PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

1. $\lim\limits_{x\to0}\dfrac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

(1) is equal to 1 (2) does not exist
(3) is equal to – 1 (4) is equal to 2

Ans. (4)

Sol. $\lim\limits_{x\to0}\dfrac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

$= \lim\limits_{x\to0}\dfrac{\frac{(2|\sin x|)^2}{2!}}{x^2} = \lim\limits_{x\to0}\dfrac{4\sin^2 x}{2x^2} = 2$

2. Let $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}, \vec{b}=4\hat{i}+\hat{j}+7\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}+4\hat{k}$ be three vectors. If a vectors $\vec{p}$ satisfies $\vec{p}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{p}.\vec{a}=0$, then $\vec{p}.\left(\hat{i}-\hat{j}-\hat{k}\right)$ is equal to

(1) 24 (2) 36
(3) 32 (4) 28

Ans. (3)

Sol. $\vec{p}\times\vec{b}=\vec{c}\times\vec{b} \Rightarrow (\vec{p}-\vec{c})\times\vec{b}=0$

$\Rightarrow \vec{p}-\vec{c}=\lambda\vec{b}$

$\Rightarrow \vec{p}=\vec{c}+\lambda\vec{b}=\left(\hat{i}-3\hat{j}+4\hat{k}\right)+\lambda\left(4\hat{i}+\hat{j}+7\hat{k}\right)$

$= \hat{i}(1+4\lambda)+\hat{j}(-3+\lambda)+\hat{k}(4+7\lambda)$

Now $\vec{p}\cdot\vec{a}=0$

$\Rightarrow 3(1+4\lambda)+1(-3+\lambda)-2(4+7\lambda)=0$

$\Rightarrow 3-3-8+12\lambda+\lambda-14\lambda=0$

$\lambda=-8$

Now $\vec{p}\cdot\left(\hat{i}-\hat{j}-k\right)$

$= (1+4\lambda)-(-3+\lambda)-(4+7\lambda)$

$= 1+4\lambda-(-3+\lambda)-4-7\lambda$

$= -4\lambda = 32$

3. If $f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all $x \in R$, then $2f(0) + f'(0)$ is equal to:

(1) 18 (2) 24
(3) 48 (4) 42

Ans. (4)

Sol. $f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$

$f(0) = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} = -1(-4-0) + (8-0) = 12$

$\Rightarrow$ $2f(0) = 24$

$f'(x) = \begin{vmatrix} 3x^2 & 2x^2+1 & 1+3x \\ 6x & 2x & x^3+6 \\ 3x^2-1 & 4 & x^2-2 \end{vmatrix} + \begin{vmatrix} x^3 & 4x & 1+3x \\ 3x^2+2 & 2 & x^3+6 \\ x^3-x & 0 & x^2-2 \end{vmatrix} + \begin{vmatrix} x^3 & 2x^2+1 & 3 \\ 3x^2+2 & 2x & 3x^2 \\ x^3-x & 4 & 2x \end{vmatrix}$

$f'(0) = \begin{vmatrix} 0 & 1 & 1 \\ 0 & 0 & 6 \\ -1 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 \\ 2 & 2 & 6 \\ 0 & 0 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 3 \\ 2 & 0 & 0 \\ 0 & 4 & 0 \end{vmatrix}$

$= -1(0+6) + 0 + 1(0-0) + 3(8-0) = -6 + 24 = 18$

$2f(0) + f'(0) = 24 + 18 = 42$

4. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = \frac{(\tan x) + y}{\sin x(\sec x - \sin x \tan x)}, x \in \left(0, \frac{\pi}{2}\right)$ satisfying the condition $y\left(\frac{\pi}{4}\right) = 2$. Then, $y\left(\frac{\pi}{3}\right)$ is

(1) $\frac{\sqrt{3}}{2}(2 + \log_e 3)$ (2) $\sqrt{3}\left(2 + \log_e \sqrt{3}\right)$
(3) $\sqrt{3}(2 + \log_e 3)$ (4) $\sqrt{3}(1 + 2\log_e 3)$

Ans. (2)

Sol. $\frac{dy}{dx} = \frac{\tan x + y}{\sin x(\sec x - \sin x \tan x)}$

$\frac{dy}{dx} - \frac{y}{\sin x(\sec x - \sin x \tan x)}$

$= \frac{\tan x}{\sin x(\sec x - \sin x \tan x)}$

Now $\sin x(\sec x - \sin x \tan x)$

$= \sin\left(\frac{1-\sin^2 x}{\cos x}\right) = \sin x \cos x$

I.F. = $e^{-\int \frac{1}{\sin x \cos x}dx} = e^{-\int \frac{2}{\sin 2x}dx} = e^{-2\frac{(\ln|\cos ec2x-\cot 2x|)}{2}}$

$= \frac{1}{\text{cosec}2x - \cot 2x} = \frac{\sin 2x}{2\sin^2 x} = \cot x$

$\Rightarrow$ solution is

$y\cot x = \int \frac{1}{\sin x \cos x}dx$

$\Rightarrow \quad y\cot x = \frac{2\log|\text{cosec}2x - \cot 2x|}{2} + c$

$\Rightarrow$ ycotx = log|cosec2x – cot2x| + c

but $y\left(\frac{\pi}{4}\right) = 2$

$\Rightarrow$ 2 = log|1 – 0| + c $\Rightarrow$ c = 2

Now $y\left(\frac{\pi}{3}\right) = \frac{\log\left|\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right| + 2}{\frac{1}{\sqrt{3}}} = \sqrt{3}\left(\log\sqrt{3} + 2\right)$

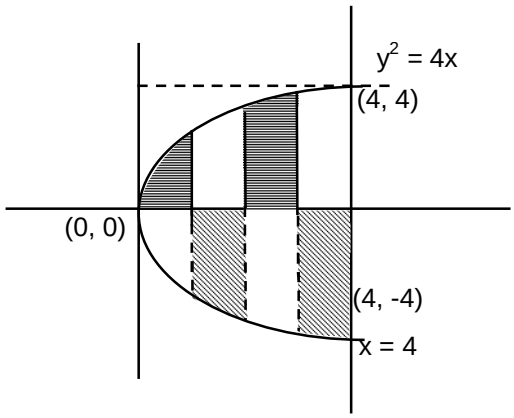
5. The area of the region $\left\{(x,y) : y^2 \le 4x, x < 4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0, x \ne 3\right\}$ is

(1) $\frac{8}{3}$ (2) $\frac{64}{3}$

(3) $\frac{16}{3}$ (4) $\frac{32}{3}$

Ans. (4)

Sol. Given $y^2 \le 4x$; x < 4 and $\frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0$, $x \ne 3$



Case (1) : When y > 0

$\frac{(x)(x-1)(x-2)}{(x-3)(x-4)} > 0$

$x \in (0, 1) \cup (2, 3)$

Case (2) : When y < 0

$x \in (1, 2) \cup (3, 4)$

Req. area = $\int_0^4 2\sqrt{x}dx = \frac{32}{3}$

*6. The sum of the series $\frac{1}{1-3.1^2+1^4} + \frac{2}{1-3.2^2+2^4} + \frac{3}{1-3.3^2+3^4}$ + ……. up to 10-terms is

(1) $\frac{45}{109}$ (2) $-\frac{55}{109}$

(3) $-\frac{45}{109}$ (4) $\frac{55}{109}$

Ans. (2)

Sol. $\frac{1}{1-3.1^2+1^4}+\frac{2}{1-3.2^2+2^4}+\frac{3}{1-3.3^2+3^4}+.....$ upto 10 terms

$$\sum_{r=1}^{10}\frac{r}{r^4-3r^2+1}$$

$$T_r=\frac{r}{r^4-3r^2+1}=\frac{r}{(r^2-r-1)(r^2+r-1)}$$

$$T_r=\frac{(r^2+r-1)-(r^2-r-1)}{2(r^2-r-1)(r^2+r-1)}$$

$$T_r=\frac{1}{2}\left[\frac{1}{r^2-r-1}-\frac{1}{r^2+r-1}\right]$$

$$T_1=\frac{1}{2}\left[\frac{1}{-1}-1\right]$$

$$T_2=\frac{1}{2}\left[1-\frac{1}{5}\right]$$

$$T_3=\frac{1}{2}\left[\frac{1}{5}-\frac{1}{11}\right]$$

.....

