PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

1. Consider the system of linear equations $x + y + z = 5$, $x + 2y + \lambda^2 z = 9$, $x + 3y + \lambda z = \mu$, where $\lambda, \mu \in \mathbb{R}$. Then, which of the following statement is NOT correct?
   (1) System has infinite number of solutions if $\lambda = 1$ and $\mu = 13$
   (2) System is inconsistent if $\lambda = 1$ and $\mu \neq 13$
   (3) System is consistent if $\lambda \neq 1$ and $\mu = 13$
   (4) System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$

Ans. (4)

Sol. $x + y + z = 5$, $x + 2y + \lambda^2 z = 9$, $x + 3y + \lambda z = \mu$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda^2 \\ 1 & 3 & \lambda \end{vmatrix} \Rightarrow (2\lambda + 1)(1 - \lambda)$$

$$D_1 = \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & \lambda^2 \\ 4 & 3 & \lambda \end{vmatrix} = (\mu - 15)\lambda^2 + \lambda - 2\mu + 27$$

$D_2 = (5 - \mu)\lambda^2 + 4\lambda + \mu - 9$

$D_3 = \mu - 13$

For $\lambda = -\frac{1}{2}$. It doesn't give unique solution

2. For $\alpha, \beta \in (0, \pi/2)$, let $3 \sin(\alpha + \beta) = 2 \sin(\alpha - \beta)$ and a real number k be such that $\tan\alpha = k \tan\beta$. Then, the value of k is equal to
   (1) $-\frac{2}{3}$ (2) –5
   (3) $\frac{2}{3}$ (4) 5

Ans. (BONUS)

3. Let $A(\alpha, 0)$ and $B(0, \beta)$ be the points on the line $5x + 7y = 50$. Let the point P divide the line segment AB internally in the ratio 7 : 3. Let $3x - 25 = 0$ be a directrix of the ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the corresponding focus be S. If from S, the perpendicular on the x-axis passes through P, then the length of the latus rectum of E is equal to
   (1) $\frac{25}{3}$ (2) $\frac{32}{9}$
   (3) $\frac{25}{9}$ (4) $\frac{32}{5}$

Ans. (4)

Sol. Point P(3, 5)

ae = 3, $\frac{a}{e}=\frac{25}{3}$

$\Rightarrow$ a = 5, b = 4

Length of LR = $\frac{2b^2}{a}=\frac{32}{5}$

4. Let $\vec{a}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$, $\alpha, \beta \in \mathbb{R}$. Let a vector $\vec{b}$ be such that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{b}|^2=6$. If $\vec{a}\cdot\vec{b}=3\sqrt{2}$, then the value of $\left(\alpha^2+\beta^2\right)|\vec{a}\times\vec{b}|^2$ is equal to

(1) 90 (2) 75

(3) 95 (4) 85

Ans. (1)

Sol. $\vec{a}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$; $|\vec{b}|=\sqrt{6}$

$\vec{a}\cdot\vec{b}=\frac{|\vec{a}||\vec{b}|}{\sqrt{2}}=3\sqrt{2} \Rightarrow \sqrt{\alpha^2+\beta^2+1}=\sqrt{6}=|\vec{a}|$

$\alpha^2 + \beta^2 = 5$, then $\left(\alpha^2+\beta^2\right)|\vec{a}\times\vec{b}|^2=\left(\alpha^2+\beta^2\right)\frac{|\vec{a}|^2|\vec{b}|^2}{2}=90$

5. Let $f(x) = (x + 3)^2 (x – 2)^3$, $x \in [–4, 4]$. If M and m are the maximum and minimum values of f, respectively in [–4, 4], then the value of M – m is

(1) 600 (2) 392

(3) 608 (4) 108

Ans. (3)

Sol. $f(x) = (x + 3)^2(x – 2)^3$

M = f(4) = 392

m = f(–4) = $(–6)^3$ = –216

M – m = 608

6. Let a and b be be two distinct positive real numbers. Let 11th term of a GP, whose first term is a and third term is b, is equal to p^{th} term of another GP, whose first term is a and fifth term is b. Then p is equal to

(1) 20 (2) 25

(3) 21 (4) 24

Ans. (3)

Sol. Let first G.P. be a, ar_1, ar_1^2 …..

