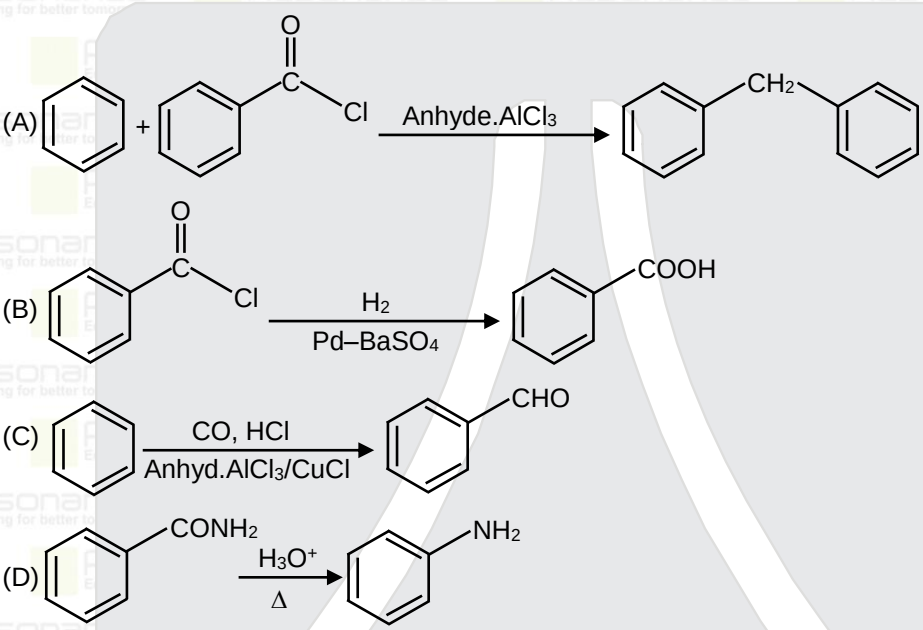
O_2, O_2^{-1}, NO, CN^{-1}, O_2^{2-}

Ans. **NTA (2)**

Sol.

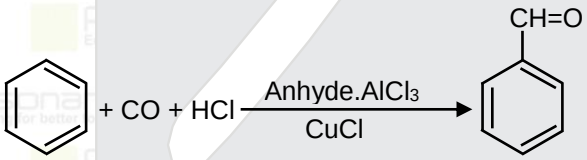
Species	O_2	O_2^-	NO	CN^-	O_2^{2-}
No. of unpaired electron	2	1	1	0	0

82. The number of the correct reaction(s) among the following is _____.



Ans. **NTA (1)**

Sol.



83. The de-Broglie's wavelength of an electron in the 4^{th} orbit is _____ πa_0. (a_0 = Bohr's radius)

Ans. **NTA (8)**

Sol. For H atom in 4^{th} orbit

$r_4 = (r_0)n^2 \quad \Rightarrow \quad r_4 = (4)^2 90$

$r_4 = 16 90$

As $n\lambda = 2\Pi r$

For 4^{th} orbit of H atom

$4\lambda = 2\Pi r_4$

$4\lambda = 2(16 90)$

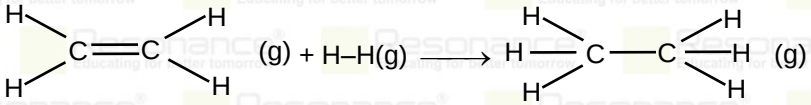
$\lambda = 8\Pi 90$

Ans. = 8

84. The enthalpy of formation of ethane (C_2H_6) from ethylene by addition of hydrogen where the bond-energies of C–H, C–C, C=C, H–H are 414 kj, 347 kj, 615 kj and 435 kj respectively is _____ kj

Ans. **NTA (125)**

Sol. $C_2H_4(g) + H_2(g) \longrightarrow C_6H_6(g)\ \Delta H^\circ_F = ?$



$\Delta H^0_{rxn} = 4E_{C-H} + E_{C=C} + E_{H-H} - 6E_{C-H} - E_{C-C}$

$= E_{C=C} + E_{H-H} - 2E_{C-H} - E_{C-C}$

$= 615 + 435 - 2 \times 414 - 347$

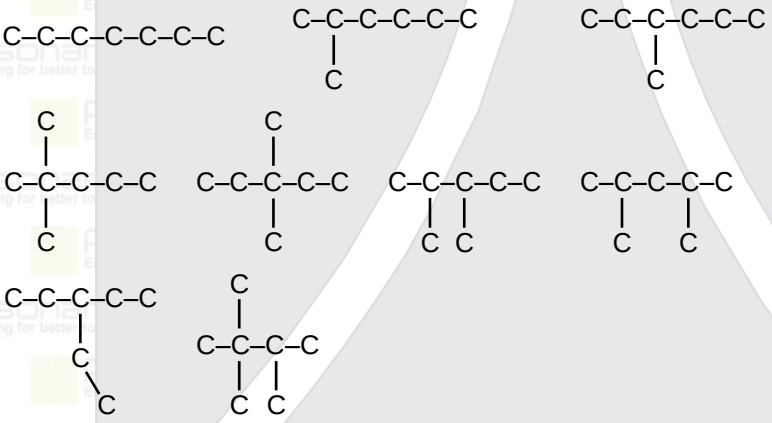
$= -125$ kJ

Ans. = 125

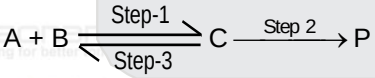
85. The number of different chain isomers for C_7H_{16} is _____.