.....

$$T_{10}=\frac{1}{2}\left[\frac{1}{89}-\frac{1}{109}\right]$$

$$S=\frac{1}{2}\left[-1-\frac{1}{109}\right]$$

$$S=\frac{1}{2}\left[\frac{-110}{109}\right]$$

$$S=\frac{-55}{109}$$

*7. If the foci of a hyperbola are same as that of the ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$ and the eccentricity of the hyperbola is $\frac{15}{8}$ times the eccentricity of the ellipse, then the smaller focal distance of the point $\left(\sqrt{2},\frac{14}{3}\sqrt{\frac{2}{5}}\right)$ on the hyperbola, is equal to

(1) $7\sqrt{\frac{2}{5}}+\frac{8}{3}$ (2) $7\sqrt{\frac{2}{5}}-\frac{8}{3}$

(3) $14\sqrt{\frac{2}{5}}-\frac{4}{3}$ (4) $14\sqrt{\frac{2}{5}}-\frac{16}{3}$

Ans. (2)

Sol. foci = (0, ± be) = $\left(0,\pm5\sqrt{1-\frac{9}{25}}\right)$ = (0, ±4)

$\Rightarrow$ be = 4

Now e(for hyp.) = $\frac{15}{8}\times\frac{4}{5}=\frac{3}{2}$

$\Rightarrow \quad \frac{3}{2}\times b = 4$

$b = \frac{8}{3}$

Now smaller focal distance

$= (ey - b) = \frac{3}{2}\times\frac{14}{3}\sqrt{\frac{2}{5}}-\frac{8}{3}=7\sqrt{\frac{2}{5}}-\frac{8}{3}$

8. If the system of linear equations
$x - 2y + z = -4$
$2x + \alpha y + 3z = 5$
$3x - y + \beta z = 3$
has infinitely many solutions, then $12\alpha + 13\beta$ is equal to
(1) 54 (2) 60
(3) 64 (4) 58

Ans. (4)

Sol. $D = \begin{vmatrix} 1 & -2 & 1 \\ 2 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = 0$

$1(\alpha\beta + 3) + 2(2\beta - 9) + 1(-2 - 3\alpha) = 0$
$\alpha\beta + 3 + 4\beta - 18 - 2 - 3\alpha = 0$
$\alpha\beta + 4\beta - 3\alpha - 17 = 0$
$\alpha(\beta - 3) + 4(\beta - 3) - 5 = 0$
$(\alpha + 4)(\beta - 3) = 5$

$D_1 = \begin{vmatrix} -4 & -2 & 1 \\ 5 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = 0$

$-4\alpha\beta - 3\alpha + 10\beta - 35 = 0$

$D_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$

$13\beta = 54 \Rightarrow \beta = \frac{54}{13}$

$D_3 = \begin{vmatrix} 1 & -2 & -4 \\ 2 & \alpha & 5 \\ 3 & -1 & 3 \end{vmatrix} = 0$

$\Rightarrow 15\alpha - 5 = 0 \Rightarrow \alpha = \frac{1}{3}$

Now $12\alpha + 13\beta = 12\left(\frac{1}{3}\right)+13\left(\frac{54}{13}\right) = 4 + 54 = 58$

9. The solution curve of the differential equation $y\frac{dx}{dy} = x\left(\log_e x - \log_e y + 1\right)$, x > 0, y > 0 passing through the point (e, 1) is

(1) $\left|\log_e \frac{y}{x}\right| = y^2$ (2) $\left|\log_e \frac{y}{x}\right| = x$

(3) $2\left|\log_e \frac{x}{y}\right| = y + 1$ (4) $\left|\log_e \frac{x}{y}\right| = y$

Ans. (4)

Sol. $\frac{dx}{dy} = \frac{x}{y}\left(\log \frac{x}{y} + 1\right)$

x = ty

$\frac{dx}{dy} = t + y\frac{dt}{dy}$

$t + y\frac{dt}{dy} = t(\log t + 1)$

$y\frac{dt}{dy} = t\log t$

$\frac{dt}{t\log t} = \frac{dy}{y}$

Integrating both side

$\int \frac{dt}{t\log t} = \int \frac{dy}{y}$

log|log t| = log|y| + logc = logc|y|

|logt| = c|y| = cy

$\left|\log \frac{x}{y}\right| = cy$

(e, 1) satisfies the curve

$\left|\log \frac{e}{1}\right| = c \Rightarrow c = 1$

$\Rightarrow \left|\log \frac{x}{y}\right| = y$

10. Three rotten apples are accidently mixed with fifteen good apples. Assuming the random variable x to be the number of rotten apples in a draw of two apples, the variance of x is

(1) $\frac{47}{153}$ (2) $\frac{37}{153}$

(3) $\frac{57}{153}$ (4) $\frac{40}{153}$

Ans. (4)

Sol.

x_i	0	1	2
p_i	$\frac{^{15}C_2}{^{18}C_2}$	$\frac{^{15}C_1 \times {}^3C_1}{^{18}C_2}$	$\frac{^3C_2}{^{18}C_2}$

$\sum p_i x_i = \frac{1}{3}$ and $\sum p_i x_i^2 = \frac{57}{153}$

Now $\sum p_i x_i^2 - \left(\sum p_i x_i\right)^2 = \frac{57}{153} - \frac{1}{9} = \frac{40}{153}$

11. let a be the sum of all coefficients in the expansion of $\left(1-2x+2x^2\right)^{2023}\left(3-4x^2+2x^3\right)^{2024}$ and $b = \lim\limits_{x\to 0}\left(\frac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}dt}{x^2}\right)$. If the equations $cx^2 + dx + e = 0$ and $2bx^2 + ax + 4 = 0$ have a common root, where c, d, e $\in$ R, then d : c : e equals

(1) 4 : 1 : 4 (2) 1 : 1 : 4

(3) 2 : 1 : 4 (4) 1 : 2 : 4

Ans. (2)

Sol. a is sum of all coeff. in exp

$(1 - 2x + 2x^2)^{2023}(3 - 4x^2 + 2x^3)^{2024}$

Put x = 1

a = 1

$b = \lim\limits_{x\to 0}\frac{\int_0^x \frac{\ln(1+t)}{t^{2024}+1}dt}{x^2}$

Applying L' Hospital

$b = \lim\limits_{x\to 0}\frac{\ln(1+x)}{(x^{2024}+1)2x} = \frac{1}{2}$

Now $2bx^2 + ax + 4 = 0$

$\Rightarrow x^2 + x + 4$ D < 0

Hence both roots are common

$\Rightarrow \frac{c}{2b} = \frac{d}{a} = \frac{e}{4} = \frac{c}{1} = \frac{d}{1} = \frac{e}{4}$

d : c : e : : 1 : 1 : 4

12. the distance of the point Q(0, 2, – 2) form the line passing through the point P(5, – 4, 3) and perpendicular to the lines $\vec{r} = \left(-3\hat{i}+2\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+5\hat{k}\right), \lambda \in R$ and $\vec{r} = \left(\hat{i}-2\hat{j}+\hat{k}\right)+\mu\left(-\hat{i}+3\hat{j}+2\hat{k}\right), \mu \in R$ is:

(1) $\sqrt{54}$ (2) $\sqrt{86}$

(3) $\sqrt{74}$ (4) $\sqrt{20}$

Ans. (3)

Sol. $\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix}$

$= -9\hat{i} - 9\hat{j} + 9\hat{k}$

Dr's = 1, 1, – 1

Equation of line

$$\frac{x-5}{1}=\frac{y+4}{1}=\frac{z-3}{-1}=\lambda$$

$x=\lambda+5,\ y=\lambda-4,\ z=-\lambda+3$

Dr's of PQ = $\lambda + 5, \lambda - 6, -\lambda + 5$

$\Rightarrow$ $1(\lambda + 5) + 1(\lambda - 6) - 1(-\lambda + 5) = 0$

$\lambda + 5 + \lambda - 6 + \lambda - 5 = 0$

$3\lambda = 6,\ \lambda = 2$

$\Rightarrow$ Q(7, –2, 1)

$\Rightarrow$ $PQ=\sqrt{49+16+9}=\sqrt{74}$.

*13. Let $\alpha, \beta, \gamma, \delta \in Z$ and let A(α, β), B(1, 0), C(γ, δ) and D(1, 2) be the vertices of a parallelogram ABCD. If AB = $\sqrt{10}$ and the points A and C lie on the line 3y = 2x + 1, then $2(\alpha + \beta + \gamma + \delta)$ is equal to

(1) 12 (2) 8

(3) 10 (4) 5

Ans. (2)

Sol. By mid point theorem

$$\left(\frac{\alpha+\gamma}{2},\frac{\beta+\delta}{2}\right)=\left(\frac{1+1}{2},\frac{2+0}{2}\right)$$

$\alpha + \gamma = 2,\ \beta + \delta = 2$

$\Rightarrow$ $2(\alpha + \beta + \gamma + \delta) = 8$.

*14. For $\alpha, \beta, \gamma \neq 0$, if $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$, then γ equals

(1) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (2) $\sqrt{3}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

Ans. (4)

Sol. $\sin^{-1}\beta+\sin^{-1}\gamma=\pi-\sin^{-1}\alpha$

Taking sine both sides

$$\beta\sqrt{1-\gamma^2}+\gamma\sqrt{1-\beta^2}=\alpha$$

$$\gamma\sqrt{1-\beta^2}=\alpha-\beta\sqrt{1-\gamma^2}$$

Squaring both sides

$\Rightarrow$ $\gamma^2\left(1-\beta^2\right)=\alpha^2+\beta^2\left(1-\gamma^2\right)-2\alpha\beta\sqrt{1-\gamma^2}$

$\Rightarrow$ $\gamma^2-\beta^2\gamma^2=\alpha^2+\beta^2-\beta^2\gamma^2-2\alpha\beta\sqrt{1-\gamma^2}$

$\Rightarrow$ $2\alpha\beta\sqrt{1-\gamma^2}=\alpha^2+\beta^2-\gamma^2$ … (i)

And $(\alpha+\beta+\gamma)(\alpha-\gamma+\beta)=3\alpha\beta$

$$(\alpha+\beta)^2-\gamma^2=3\alpha\beta$$

$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta - \gamma^2 = 3\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 - \gamma^2 = \alpha\beta$ … (ii)

$2\alpha\beta\sqrt{1-\gamma^2} = \alpha\beta$

$\sqrt{1-\gamma^2} = \frac{1}{2}$

$\Rightarrow 1-\gamma^2 = \frac{1}{4}$

$\Rightarrow \gamma^2 = \frac{3}{4}$

$\Rightarrow \gamma = \frac{\sqrt{3}}{2}$.

15. Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each drawing. Then the probability, that first drawn marble is red and second drawn marble is white, is

(1) $\frac{2}{25}$ (2) $\frac{4}{75}$

(3) $\frac{4}{25}$ (4) $\frac{2}{3}$

Ans. (2)

Sol. 10 Red
30 white
20 Blue
15 orange
Total 75 enable

Prob. = $\frac{10}{75} \times \frac{30}{75} = \frac{4}{75}$.

16. Let S be the set of positive integral values of a for which $\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32} < 0, \forall x \in R$..

Then, the number of elements in S is:

(1) ∞ (2) 3

(3) 0 (4) 1

Ans. (3)

Sol. $\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32} < 0 \quad \forall x \in R$

In $x^2 - 8x + 32 \Rightarrow 0 = 64 - 128 < 0$

$\Rightarrow x^2 - 8x + 32 > 0 \ \forall \ x \in R$

$\Rightarrow ax^2 + 2(a + 1)x + 9a + 4 < 0 \ \forall \ x \in R$

$\Rightarrow \left.\begin{matrix} a < 0 \\ D < 0 \end{matrix}\right] \cap$

$\Rightarrow$ no +ve integral of a

$\Rightarrow$ number of element in S = 0.

17. Let g(x) be a linear function and $f(x) = \begin{cases} g(x) & ,x \le 0 \\ \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}} & ,x > 0 \end{cases}$, is continuous at x = 0. If $f'(1) = f(-1)$, then the value g(3) is

(1) $\frac{1}{3}\log_e\left(\frac{4}{9e^{1/3}}\right)$ (2) $\log_e\left(\frac{4}{9e^{1/3}}\right)$

(3) $\frac{1}{3}\log_e\left(\frac{4}{9}\right)+1$ (4) $\log_e\left(\frac{4}{9}\right)-1$

Ans. (2)

Sol. g(0) = 0

$$f'(x) = e^{\frac{1}{x}\ln\left(\frac{1+x}{2+x}\right)}\left[\frac{1}{x}\cdot\frac{2+x}{1+x}\frac{1}{(x+2)^2}+\ln\left(\frac{1+x}{2+x}\right)\times-\frac{1}{x^2}\right]$$

$$f'(1) = e^{\ln\left(\frac{2}{3}\right)}\left[\frac{3}{2}\times\frac{1}{g}+\ln\left(\frac{2}{3}\right)\times-1\right]$$

$$= \frac{2}{3}\left[\frac{1}{6}-\ln\frac{2}{3}\right] = g(-1)$$

g(0) = 0

g(x) = ax, g(–1) = –a

$$\frac{1}{9}-\frac{2}{3}\ln\frac{2}{3} = -a$$

$$a = \frac{2}{3}\ln\frac{2}{3}-\frac{1}{9}$$

$$g(x) = \left(\frac{2}{3}\ln\frac{2}{3}-\frac{1}{9}\right)x$$

$$g(3) = 2\ln\frac{2}{3}-\frac{1}{3}$$

$$= \ln\frac{4}{9}-\ln e^{1/3}$$

$$= \ln\left(\frac{4}{9e^{1/3}}\right).$$

*18. If one of the diameters of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle C, whose center is the point of intersection of the lines 2x + 3y = 12 and 3x – 2y = 5, then the radius of the circle C is:

(1) $\sqrt{20}$ (2) 4

(3) 6 (4) $3\sqrt{2}$

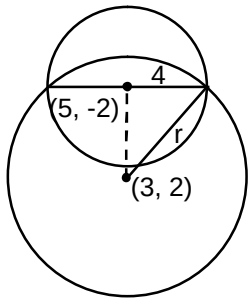
Ans. (3)

Sol. $2x + 3y = 12,\ 3x - 2y = 5$, centre $(3, 2)$

$= \sqrt{25 + 4 - 13} = 4$, $(5, -2)$

$r^2 = 16 + \left(\sqrt{4 + 16}\right)^2 = 36$

$r = 6$



*19. For $0 < c < b < a$, let $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ and $\alpha \neq 1$ be one of its root. Then, among the two statements

(I) If $\alpha \in (-1, 0)$, then b cannot be the geometric mean of a and c

(II) If $\alpha \in (0, 1)$, then b may be the geometric mean of a and c

(1) neither (I) nor (II) is true (2) only (II) is true

(3) only (I) is true (4) both (I) and (II) are true

Ans. (4)

Sol. $\alpha = \dfrac{c + a - 2b}{a + b - 2c}$

Statement I $-1 < \dfrac{c + a - 2b}{a + b - 2c} < 0$

$-a - b + 2c < c + a - 2b < 0$

$c + a < 2b$

$c + a < 2\sqrt{ac}$

$\left(\sqrt{c} - \sqrt{a}\right)^2 < 0$ statement I is correct

Statement II : $0 < \dfrac{c + a - 2b}{a + b - 2c} < 1$

$0 < c + a - 2\sqrt{ac}$

$0 < \left(\sqrt{a} - \sqrt{c}\right)^2$ is correct

20. If $f(x) = \dfrac{4x+3}{6x-4}, x \neq \dfrac{2}{3}$ and $(fof)(x) = g(x)$, where $g : R - \left\{\dfrac{2}{3}\right\} \to R - \left\{\dfrac{2}{3}\right\}$, then (gogog)(4) is equal to

(1) 4 (2) -4

(3) $-\dfrac{19}{20}$ (4) $\dfrac{19}{20}$

Ans. (1)

Sol. $f(x) = \dfrac{4x + 3}{6x - 4}$

$g(x) = ff(x) = \dfrac{4f(x) + 3}{6f(x) - 4}$

$$= \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}$$

$$= \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{34x}{34} = x$$

Now g g g(4) = 4.

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

21. Let Q and R be the feet of perpendiculars from the point P(a, a, a) on the lines x = y, z = 1 and x = – y, z = – 1 respectively. If $\angle QPR$ is a right angle, then $12a^2$ is equals to_____.

Ans. 12

Sol. P(a, a, a)

$$\frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = \lambda$$

$x = \lambda, y = \lambda, z = 1$

similarly R $(\mu, -\mu, -1)$

Now PQ $\perp L_1$

$\Rightarrow 1(\lambda - a) + 1(\lambda - a) + 0(1-a) = 0$

$\lambda = a$

$\Rightarrow$ Q(a, a, 1)

Again RP $\perp L_2$

$1(\mu - a) - 1(-\mu - a) + 0(-1-a) = 0$

$\mu - a + \mu + a = 0$

$\mu = 0$

$\Rightarrow$ R(0, 0, –1)

Dr's of PQ = (0, 0, a–1)

PR = (a, a, a +1)

$\Rightarrow a = \pm 1$

$\Rightarrow 12a^2 = 12.$

22. Let S = (– 1, ∞) and $f : S \to R$ be defined as $f(x) = \int_{-1}^{x} (e^t - 1)^{11}(2t-1)^5(t-2)^7(t-3)^{12}(2t-10)^{61}dt$.

Let p = sum of squares of the values of x, where $f(x)$ attains local maxima on S, and q = sum of the values of x, where $f(x)$ attains local minima on S. then, the value of $p^2 + 2q$ is _____.

Ans. 27

Sol. $f'(x) = (e^x - 1)^{11}(2x-1)^5(x-2)^7(x-3)^{12}(2x-10)^{61}$

Local maxima at x = 0, 2 $\Rightarrow p = 0^2 + 2^2 = 4$

Local minima at x = 1/2, 5 $\Rightarrow$ q = $\frac{1}{2}+5=\frac{11}{2}$

$p^2+2q=(4)^2+2\left(\frac{11}{2}\right)=27$

*23. Let the foci and length of the latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ be $(\pm 5,\ 0)$ and $\sqrt{50}$, respectively. Then, the square of the eccentricity of the hyperbola $\frac{x^2}{b^2}-\frac{y^2}{a^2b^2}=1$ equals

Ans. 51

Sol. $(\pm 5,\ 0)=(\pm ae,\ 0)$ & $\frac{2b^2}{a}=\sqrt{50}$

$\Rightarrow$ ae = 5, $b^2=\frac{\sqrt{50}a}{2}$

$\Rightarrow$ $a^2-b^2=25$

$\Rightarrow$ $a^2-\frac{\sqrt{50}a}{2}=25$

$\Rightarrow$ $2a^2-5\sqrt{2}a-50=0$

$2a^2-10\sqrt{2}a+5\sqrt{2}a-50=0$

$2a\left(a-5\sqrt{2}\right)+5\sqrt{2}\left(a-5\sqrt{2}\right)=0$

$\Rightarrow$ $a=5\sqrt{2}$

$\Rightarrow$ $b^2=\frac{\sqrt{50}\times 5\sqrt{2}}{2}=25$

$\Rightarrow$ b = 5

Now e for $\frac{x^2}{b^2}-\frac{y^2}{a^2b^2}=1$

Is $\sqrt{1+\frac{a^2b^2}{b^2}}$

$=\sqrt{1+a^2}=\sqrt{51}$, e^2 = 51

24. Let A = {1, 2, 3, 4} and R = {(1, 2), (2, 3), (1, 4)} be a relation on A. Let S be the equivalence relation on A such that R $\subset$ S and the number of elements in S is n. Then, the minimum value of n is _____.