$ar_1^2=b \Rightarrow r_1^2=\frac{b}{a}$

$T_{11}=a\cdot r_1^{10}$

Let second G.P. be $a, ar_2^2, ar_2^2, \ldots .. ar_1^{10}=ar_2^{p-1}$

$ar_2^4=b \Rightarrow r_2^4=\left(\frac{b}{a}\right) \Rightarrow ar_1^{10}=ar_2^{p-1} \Rightarrow p = 21$

7. If $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$ is the locus of a point, which moves such that it is always equidistant from the lines $x + 2y + 7 = 0$ and $2x - y + 8 = 0$, then the value of $g + c + h - f$ equals

(1) 14 (2) 6
(3) 8 (4) 29

Ans. (1)

Sol. $\frac{x+2y+7}{\sqrt{5}} = \pm\frac{(2x-y+8)}{\sqrt{5}} \Rightarrow$ Locus is $(x + 2y + 7)^2 - (2x - y + 8)^2 = 0$

$\Rightarrow x^2 - y^2 - \frac{8}{3}xy + 6x - \frac{44}{3}y + 5 = 0$

$\Rightarrow g = 3, c = 5, h = -\frac{8}{6}, f = -\frac{44}{6}$

$\Rightarrow g + c + h - f = 14$

8. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{b}| = 1$ and $|\vec{b} \times \vec{a}| = 2$. Then $|(\vec{b} \times \vec{a}) - \vec{b}|^2$ is equal to

(1) 3 (2) 5
(3) 1 (4) 4

Ans. (2)

Sol. $|(\vec{b} \times \vec{a}) - \vec{b}|^2 = ((\vec{b} \times \vec{a}) - \vec{b}) \cdot ((\vec{b} \times \vec{a}) - \vec{b}) = |\vec{b} \times \vec{a}|^2 + |\vec{b}|^2 = 5$

9. Let $y = f(x)$ be a thrice differentiable function in $(-5, 5)$. Let the tangents to the curve $y = f(x)$ at $(1, f(1))$ and $(3, f(3))$ make angles $\pi/6$ and $\pi/4$, respectively with positive x-axis. If $27\int_1^3 \left((f'(t))^2 + 1\right) f''(t)\,dt = \alpha + \beta\sqrt{3}$ where α, β are integers, then the value of $\alpha + \beta$ equals

(1) –14 (2) 26
(3) –16 (4) 36

Ans. (2)

Sol. $f'(1) = \frac{1}{\sqrt{3}}, f'(3) = 1$

$27\int_1^3 \left((f'(t))^2 + 1\right) f''(t)\,dt \Rightarrow 27\left(\frac{(f'(t))^3}{3} + f'(t)\right)\Bigg|_1^3$

$\Rightarrow 27\left[\left(\frac{1}{3} + 1\right) - \left(\frac{1}{9\sqrt{3}} + \frac{1}{\sqrt{3}}\right)\right] = 36 - 10\sqrt{3}$

$\alpha = 36$; $\beta = -10 \Rightarrow \alpha + \beta = 26$

10. Let P be a point on the hyperbola $H: \frac{x^2}{9} - \frac{y^2}{4} = 1$, in the first quadrant such that the area of triangle formed by P and the two foci of H is $2\sqrt{13}$. Then, the square of the distance of P from the origin is

(1) 18 (2) 26
(3) 22 (4) 20

Ans. (3)

Sol. focii $(\pm\sqrt{13}, 0)$

Area = $\frac{1}{2}\times \text{base}\times \text{height} = \frac{1}{2}\times 2\sqrt{13}\times y = 2\sqrt{13}$

$y = 2$

$\frac{x^2}{9}-\frac{2^2}{4}=1 \Rightarrow x^2 = 18$

Square of distance = $\left(\sqrt{18}\right)^2 + 2^2 = 22$

11. Bag A contains 3 white, 7 red balls and Bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from the bag A, if the ball drawn is white, is

(1) 1/4 (2) 1/9

(3) 1/3 (4) 3/10

Ans. (3)

Sol.