Ans. **NTA (9)**

Sol.



86. Consider the following transformation involving first order elementary reaction in each step at constant temperature as shown below.



Some details of the above reactions are listed below.

Step	Rate constant (sec^{-1})	Activation energy ($kJ\ mol^{-1}$)
1	k_1	300
2	k_2	200
3	k_3	Ea_3

It the overall rate constant of the above transformation (k) is given as $k = \frac{k_1 k_2}{k_3}$ and the overall activation energy (E_a) is 400 kJ mol^{-1}, then the value of Ea_3 is _____ kJ mol^{-1} (nearest integer)

Ans. **NTA (100)**

Sol. $k = Ae^{-\frac{Ea}{K_T}}$

$(k)_{over} = \frac{k_1 \times k_2}{k_3}$

$Ae^{\frac{-(Ea)_{over}}{R_T}} = \frac{Ae^{-\frac{Ea_1}{R_T}} \times Ae^{-\frac{Ea_2}{R_T}}}{Ae^{-\frac{Ea_3}{R_T}}}$

$e^{\frac{-(Ea)_{over}}{R_T}} = e^{\frac{[Ea_1 + Ea_2 - Ea_3]}{R_T}}$

$(Ea)_{over} = (Ea_1 + Ea_2 - Ea_3)$

$400 = 300 + 200 - Ea_3$

$Ea_3 = 100$ kJ/mole.

87. Only 2 mL of $KMnO_4$ solution of unknown molarity is required to reach the end point of a titration of 20 mL or oxalic acid (2 M) in acidic medium. The molarity of $KMnO_4$ solution should be _____ M.

Ans. **NTA (50)**

Sol. $\underset{VF=5}{\overset{+7}{MnO_4^-}} + \underset{VF=2}{\overset{+3}{C_2O_4^{2-}}} \longrightarrow \overset{+2}{Mn^{2+}} + 2\overset{+4}{CO_2}$

mili equivalent of $KMnO_4$ = mili equivalent of $C_2O_4^{2-}$

$\Rightarrow \quad 5(M \times 2) = 2(2 \times 20)$

$M = \frac{80}{10} = 8$ M

Ans. 8

88. 2.5 g of a non-volatile, non-electrolyte is dissolved in 100 g of water at 25ºC, The solution showed a boiling point elevation by 2ºC. Assuming the solute concentration is negligible with respect to the solvent concentration, the vapour pressure of the resulting aqueous solution is _____ mm of Hg (Nearest integer)

(Given : Molal boiling point elevation constant of water (K_b) = 0.52 K. kg mol^{-1}, 1 atm pressure = 760 mm of Hg, molar mass of water = 18 g mol^{-1}]

Ans. **NTA (707)**

Sol. $\Delta T_b = k_b \times m$

$2 = 0.52 \left\{\frac{2.5 \times 1000}{M_{Solute} \times 100}\right\}$

$M_{solute} = 6.5$

$\frac{P^0 - P_5}{P^0} = \frac{n_{Solute}}{n_{Solvent}}$

$\Rightarrow \frac{760 - P_5}{P^0} = \frac{2.5 \times 18}{6.5 \times 100} = 0.06923$

$P_5 = 707.38$ mm of Hg

Ans. = 707

89. Consider the following reaction

$MnO_2 + KOH + O_2 \rightarrow A + H_2O$.

Product 'A' in neutral or acidic medium disproportionate to give products 'B' and 'C' along with water. The sum of spin-only magnetic moment values of B and C is _____ BM. (nearest integer) Given atomic number of Mn is 25)

Ans. **NTA (4)**

Sol. $MnO_2 + KOH + O_2 \longrightarrow \underset{(A)}{K_2MnO_4} + H_2O$

$MnO_4^{2-} + 4H^+ \longrightarrow \underset{(B)}{MnO_4^-} + \underset{(C)}{MnO_2} + H_2O$

Species	Number of paired e⁻
(B) MnO_4^-	0
(C) MnO_2	3

Spin only Magnetic moment = $\sqrt{n(n+2)}$ BM

= $\sqrt{15}$ BM

Ans. = 4

90. X g of ethalamine is subjected to reaction with $NaNO_2$/HCl followed by water, evolved dinitrogen gas which occupied 2.24 L volume at STP. X is _____ × 10^{-1} g.

Ans. **NTA (45)**

Sol. $C_2H_5NH_2 + NaNO_2 \longrightarrow C_2H_5OH + N_2(g)$

Mole of $N_2(g) = \frac{2.24}{22.4} = 0.1$

Mole of $C_2H_5NH_2 = 0.1$

Mass of $C_2H_5NH_2 = 0.1 \times 45$

= 4.5 gram

= 45 ×10^{-1} gram.