Ans. 16

Sol. S = {(1, 2), (2, 1), (2, 3), (3, 2), (1, 4), (4, 1), (1, 1), (2, 2), (3, 3), (4, 4), (1, 3), (3, 1), (2, 4), (4, 2), (3, 4) (4, 3)}

25. If the integral $525\int_0^{\frac{\pi}{2}} \sin 2x \cos^{\frac{11}{2}} x\left(1+\cos^{\frac{5}{2}} x\right)^{\frac{1}{2}} dx$ is equal to $\left(n\sqrt{2}-64\right)$, then n is equal to _____.

Ans. 176

Sol. $525\int_0^{\pi/2} 2\sin x\cos^{13/2} x\left(1+\cos^{5/2} x\right)^{1/2} dx$

$1+\cos^{5/2} x = t^2$

$\frac{5}{2}\cos^{3/2} x \times -\sin x dx = 2t\ dt$

$-525\int_{\sqrt{2}}^{1} \frac{8t}{5}\left(t^2-1\right)^2 t\ dt$

$-105\times 8\int_{\sqrt{2}}^{1} t^2\left[t^4-2t^2+1\right] dt$

$-105\times 8\left[\frac{t^7}{7}-\frac{2t^5}{5}+\frac{t^3}{3}\right]_{\sqrt{2}}^{1}$

n = 176

*26. If α denotes the number of solutions of $|1 - i|^x = 2^x$ and $\beta = \left(\frac{|z|}{\arg(z)}\right)$, where $z = \frac{\pi}{4}(1+i)^4\left[\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i}+\frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right], i = \sqrt{-1}$, then the distance of the point (α, β) from the line $4x - 3y = 7$ is _____ .

Ans. 3

Sol. $|1-i|^x = 2^x \Rightarrow 2^{x/2} = 2^x$

$\Rightarrow$ x = 0

$\Rightarrow$ only one solution

$\Rightarrow$ $\alpha = 1$

$z = \frac{\pi}{4}(1+i)^4\left[\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i}+\frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right]$

$= \frac{\pi}{4}(-4)\left(\frac{1+\pi+\pi+1}{\sqrt{\pi}+\pi i+i-\sqrt{\pi}}\right)$

$= -\pi\left(\frac{2}{i}\right) = 2i\pi$

$\beta = \left(\frac{|z|}{\arg z}\right) = \frac{2\pi}{\frac{\pi}{2}} = 4$

Distance of $(\alpha, \beta) = (1, 4)$ from $4x - 3y - 7 = 0$

$$= \left|\frac{4-12-7}{\sqrt{4^2+3^2}}\right| = 3$$

*27. The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal to _____.

Ans. 3734

Sol. Letters used in word
D S R B U O N I I I T T
4 different letters $\Rightarrow {}^9C_4\, 4! = 3024$

2 same 2 different $\Rightarrow {}^2C_1\, {}^8C_2\, \frac{4!}{2!} = 672$

2 same 2 same $\Rightarrow \frac{4!}{2!\,2!} = 6$

3 same 1 different $\Rightarrow {}^8C_1 \frac{4!}{3!} = 32$

Total words = 3734

*28. In the expansion of $(1 + x)(1 - x^2)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$, $x \neq 0$, then sum of the coefficients of x^3 and x^{-13} is equal to _____.

Ans. 118

Sol. $(1+x)\left(1-x^2\right)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$

$$= (1+x)(1+x)(1-x)\left(\frac{x^3+3x^2+3x+1}{x^3}\right)^5$$

$$= (1+x)^2(1-x)\left(\frac{(x+1)^3}{x^3}\right)^5$$

$$= (1+x)^2(1-x)\frac{(x+1)^{15}}{x^{15}}$$

$$= \frac{(1-x)(1+x)^{17}}{x^{15}}$$

Coeff. Of x^3 in expression $\frac{(1-x)(1+x)^{17}}{x^{15}}$

= coeff of x^3 in $\frac{(1+x)^{17}}{x^{15}} - \frac{(1+x)^{17}}{x^{14}}$

= coeff of x^{18} in $(1 + x)^{17}$ – coeff of x^{17} in $(1 + x)^{17}$

$= 0 - {}^{17}C_{17} = -1$

Coeff. Of x^{-13} in expression $\frac{(1-x)(1+x)^{17}}{x^{15}}$

= coeff of x^{-13} in $\left[\frac{(1+x)^{17}}{x^{15}}-\frac{(1+x)^{17}}{x^{14}}\right]$

= coeff of x^2 in $(1+x)^{17}$ – coeff of x in $(1+x)^{17}$

= $^{17}C_2 - {}^{17}C_1 = 119$

Sum of coefficient = 119 – 1 = 118.

29. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|=1, |\vec{b}|=4$, and $\vec{a}.\vec{b}=2$. If $\vec{c}=(2\vec{a}\times\vec{b})-3\vec{b}$ and the angle between $\vec{b}$ and $\vec{c}$ is α, then $192\sin^2\alpha$ is equal to _____ .

Ans. 48

Sol. Let angle between $\vec{a}$ & $\vec{b}$ is θ

$$\vec{a}\cdot\vec{b}=|\vec{a}|\,|\vec{b}|\cos\theta=2$$

$$\Rightarrow \quad \cos\theta=\frac{1}{2}$$

$$\vec{c}=2\vec{a}\times\vec{b}-3\vec{b}$$

$$\Rightarrow \quad |\vec{c}|^2=|2\vec{a}\times\vec{b}-3\vec{b}|^2$$

$$=4(\vec{a}\times\vec{b})^2+9|\vec{b}|^2-12(\vec{a}\times\vec{b})\cdot\vec{b}$$

$$= 4\times12 + 9\times16 - 0$$

$$= 192$$

And $\vec{c}+3\vec{b}=2\vec{a}\times\vec{b}$

$$|\vec{c}+3\vec{b}|^2=|2\vec{a}\times\vec{b}|^2$$

$$|\vec{c}|^2+9|\vec{b}|^2+6\vec{b}\cdot\vec{c}=4(\vec{a}\times\vec{b})^2$$

$$|\vec{c}|^2+144+6\cdot4\cdot8\sqrt{3}\cos\alpha=4\cdot12$$

$$192+144+192\sqrt{3}\cos\alpha=48$$

$$\cos\alpha=\frac{-288}{192\sqrt{3}}=-\frac{\sqrt{3}}{2}$$

$$\cos^2\alpha=\frac{3}{4}\Rightarrow\sin^2\alpha=\frac{1}{4}$$

$$\Rightarrow \quad 192\sin^2\alpha=\frac{192}{4}=48.$$

30. Let $f : R \to R$ be a function defined by $f(x) = \frac{4^x}{4^x+2}$ and $M = \int_{f(a)}^{f(1-a)} x\sin^4\left(x(1-x)\right)dx$, $N = \int_{f(a)}^{f(1-a)} \sin^4\left(x(1-x)\right)dx; a \neq \frac{1}{2}$. If $\alpha M = \beta N$, $\alpha, \beta \in N$, then the least value of $\alpha^2 + \beta^2$ is equal to _____

Ans. 5

Sol. $f(x) + f(1-x) = \frac{4^x}{4^x+2} + \frac{4^{1-x}}{4^{1-x}+2}$

$= \frac{4^x}{4^x+2} + \frac{4}{4+2.4^x}$

$= \frac{4^x}{4^x+2} + \frac{2}{2+4^x} = 1$

$M = \int_{f(a)}^{f(1-a)} x\sin^4\left(x(1-x)\right)\,dx$

$x \to a + b - x \Rightarrow x = f(a) + f(1-a) - x$

$= 1 - x$

$M = \int_{f(a)}^{f(1-a)} (1-x)\sin^4\left((1-x)x\right)dx$

$2M = \int_{f(a)}^{f(1-a)} \sin^4\left(x(1-x)\right)dx$

$= N$

$\alpha = 2, \beta = 1 \Rightarrow \alpha^2 + \beta^2 = 5.$

PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

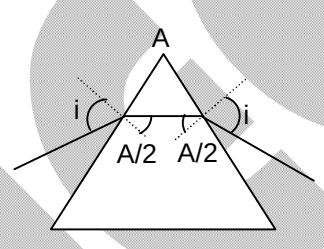
31. The refractive index of a prism with apex angle A is $\cot A/2$. The angle of minimum deviation is :

(1) $\delta_m = 180° - 4A$ (2) $\delta_m = 180° - 3A$

(3) $\delta_m = 180° - 2A$ (4) $\delta_m = 180° - A$

Ans. (3)

Sol. $\sin i = \mu \sin\frac{A}{2} = \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}\sin\frac{A}{2}$



$i = 90° - \frac{A}{2}$

$\sin i = i + i - A = 180° - 2A$

*32. A coin is placed on a disc. The coefficient of friction between the coin and the disc is μ. If the distance of the coin from the center of the disc is r, the maximum angular velocity which can be given to the disc, so that the coin does not slip away, is :

(1) $\frac{\mu}{\sqrt{rg}}$ (2) $\sqrt{\frac{\mu g}{r}}$

(3) $\frac{\mu g}{r}$ (4) $\sqrt{\frac{r}{\mu g}}$

Ans. (2)