A	B
3 w	3 w
7 R	2 R

Using Baye's Theorem

Probability = $\frac{\frac{1}{2}\times\frac{3}{10}}{\frac{1}{2}\times\frac{3}{10}+\frac{3}{5}\times\frac{1}{2}} = \frac{3}{3+6} = \frac{1}{3}$

12. Let $f:\mathbb{R}\to\mathbb{R}$ be defined as $f(x) = ae^{2x} + be^x + cx$. If $f(0) = -1$, $f'(\log_e 2) = 21$ and $\int_0^{\log_e 4}\left(f(x)-cx\right)dx = \frac{39}{2}$, then the value of $|a + b + c|$ equals

(1) 16 (2) 10

(3) 12 (4) 8

Ans. (4)

Sol. $f(x) = ae^{2x} + be^x + cx$

$f(0) = a + b = -1$ ..... (1)

$f'(x) = 2ae^{2x} + be^x + c$

$f'(\log_e 2) = 8a + 2b + c = 21$ ..... (2)

and $\int_0^{\log_e 4}\left(ae^{2x}+be^x\right)dx = \frac{39}{2}$

$\Rightarrow \left[\frac{a}{2}e^{2x}+be^x\right]_0^{\log_e 4} = \frac{39}{2}$

$8a + 4b - \frac{a}{2} - b = \frac{39}{2}$

$\Rightarrow 15a + 6b = 39$ ..... (3)

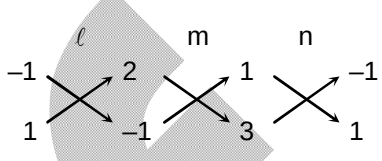
by solving these equations, we get $|a + b + c| = 8$

13. Let $L_1 : \vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + 2\hat{k}\right)$, $\lambda \in \mathbb{R}$. $L_2 : \vec{r} = \left(\hat{j} - \hat{k}\right) + \mu\left(3\hat{i} + \hat{j} + p\hat{k}\right), \mu \in \mathbb{R}$, and $L_3 : \vec{r} = \delta\left(\ell\hat{i} + m\hat{j} + n\hat{k}\right), \delta \in \mathbb{R}$ be three lines such that L_1 is perpendicular to L_2 and L_3 is perpendicular to both L_1 and L_2. Then, the point which lies on L_3 is

(1) (–1, 7, 4) (2) (–1, –7, 4)
(3) (1, 7, –4) (4) (1, –7, 4)

Ans. (1)

Sol. $L_1 \perp L_2$
$\Rightarrow 3 - 1 + 2p = 0 \Rightarrow P = -1$
$L_3 \perp L_1$ and $L_3 \perp L_2$
$\Rightarrow \ell - m + 2n = 0$; $3\ell + m - n = 0$
$\Rightarrow$ (–1, 7, 4)



14. Let a and b be real constants such that the function f defined by $f(x) = \begin{cases} x^2 + 3x + a & ; \ x \le 1 \\ bx + 2 & ; \ x > 1 \end{cases}$ be differentiable on $\mathbb{R}$. Then the value of $\int_{-2}^{2} f(x)dx$ equals

(1) 15/6 (2) 19/6
(3) 21 (4) 17

Ans. (4)

Sol. $f(x) = \begin{cases} x^2 + 3x + a & ; \ x \le 1 \\ bx + 2 & ; \ x > 1 \end{cases}$ for differentiability it should be continuous $f(1^-) = f(1^+)$
$\Rightarrow a + 2 = b$
$f'(x) = \begin{cases} 2x + 3 & x \le 1 \\ b & x > 1 \end{cases}$
$2(1) + 3 = b \Rightarrow b = 5$ then $a = 3$

$$I = \int_{-2}^{2} f(x)dx = \int_{-2}^{1} (x^2 + 3x + 3)dx + \int_{1}^{2} (5x + 2)dx = \left[\frac{x^3}{3} + \frac{3x^2}{2} + 3x\right]_{-2}^{1} + \left[\frac{5x^2}{2} + 2x\right]_{1}^{2}$$

$$= 3 + \left(-\frac{9}{2}\right) + 9 + \frac{15}{2} + 2 = 17$$

15. Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be a function satisfying $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ for all x, y, $f(y) \neq 0$. If $f'(1) = 2024$, then

(1) $xf'(x) - 2024f(x) = 0$ (2) $xf'(x) + 2024f(x) = 0$
(3) $xf'(x) + f(x) = 2024$ (4) $xf'(x) - 2023f(x) = 0$

Ans. (1)

Sol. $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$; $f(y) \neq 0$

Taking 'y' as constant, we get $\frac{1}{y}f'\left(\frac{x}{y}\right)=\frac{f'(x)}{f(y)}$ (Put x = 1)

$\Rightarrow \frac{1}{y}f'\left(\frac{1}{y}\right)=\frac{2024}{f(y)}$ (Taking $\frac{1}{y}=t$)

$tf'(t) = 2024f(t)$; $\left(f\left(\frac{1}{y}\right)=\frac{1}{f(y)}, f(1)=1\right)$

$\Rightarrow tf'(t) - 2024f(t) = 0 \Rightarrow xf'(x) - 2024f(x) = 0$

16. If z is a complex number, then the number of common roots of the equations $z^{1985}+z^{100}+1=0$ and $z^3+2z^2+2z+1=0$, is equal to
(1) 1 (2) 2
(3) 0 (4) 4

Ans. (2)

Sol. $z^3+2z^2+2z+1=0 \Rightarrow z(z^2+z+1)+1(z^2+z+1)=0$
$z = -1, \omega, \omega^2$
$z^{1985}+z^{100}+1=0$, ω and ω^2 satisfy the above equation
Number of common roots = 2

17. Suppose 2 – p, p, 2 – α, α are the coefficients of four consecutive terms in the expansion of $(1 + x)^n$. Then the value of $p^2 - \alpha^2 + 6\alpha + 2p$ equals
(1) 4 (2) 10
(3) 8 (4) 6

Ans. (BONUS)

18. If the domain of the function $f(x)=\log_e\left(\frac{2x+3}{4x^2+x-3}\right)+\cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha, \beta]$, then the value of
$5\beta - 4\alpha$ is equal to
(1) 10 (2) 12
(3) 11 (4) 9

Ans. (2)

Sol. $f(x)=\log_e\left(\frac{2x+3}{4x^2+x-3}\right)+\cos^{-1}\left(\frac{2x-1}{x+2}\right)$

$-1\le\frac{2x-1}{x+2}\le 1 \Rightarrow x\in\left[-\frac{1}{3}, 3\right]$ and $\frac{2x+3}{4x^2+x-3}\ge 0$

$x\in\left[-\frac{3}{2}, -1\right)\cup\left(\frac{3}{4}, \infty\right)$

Required solution is $\left(\frac{3}{4}, 3\right]$

$\Rightarrow 5\beta - 4\alpha \Rightarrow 15 - 3 = 12$

19. Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by $f(x)=\frac{x}{\left(1+x^4\right)^{1/4}}$, and g(x) = f(f(f(f(x)))).

Then $18\int_0^{\sqrt{2\sqrt{5}}} x^2 g(x)\,dx$ is equal to

(1) 33 (2) 36
(3) 42 (4) 39

Ans. (4)

Sol. $f(f(x))=f\left(\frac{x}{\left(1+x^4\right)^{1/4}}\right)=\frac{\frac{x}{(1+x^4)^{\frac{1}{4}}}}{\left(1+\frac{x^4}{1+x^4}\right)^{\frac{1}{4}}}=\frac{x}{(1+2x^4)^{\frac{1}{4}}}$

$f(f(f(x)))=f\left(\frac{x}{\left(1+2x^4\right)^{1/4}}\right)=\frac{\frac{x}{(1+2x^4)^{\frac{1}{4}}}}{\left(1+\frac{x^4}{1+2x^4}\right)^{\frac{1}{4}}}=\frac{x}{(1+3x^4)^{\frac{1}{4}}}$

$f(f(f(f(x))))=\frac{x}{\left(1+4x^4\right)^{1/4}}$

$I=18\int_0^{\sqrt{2\sqrt{5}}} x^2\left(\frac{x}{(1+4x^4)^{\frac{1}{4}}}\right)$ (Put $1+4x^4=t\Rightarrow 16x^3\,dx=dt$)