Sol. $\mu mg = m\omega^2 r$

$\omega = \sqrt{\frac{\mu g}{r}}$

*33. The relation between time 't' and distance 'x' is $t = \alpha x^2 + \beta x$, where α and β are constants. The relation between acceleration (a) and velocity (v) is :

(1) $a = -2\alpha v^3$ (2) $a = -4\alpha v^3$

(3) $a = -3\alpha v^3$ (4) $a = -5\alpha v^3$

Ans. (1)

Sol. $t = \alpha x^2 + \beta x$

$1 = 2\alpha x v + \beta v \Rightarrow v = \frac{1}{\beta + 2\alpha x}$

$0 = 2\alpha v^2 + 2\alpha x a + \beta a$

$a(\beta + 2\alpha x) = -2\alpha v^2$

$a = -2\alpha v^3$

*34. The parameter that remains the same for molecules of all gases at a given temperature is :
(1) kinetic energy (2) mass
(3) speed (4) momentum

Ans. (1)

Sol. Total translational K.E. = $\frac{3}{2}KT$
So, independent of type of gas

35. When a metal surface is illuminated by light of wavelength λ, the stopping potential is 8V. When the same surface is illuminated by light of wavelength 3λ, stopping potential is 2V. The threshold wavelength for this surface is:
(1) 9λ (2) 3λ
(3) 4.5λ (4) 5λ

Ans. (1)

Sol. $\frac{hC}{\lambda} - \phi = 8e$

$\frac{hC}{3\lambda} - \phi = 2e$

So, $\phi = \frac{hC}{9\lambda}$

So, $\lambda_{th} = 9\lambda$

36. A coil is placed perpendicular to a magnetic field of 5000T . When the field is changed to 3000T in 2s , an induced emf of 22V is produced in the coil. If the diameter of the coil is 0.02m , then the number of turns in the coil is:
(1) 70 (2) 140
(3) 35 (4) 7

Ans. (4)

Sol. $22 = \frac{N\pi(0.01)^2 \times 2000}{2}$

So, $N = \frac{22 \times 10 \times 7}{22} = 70$

*37. The fundamental frequency of a closed organ pipe is equal to the first overtone frequency of an open organ pipe. If length of the open pipe is 60 cm, the length of the closed pipe will be:
(1) 30 cm (2) 45 cm
(3) 60 cm (4) 15 cm

Ans. (4)

Sol. $\frac{v}{4\ell_C} = \frac{2v}{2\ell_0}$

$\Rightarrow \ell_0 = 4\ell_C$

$\Rightarrow \ell_C = 15$ cm

38. Two conductors have the same resistances at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients for their series and parallel combinations are :

(1) $\alpha_1+\alpha_2,\ \frac{\alpha_1+\alpha_2}{2}$ (B) $\frac{\alpha_1+\alpha_2}{2},\ \frac{\alpha_1+\alpha_2}{2}$

(3) $\frac{\alpha_1+\alpha_2}{2},\ \alpha_1+\alpha_2$ (D) $\alpha_1+\alpha_2,\ \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$

Ans. (2)

Sol. $R_S = R(1+\alpha_1 T)+R(1+\alpha_2 T)$

$$= 2R + 2R\frac{(\alpha_1+\alpha_2)}{2}T$$

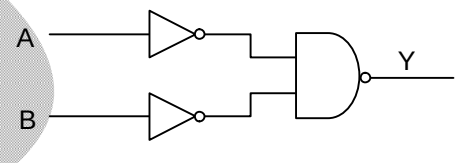
$$= 2R\left(1+\frac{(\alpha_1+\alpha_2)}{2}T\right)$$

$$R_P = \frac{R(1+\alpha_1 T)R(1+\alpha_2 T)}{R(1+\alpha_1 T)+R(1+\alpha_2 T)}$$

$$= \frac{R^2(1+\alpha_1 T+\alpha_2 T)}{2R\left(1+\frac{\alpha_1+\alpha_2}{2}T\right)} = \frac{R}{2}\left[1+(\alpha_1+\alpha_2)T-\frac{\alpha_1+\alpha_2}{2}T\right]$$

$$= \frac{R}{2}\left[1+\frac{\alpha_1+\alpha_2}{2}T\right]$$

39. Identify the logic operation performed by the given circuit.



(1) AND
(2) NOR
(3) NAND
(4) OR

Ans. (4)

Sol. $\overline{\overline{A}\cdot\overline{B}} = A+B$

40. In a plane EM wave, the electric field oscillates sinusoidally at a frequency of 5×10^{10}Hz and an amplitude of 50Vm^{-1}. The total average energy density of the electromagnetic field of the wave is:
[Use $\varepsilon_0 = 8.85\times10^{-12}\ C^2/Nm^2$]

(1) 1.106×10^{-8}Jm^{-3} (2) 2.212×10^{-10}Jm^{-3}

(3) 4.425×10^{-8}Jm^{-3} (4) 2.212×10^{-8}Jm^{-3}

Ans. (1)

Sol. $\frac{dU_T}{dV} = \frac{\varepsilon_0 E_0^2}{2} = \frac{8.85\times10^{-12}(50)^2}{2} = 1.106\times10^{-8}\ J/m^3$

41. If the wavelength of the first member of Lyman series of hydrogen is λ. The wavelength of the second member will be

(1) $\frac{5}{27}\lambda$ (2) $\frac{27}{32}\lambda$

(3) $\frac{32}{27}\lambda$ (4) $\frac{27}{5}\lambda$

Ans. (2)

Sol. $\frac{1}{\lambda} = R_H\left(1-\frac{1}{4}\right) = \frac{3R_H}{4}$

$\frac{1}{\lambda'} = R_H\left(1-\frac{1}{9}\right) = \frac{8R_H}{9} = \frac{8}{9}\left(\frac{4}{3\lambda}\right)$

$\lambda' = \frac{27}{32}\lambda$

*42. Four identical particles of mass m are kept at the four corners of a square. If the gravitational force exerted on one of the masses by the other masses is $\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$, the length of the sides of the square is

(1) 2L (2) 4L

(3) 3L (4) $\frac{L}{2}$

Ans. (2)

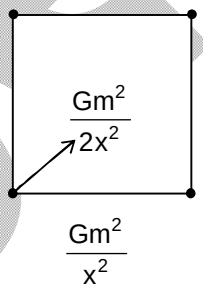
Sol. $F = \frac{Gm^2}{2x^2} + \frac{Gm^2}{x^2}\sqrt{2}$

$= \frac{Gm^2}{x^2}\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right) = \frac{Gm^2}{x^2}\left(\frac{2\sqrt{2}+1}{2}\right)$

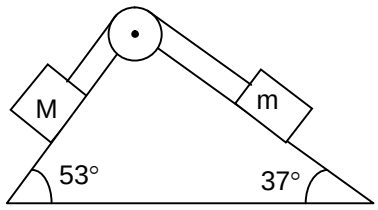
$= \frac{Gm^2}{\ell^2}\left(\frac{2\sqrt{2}+1}{32}\right)$

$x^2 = 16\ell^2$

So, $x = 4\ell$



*43. In the given arrangement of a doubly inclined plane two blocks of masses M and m are placed. The blocks are connected by a light string passing over an ideal pulley as shown. The coefficient of friction between the surface of the plane and the blocks is 0.25. The value of m, for which M=10kg will move down with an acceleration of 2 m/s^2, is: (take g = 10 m/s^2 and tan 37° = 3/4)



(1) 6.5 kg (2) 4.5 kg

(3) 2.25 kg (4) 9 kg

Ans. (2)

Sol. $100\times\frac{4}{5} - T - 100\times\frac{3}{5}\times\frac{1}{4} = 10a$

$\Rightarrow 80 - T - 15 = 10a$

$\Rightarrow 65 - T = 10a$

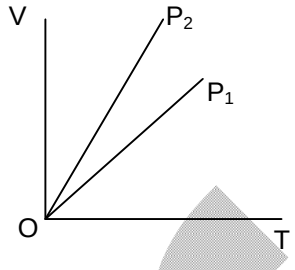
$T - 10m\frac{3}{5} - 10m\frac{4}{5}\times\frac{1}{4} = ma$

$\Rightarrow T - 6m - 2m = ma$

$\Rightarrow T - 8m = ma$

So, $65 - 8m = 2(10 + m) = 20 + 2m$
$45 = 10\ m$
$m = 4.5\ kg$

*44. The given figure represents two isobaric processes for the same mass of an ideal gas, then



(1) $P_1 > P_2$
(2) $P_2 > P_1$
(3) $P_1 = P_2$
(4) $P_2 \geq P_1$

Ans. (1)

Sol. $V = \frac{nR}{P}T$

So, $P_2 < P_1$

45. If the percentage errors in measuring the length and the diameter of a wire are 0.1% each. The percentage error in measuring its resistance will be:
(1) 0.2%
(2) 0.144%
(3) 0.3%
(4) 0.1%

Ans. (3)

Sol. $R = \rho\frac{\ell}{A}$

$\frac{dR}{R} = \frac{d\ell}{\ell} + 2\frac{dd}{d} = 0.3\%$

46. Two charges q and 3q are separated by a distance 'r' in air. At a distance x from charge q, the resultant electric field is zero. The value of x is :

(1) $\frac{r}{(1+\sqrt{3})}$
(2) $\frac{(1+\sqrt{3})}{r}$
(3) $r(1+\sqrt{3})$
(4) $\frac{r}{3(1+\sqrt{3})}$

Ans. (1)

Sol. $\frac{kq}{x^2} = \frac{k3q}{(r-x)^2}$

$(r - x) = \sqrt{3}x$

$x = \frac{r}{\sqrt{3}+1}$

47. A force is represented by $F = ax^2 + bt^{\frac{1}{2}}$
where x = distance and t = time. The dimensions of b^2 / a are:
(1) $\left[ML^2T^{-3}\right]$
(2) $\left[ML^3T^{-3}\right]$
(3) $\left[ML^{-1}T^{-1}\right]$
(4) $\left[MLT^{-2}\right]$

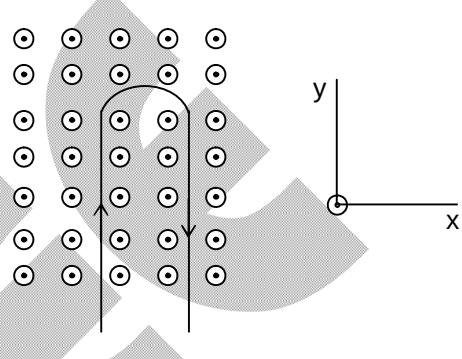
Ans. (2)

Sol. $a = \frac{\left[ML T^{-2}\right]}{L^2} = \left[ML^{-1}T^{-2}\right]$

$b = \frac{\left[ML T^{-2}\right]}{T^{1/2}} = \left[ML T^{-5/2}\right]$

$\frac{b^2}{a} = \frac{\left[M^2L^2T^{-5}\right]}{\left[ML^{-1}T^{-2}\right]} = \left[ML^3T^{-3}\right]$

48. A rigid wire consists of a semicircular portion of radius R and two straight sections. The wire is partially immerged in a perpendicular magnetic field $B = B_0\hat{j}$ as shown in figure. The magnetic force on the wire if it has a current i is:



(1) $-2iBR\hat{j}$

(2) $2iBR\hat{j}$

(3) $-iBR\hat{j}$

(4) $iBR\hat{j}$

Ans. (1)

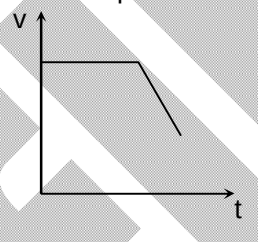
Sol. By taking $\vec{B} = B_0\hat{k}$

$\vec{F} = 2iB_0R(-\hat{j})$

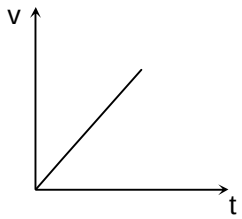
Although question should be bonus

*49. A small steel ball is dropped into a long cylinder containing glycerine. Which one of the following is the correct representation of the velocity time graph for the transit of the ball?

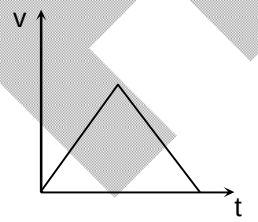
(A)



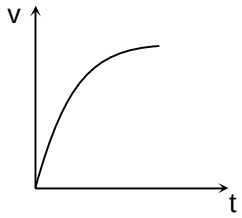
(B)



(C)



(D)



Ans. (4)

Sol. Speed will increase expontially

*50. An artillery piece of mass M_1 fires a shell of mass M_2 horizontally. Instantaneously after the firing, the ratio of kinetic energy of the artillery and that of the shell is:

(1) $M_1/(M_1+M_2)$ (2) $M_2/(M_1+M_2)$

(3) $\frac{M_1}{M_2}$ (4) $\frac{M_2}{M_1}$

Ans. (4)

Sol. $\frac{K_{M_1}}{K_{M_2}} = \frac{P^2 * 2M_2}{2M_1 * P^2} = \frac{M_2}{M_1}$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

*51. The depth below the surface of sea to which a rubber ball be taken so as to decrease its volume by 0.02% is _____m .

(Take density of sea water = 10^3 kgm^{-3}, Bulk modulus of rubber = 9×10^8 Nm^{-2}, and g = 10ms^{-2})

Ans. 18

Sol. $\Delta P = B\frac{\Delta V}{V}$

$\rho gh = B \times \frac{0.02}{100}$

$h = \frac{B}{\rho g} \times \frac{2}{10000} = \frac{9\times10^8\times2}{10^3\times10\times10^4} = 18$ m

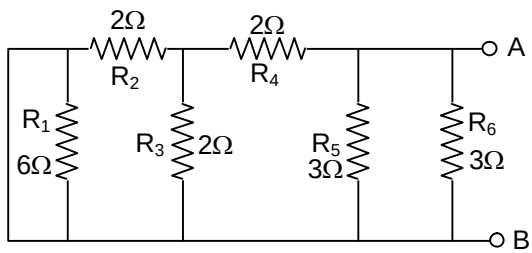
52. An electron moves through a uniform magnetic field $\vec{B} = B_0\hat{i} + 2B_0\hat{j}$ T . At a particular instant of time, the velocity of electron is $\vec{u} = 3\hat{i} + 5\hat{j}$ m/s . If the magnetic force acting on electron is $\vec{F} = 5e\hat{k}$ N, where e is the charge of electron, then the value of B_0 is _____T .

Ans. 5

Sol. $5e\hat{k} = e(3\hat{i} + 5\hat{j}) \times (B_0\hat{i} + 2B_0\hat{j})$

$5 = 6B_0 - 5B_0 = B_0$

53. Equivalent resistance of the following network is _____Ω.



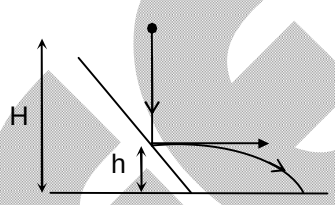
Ans. 1

Sol. $\frac{(3)\left(\frac{3}{2}\right)}{3+\left(\frac{3}{2}\right)}=\frac{9}{9}=1$

*54. A body starts falling freely from height H hits an inclined plane in its path at height h . As a result of this perfectly elastic impact, the direction of the velocity of the body becomes horizontal. The value of $\frac{H}{h}$ for which the body will take the maximum time to reach the ground is _____.

Ans. 2

Sol. $t=\sqrt{\frac{2(H-h)}{g}}+\sqrt{\frac{2h}{g}}$



$\frac{dt}{dh}=0$

$\frac{H}{h}=2$

55. A parallel plate capacitor with plate separation 5mm is charged up by a battery. It is found that on introducing a dielectric sheet of thickness 2mm , while keeping the battery connections intact, the capacitor draws 25% more charge from the battery than before. The dielectric constant of the sheet is _____.

Ans. 2

Sol. $\frac{5}{4}\frac{\varepsilon_0 A}{(5)}=\frac{\varepsilon_0 A}{5-2+\frac{2}{k}}$

$4=3+\frac{2}{k}$

$\Rightarrow k=2$

56. Two waves of intensity ratio 1 : 9 cross each other at a point. The resultant intensities at that point, when (a) Waves are incoherent is I_1 (b) Waves are coherent is I_2 and differ in phase by 60∘ . If $\frac{I_1}{I_2}=\frac{10}{x}$, then x = _____.

Ans. 13

Sol. $I_1=I_0+9I_0=10I_0$

$I_2=I_0+9I_0+2\sqrt{(I_0)(9I_0)}\,\frac{1}{2}=13I_0$

So, $\frac{I_1}{I_2}=\frac{10}{13}$

*57. The mass defect in a particular reaction is 0.4g . The amount of energy liberated is $n\times10^7$ kWh, where n = _____. (speed of light = 3×10^8m/s)

Ans. 1

Sol. $E = (0.4)\,(3 \times 10^8)^2 \times 10^{-3}$
$= 3.6 \times 10^{13}\ J = 1 \times 10^7\ kWh$

*58. A solid circular disc of mass 50kg rolls along a horizontal floor so that its center of mass has a speed of 0.4m/s . The absolute value of work done on the disc to stop it is _____ J.

Ans. 6

Sol. Total kinetic energy = $\frac{1}{2}\left(\frac{3}{2}mR^2\right)\omega^2 = \frac{3}{4}\times 50\times(0.4)^2 = 6\ J$

59. A small square loop of wire of side ℓ is placed inside a large square loop of wire of side L($L = \ell^2$). The loops are coplanar and their centers coincide. The value of the mutual inductance of the system is $\sqrt{x}\times 10^{-7}H$, where x = _____

Ans. 128

Sol. $$M = \frac{4\frac{\mu_0 I}{4\pi\frac{L}{2}}\left(2\frac{1}{\sqrt{2}}\ell^2\right)}{I}$$

$$= 4\frac{\mu_0}{\pi}\frac{\ell^2}{\sqrt{2}L} = \frac{4\times 4\pi\times 10^{-7}\ell^2}{\pi\times\sqrt{2}\times 2}$$

$$= \frac{16}{\sqrt{2}}\times 10^{-7}H = \sqrt{128}\times 10^{-7}H$$

*60. A particle performs simple harmonic motion with amplitude A . Its speed is increased to three times at an instant when its displacement is $\frac{2A}{3}$. The new amplitude of motion is $\frac{nA}{3}$. The value of n is _____.