$I=\frac{18}{16}\int_1^{81}\frac{1}{t^{\frac{1}{4}}}dt=\frac{18}{16}\times\frac{4}{3}\left[t^{\frac{3}{4}}\right]_1^{81}=\frac{3}{2}(27-1)=39$

20. Let $R=\begin{pmatrix} x & 0 & 0\\ 0 & y & 0\\ 0 & 0 & z\end{pmatrix}$ be a non-zero 3×3 matrix, where

$x\sin\theta=y\sin\left(\theta+\frac{2\pi}{3}\right)=z\sin\left(\theta+\frac{4\pi}{3}\right)\neq 0$, $\theta\in(0, 2\pi)$. For a square matrix M, let trace (M) denote the sum of all the diagonal entries of M. Then, among the statements:

(I) Trace (R) = 0

(II) If trace (adj(adj(R)) = 0, then R has exactly one non-zero entry.

(1) Both (I) and (II) are true (2) Neither (I) nor (II) is true
(3) Only (II) is true (4) Only (I) is true

Ans. (2)

Sol. $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

$\Rightarrow xy + yz + zx = 0$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx) = x^2 + y^2 + z^2 \neq 0$

trace (adj(adj(R)) = $xyz(x + y + z) \neq 0$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

21. Let Y = Y(X) be a curve lying in the first quadrant such that the area enclosed by the line Y – y = Y′(x)(X – x) and the co-ordinate axes, where (x, y) is any point on the curve, is always $\frac{-y^2}{2Y'(x)}+1$, Y′(x) ≠ 0. If Y(1) = 1, then 12Y(2) equals _____

Ans. 20

Sol. Area enclosed by the line Y – y = Y′(x)(X – x) with coordinate axes is $1-\frac{y^2}{2Y'(x)}$

$$\Rightarrow \frac{1}{2}\left(y-x\frac{dy}{dx}\right)\left(x-\frac{y}{\frac{dy}{dx}}\right)=1-\frac{y^2}{\frac{2dy}{dx}}$$

$$\Rightarrow x^2\frac{dy}{dx}-2xy=-2 \Rightarrow \frac{y}{x^2}=\frac{2}{3x^3}+c$$

As Y(1) = 1 $\Rightarrow c=\frac{1}{3}$

$$\Rightarrow y=\frac{2}{3x}+\frac{x^2}{3} \Rightarrow Y(2)=\frac{1}{3}+\frac{4}{3}=\frac{5}{3} \Rightarrow 12Y(2)=20$$

22. Let a line passing through the point (–1, 2, 3) intersect the lines $L_1:\frac{x-1}{3}=\frac{y-2}{2}=\frac{z+1}{-2}$ at M(α, β, γ) and $L_2:\frac{x+2}{-3}=\frac{y-2}{-2}=\frac{z-1}{4}$ at N(a, b, c). Then, the value of $\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2}$ equals _____

Ans. 196

Sol. Let the line be $L:\frac{x+1}{p}=\frac{y-2}{q}=\frac{z-3}{r}$

L and L_1 are coplanar ⇒ 2p – 2q + r = 0

Similarly L and L_2 are coplanar ⇒ 2p – 5q – r = 0

L comes out to be $\frac{x+1}{7}=\frac{y-2}{4}=\frac{z-3}{-6}$

We get, (α, β, γ) = (13, 10, –9) and (a, b, c) = (–8, –2, 9)

$$\Rightarrow \left(\frac{\alpha+\beta+\gamma}{a+b+c}\right)^2=(14)^2=196$$

23. Consider two circles $C_1 : x^2 + y^2 = 25$ and $C_2 : (x-\alpha)^2 + y^2 = 16$, where $\alpha \in (5, 9)$. Let the angle between the two radii (one to each circle) drawn from one of the intersection points of C_1 and C_2 be $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$. If the length of common chord of C_1 and C_2 is β, then the value of $(\alpha\beta)^2$ equals _____