Ans. 7

Sol. $$v = \omega\sqrt{A^2 - \frac{4A^2}{9}} = \omega\frac{\sqrt{5}A}{3}$$

So, $v_{new} = \sqrt{5}A\omega = \omega\sqrt{x^2 - \frac{4A^2}{9}}$

$$5A^2 = x^2 - \frac{4A^2}{9}$$

$$x = \frac{7A}{3}$$

PART – C (CHEMISTRY)

SECTION - A

(One Options Correct Type)

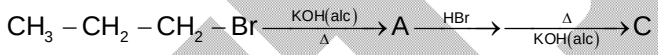
This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

61. Given below are two statments:
Statement-I: Noble gases have very high boiling points.
Statement-II: Noble gases are monoatomic gases. They are held together by strong dispersion forces. Because of this they are liquefuied at very low temperatrue. Hence, they have very high boiling points.
In the light of the above statements, choosde the ***correct answer*** from the options given below:
Option:
(1) Statement I is false but Statement II is true.
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are false

Ans. (4)

Sol. Noble gases have low boiling points because they are held together by weak dispersion forces.

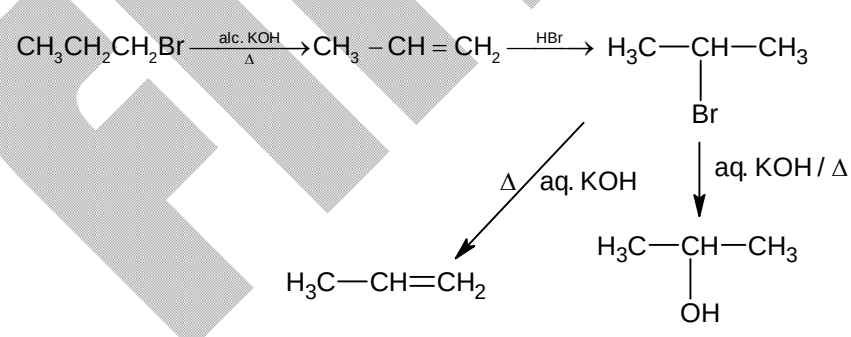
62. The prodcut (C) in the below mentioned reaction is:



Option:
(1) Propene
(2) Porpan-2-ol
(3) Propyne
(4) Propan-1-ol

Ans. (1), (2)

Sol.



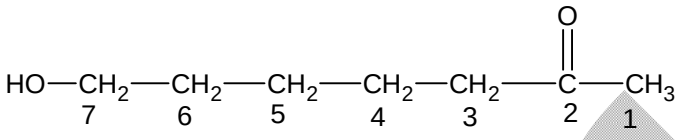
63. Identify the mixture that shows positive deviations from Raoult's Law:
Option:
(1) $CHCl_3 + (CH_3)_2CO$
(2) $CHCl_3 + C_6H_6$
(3) $(CH_3)_2CO + C_6H_5NH_2$
(4) $(CH_3)_2CO + CS_2$

Ans. (4)

Sol. Fact

*64. Given below are two statements:
Statement I: IUPAC name of HO – CH_2 – $(CH_2)_3$ – CH_2 – $COCH_3$ is 7-hydroxyheptan-2-one
Statement II: 2-oxoheptan-7-ol is the correct IUPAC name for the above compound.
In the light of the above statements, choose the ***most appropriate answer*** from the options given below:
Option:
(1) Statement I is correct but Statement II is incorrect.
(2) Statement I is incorrect but Statement II is correct
(3) Both Statement I and Statement II are correct
(4) Both Statement I and Staterment II are incorrect

Ans. (1)

Sol. 

7-hydroxylhept-2-one

*65. A species having carbon with sextet of electrons and can acts as electrophile called
(1) pentavalent carbon
(2) carbon free radical
(3) carbocation
(4) carbanion

Ans. (3)

Sol. Carbocation

66. Math List-I with List-II:

List – I		List – II	
(A)	Glucose/$NaHCO_3/\Delta$	(I)	Gluconic acid
(B)	Glucose/HNO_3	(II)	No reaction
(C)	Glucose/HI/Δ	(III)	n-hexane
(D)	Glucose/Bromine water	(IV)	Saccharic acid

Choosef the correct answer from the options given below:
Option:
(1) (A) $\rightarrow$ (IV), (B) $\rightarrow$ (I), (C) $\rightarrow$ (III), (D) $\rightarrow$ (II)
(2) (A) $\rightarrow$ (I), (B) $\rightarrow$ (IV), (C) $\rightarrow$ (III), (D) $\rightarrow$ (II)
(3) (A) $\rightarrow$ (II), (B) $\rightarrow$ (IV), (C) $\rightarrow$ (III), (D) $\rightarrow$ (I)
(4) (A) $\rightarrow$ (III), (B) $\rightarrow$ (II), (C) $\rightarrow$ (I), (D) $\rightarrow$ (IV)

Ans. (3)

Sol.

Glucose/$NaHCO_3/\Delta$	No reaction
Glucose/HNO_3	Glucaric acid
Glucose/HI/Δ	n-hexane
Glucose/Br_2 water	Gluconic acid

67. Identify the factor from the following that ***does not*** affect electrolytic conductance of a solution:
Option:
(1) The nature of solvent used
(2) Concentration of the electrolyte
(3) The nature of the electrode used
(4) The nature of the electrolyte added

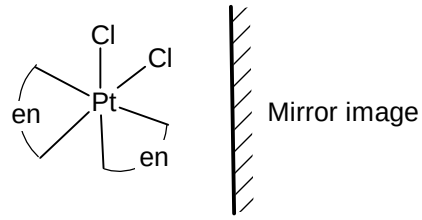
Ans. (3)

Sol. Nature of electrode used.

68. The correct statements from following are
(A) The strength of anionic ligands can be explained by crytstal field theory
(B) Valence bond theory does not give a quatitatuive interpretion of kinetic stability of coordination compounds
(C) The hybridization involved in formation of $[Ni(CN)_4]^{2-}$ complex is dsp^2
(D) The number of possible isomer(s) of cis $-[PtCl_2(en)_2]^{2-}$ is one
Choose the correct answer from the options given below:
Option:
(1) B, C only
(2) A, C only
(3) A, D only
(4) B, D only

Ans. (1)

Sol.



$cis-\left[PtCl_2(en)_2\right]^{2+}$ - two isomers

$\left[Ni(CN)_4\right]^{2-} \rightarrow dsp^2$ hybridisation

*69. The correct sequence of electron gain enthalpy of the elements listed below is
(A) Ar
(B) Br
(C) F
(D) S
Option:
(1) A > D > B > C
(2) D > C > B > A
(3) A > D > C > B
(4) C > B > D > A

Ans. (1)

Sol. C < B < D < A
F = –328
Br = –325
S = –200
Ar = +96

70. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**:
Assertion A: pK_a value of phenol is 10.0 while that of ethanol is 15.9.
Reason R: Ethanol is stronger acid than phenol.
In the light of the above statements, choose the ***correct answer*** from the options given below:
Option:
(1) A is true but R is false
(2) Both A and R are true but R is NOT the correct explanation of A
(3) Both A and R are true but R is the correct explanation of A
(4) A is false but R is true

Ans. (1)

Sol. Phenol is stronger acid than ethanol.

*71. Consider the oxides of group 14 elements
SiO_2, GeO_2, SnO_2, PbO_2, CO and GeO. The amphoteric oxides are
Option:
(1) GeO, GeO_2
(2) SiO_2, GeO_2
(3) SnO_2, CO
(4) SnO_2, PbO_2

Ans. (4)

Sol. $\underbrace{SnO_2, PbO_2}_{\text{Amphoteric}}$

72. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**:
Assertion A: Alcohols react both nucleophiles and electrophiles
Reason R: Alcohols react with active metals such as sodium, potassium and aluminium to yield corresponding alkoxyides and liberate hydrogen
In the light of the above statements, choose the ***correct answer*** from the options given below:
Option:
(1) Both A and R are true but R is NOT the correct explanation of A
(2) A is true but R is false
(3) Both A and R are true but R is the correct explanation of A
(4) A is false but R is true

Ans. (1)

Sol. Alcohols react both as nucleophile and electrophile.

73. The compound that is white in color is
Option:
(1) ammonium sulphide
(2) lead sulphate
(3) ammonium arsinomolybdate
(4) lead iodide

Ans. (2)

Sol. $PbSO_4$ white
Ammonium arsinomolybdate yellow
PbI_2 Yellow
$(NH_4)_2S$ Yellowish solid

74. Integrated rate law equation for a first order gas phase reaction is given by
(where P_i is initial pressure and P_t is total pressure at time t)
Option:
(1) $k = \frac{2.303}{t} \times \log \frac{(2P_i - P_t)}{P_i}$
(2) $k = \frac{2.303}{t} \times \log \frac{P_i}{(2P_i - P_t)}$
(3) $k = \frac{2.303}{t} \times \log \frac{2P_i}{(2P_i - P_t)}$
(4) $k = \frac{2.303}{t} \times \log \frac{P_i}{(2P_i - P_t)}$

Ans. (4)

Sol.
$$A(g) \longrightarrow 2B(g)$$
P_i 00
$P_f = P_i - p$ 2p

$$P_T = P_i - p + 2p$$
$$P_T = P_i + p$$
$$k = \frac{2.303}{t}\log\frac{P_i}{P_f}$$
$$= \frac{2.303}{t}\log\frac{P_i}{P_i - (P_T - P_i)}$$
$$k = \frac{2.303}{t}\log\frac{P_i}{2P_i - P_T}$$

*75. The linear combinaiton of aotmic orbitals to form molecular orbitals takes place only when the combining atomic orbitals
(A) have the same energy
(B) have the minimum overlap
(C) have the same symmetry about the molecular axis
(D) have different symmetry about the molecular axis
Choose the most appropriate from the option given below:
Option:
(1) A, B, C only
(2) A and C only
(3) B and D only
(4) B, C, D only

Ans. (2)

Sol. Have the same energy and have same symmetry about the molecular axis.

*76. 'Adosrption' principle is used for which of the following purification method?
Option:
(1) Chromatoraphy
(2) Sublimation
(3) Extraction
(4) Distillation

Ans. (1)

Sol. Chromatography
Adsorption principle is used in chromatography.

*77. For the given reaciton, choose the correct expression of K_C from the following:

$$Fe^{3+}_{(aq)} + SCN^-_{(aq)} \rightleftharpoons \left[FeSCN^-\right]^{2+}_{(aq)}$$

Option:
(1) $K_C = \frac{\left[FeCN^{2+}\right]^2}{\left[Fe^{3+}\right]\left[SCN^-\right]}$
(2) $K_C = \frac{\left[FeCN^{2+}\right]}{\left[Fe^{3+}\right]^2\left[SCN^-\right]^2}$
(3) $K_C = \frac{\left[Fe^{3+}\right]\left[SCN^-\right]}{\left[FeCN^{2+}\right]}$
(4) $K_C = \frac{\left[FeCN^{2+}\right]}{\left[Fe^{3+}\right]\left[SCN^-\right]}$

Ans. (4)

Sol. $K_c = \frac{\left[FeSCN^{2+}\right]}{\left[Fe^{3+}\right]\left[SCN^-\right]}$

78. Identify correct statements from below:
(A) The chromate ion is sqaure planar
(B) Dichromates are generally prepared from chroamtes
(C) The green manganate ion is diamagnetic
(D) Dark green coloured K_2MnO_4 disproportionates in a neutral or acidic medium to give permanganate
(E) With inceasing oxidation number of transition metal, ionic character of the oxides decreases.
Choose the correct answer from the options given below:
Option:
(1) B, C, D only
(2) A, D, E only
(3) A, B, C only
(4) B, D, E only

Ans. (4)

Sol. Chromate is tetrahedral in shape.
The green manganate ion is paramagnetic.

*79. Math List-I with List-II:

List – I (Technique)		List – II (Application)	
(A)	Distillation	(I)	Separation of glycerol from spent -1-lye
(B)	Fractional distillation	(II)	Aniline – water mixture
(C)	Steam distillation	(III)	Separation of crude oil fractions
(D)	Distillation under reduced pressure	(IV)	Chloroform – analine

Choose the correct answer from the options given below:
Option:
(1) (A) $\rightarrow$ (IV), (B) $\rightarrow$ (I), (C) $\rightarrow$ (II), (D) $\rightarrow$ (III)
(2) (A) $\rightarrow$ (IV), (B) $\rightarrow$ (III), (C) $\rightarrow$ (II), (D) $\rightarrow$ (I)
(3) (A) $\rightarrow$ (I), (B) $\rightarrow$ (II), (C) $\rightarrow$ (IV), (D) $\rightarrow$ (III)
(4) (A) $\rightarrow$ (II), (B) $\rightarrow$ (III), (C) $\rightarrow$ (I), (D) $\rightarrow$ (IV)

Ans. (2)

Sol. Distillation under reduced pressure – separation of glycerol from spent-lye.
Steam distillation – aniline-water mixture.

80. The metals that are employed in the battery industries are
(A) Fe
(B) Mn
(C) Ni
(D) Cr
(E) Cd
Choosef the correct answer from the options given below:
Option:
(1) A, B, C, and D only
(2) `B, C and E only
(3) B, D and E only
(4) A, B, C, D and E

Ans. (2)

Sol. Mn, Ni and Cd.

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

*81. Consider the following reaction at 298 K $\frac{3}{2}O_2(g) \rightleftharpoons O_3(g); K_p = 2.47\times10^{-29}$, $\Delta_f G^0$ for the reaction is _____ kJ. (Given R = 8.314 JK^{-1} mol^{-1})

Ans. 163

Sol. $\frac{3}{2}O_2(g) \rightleftharpoons O_3(g) \qquad K_P = 2.47\times10^{-29}$

$\Delta_r G^o = -RT\ln K_P$

$= -2.303RT\log K_P$

$= -\frac{2.303\times8.314\times298}{1000}\log\left(2.47\times10^{-29}\right)$ kJ

$= -5.705\times\left[-29+0.4\right]$

= 163.163 kJ.

82. Molar mass of the salt from NaBr, $NaNO_3$. KI and CaF_2 which does not evolve coloured vapours on heating with concentrated H_2SO_4 is _____ g mol^{-1}.
(Molar mass in g mol^{-1}: Na = 23, N = 14, K = 39, O = 16, Br = 80, I : 127, F: 19, Ca = 40)

Ans. 78

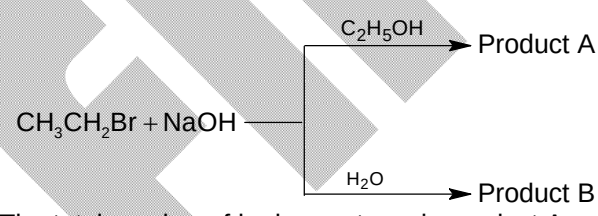
Sol. CaF_2

$= 40+2\times19$

= 40 + 38

=78

83.



The total number of hydroen atoms in product A and product B is _____.

Ans. 10

Sol. A = $CH_2 = CH_2$

B = $CH_3CH_2 - OH$

Total hydrogen in A and B = 10.

*84. Number of alkanes obtained on electrolysis of a mixture of CH_3COONa and C_2H_5COONa is _____.

Ans. 3

Sol. $CH_3COONa + CH_3CH_2COONa \xrightarrow{\text{Electrolysis}} CH_3 - CH_3 + CH_3CH_2CH_2 - CH_3 + CH_3CH_2CH_3$

*85. The number of species from the following in which the central atoms uses sp^3 hybrid orbitals in its bonding is _____.
NH_3, SO_2, SiO_2, $BeCl_2$, CO_2, H_2O, CH_4, BF_3

Ans. 4

Sol. NH_3, H_2O, CH_4, SiO_2

*86. Number of moles of methane required to produce 22 g $CO_2(g)$ after combustion is $x \times 10^{-2}$ moles. The value of x is _____.

Ans. 50

Sol. Moles of $CO_2 = \frac{22}{44} = \frac{1}{2} = 0.5$

$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

Moles of CH_4 required $= \frac{n_{CO_2}}{1} = 0.5 = 50 \times 10^{-2}$

87. The 'Spin only' Magnetic moment for $[Ni(NH_3)_6]^{2+}$ is _____ $\times 10^{-1}$ BM.
(Given = atomic number of Ni : 28)

Ans. 28

Sol. $\left[Ni(NH_3)_6\right]^{2+}$

$Ni \longrightarrow 3d^8$



$n = 2$

$\mu = \sqrt{n(n+2)}$

$= \sqrt{2(2+2)}$

$= \sqrt{8} = 2.82$

*88. The ionization energy of sodium in kJ mol^{-1}, iof electromagnetic radiation of wavelength 242 nm is just sufficient to ionize sodium atom is_____.

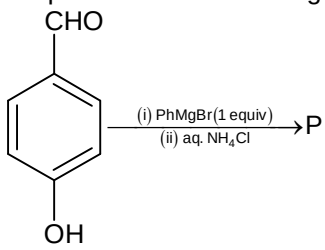
Ans. 494

Sol. $IE = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{242 \times 10^{-9}} J$

$IE = \frac{6.62 \times 10^{-34} \times 3 \times 10^8 \times 6.02 \times 10^{23}}{242 \times 10^{-9} \times 1000} kJ/mol$

$= 494 \text{ kJ mol}^{-1}$

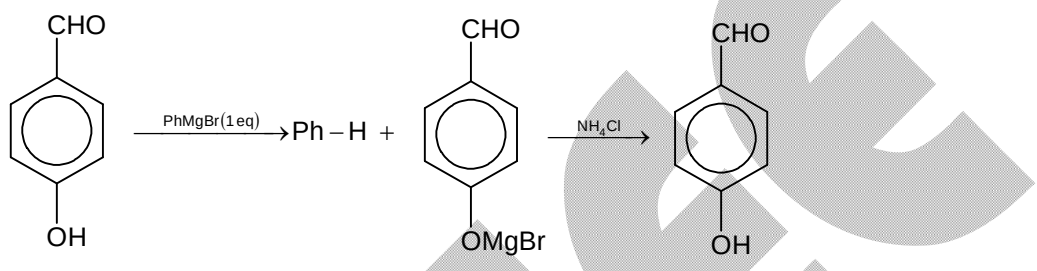
89. The product of the following reaction is P.



The number of hydroxyl groups present in the product P is _____.

Ans. 1

Sol.



90. One Faraday of electricity liberates $x \times 10^{-1}$ gram atom of copper from copper sulphate. x is _____.

Ans. 5

Sol. 2F liberate = 1 gm atom of Cu

1F liberate = $\frac{1}{2}$

= 0.5

= 5×10^{-1}