Ans. 1575

Sol. $\cos\theta = \frac{5^2 + 4^2 - \alpha^2}{2\times5\times4} = \frac{1}{8} \Rightarrow \alpha^2 = 36, \alpha = 6$

Common chord $x = \frac{15}{4}$

$\frac{\beta}{2} = \sqrt{5^2 - \left(\frac{15}{4}\right)^2} = \sqrt{\frac{175}{4^2}} = \frac{\sqrt{175}}{4} \Rightarrow \beta = \frac{\sqrt{175}}{2}$

$(\alpha\beta)^2 = \left(6\times\frac{\sqrt{175}}{2}\right)^2 = 1575$

24. Let $\alpha = \sum_{k=0}^{n}\left(\frac{\left({}^nC_k\right)^2}{k+1}\right)$ and $\beta = \sum_{k=0}^{n-1}\left(\frac{{}^nC_k\ {}^nC_{k+1}}{k+2}\right)$. If $5\alpha = 6\beta$, then n equals _____

Ans. 10

Sol. $\alpha = \sum_{k=0}^{n}\frac{{}^nC_k\ .{}^nC_k}{k+1} = \frac{1}{n+1}\sum_{k=0}^{n}{}^nC_k\ .\ {}^{n+1}C_{n-k} = \frac{{}^{2n+1}C_n}{n+1}$

$\beta = \sum_{k=0}^{n-1}\frac{{}^nC_k\ .{}^nC_{k+1}}{k+2} = \frac{1}{n+1}\sum_{k=0}^{n-1}{}^nC_k\ .\ {}^{n+1}C_{n-k-1} = \frac{{}^{2n+1}C_{n-1}}{n+1}$

$5\alpha = 6\beta \Rightarrow \frac{{}^{2n+1}C_n}{{}^{2n+1}C_{n-1}} = \frac{6}{5} \Rightarrow n = 10$

25. Let S_n be the sum to n-terms of an arithmetic progression 3, 7, 11, ... If $40 < \left(\frac{6}{n(n+1)}\sum_{k=1}^{n}S_k\right) < 42$, then n equals _____

Ans. 9

Sol. 3, 7, 11, ...

Sum = $\frac{n}{2}(6 + (n-1)(4)) = \frac{n}{2}(4n+2) = n(2n+1)$

$\sum S_k = \sum 2n^2 + \sum n = \frac{2(n)(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= n(n+1)\left(\frac{2n+1}{3} + \frac{1}{2}\right) = n(n+1)\left(\frac{4n+5}{6}\right)$

If $40 < (4n+5) < 42 \Rightarrow 35 < 4n < 37$

So, n = 9

26. In an examination of Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections : A, B and C. A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is _____

Ans. 11376

Sol.

A	B	C
1	1	1
0	1	2
1	0	2
1	2	0
2	1	0
2	0	1
0	2	1
3	0	0

$$^8C_5 \times {}^6C_5 \times {}^6C_5 + \left({}^6C_5 \times {}^8C_4 \times {}^6C_6\right)(2) + \left({}^8C_5 \times {}^6C_4 \times {}^6C_6\right)(2) + \left({}^8C_6 \times {}^6C_5 \times {}^6C_4\right)(2) + {}^8C_7 \times {}^6C_4 \times {}^6C_4$$

= 11376

27. The number of symmetric relations defined on the set {1, 2, 3, 4} which are not reflexive is _____

Ans. 960

Sol. Total required relation = $2^6(2^4 - 1) = 960$

28. The number of real solutions of the equation $x(x^2 + 3|x| + 5|x - 1| + 6|x - 2|) = 0$ is _____

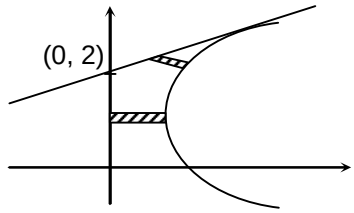
Ans. 1

Sol. $x(x^2 + 3|x| + 5|x - 1| + 6|x - 2|) = 0$
As $x^2 + 3|x| + 5|x - 1| + 6|x - 2| > 0$ (always)
So, x = 0 is the only solution

29. The area of the region enclosed by the parabola $(y - 2)^2 = x - 1$, the line $x - 2y + 4 = 0$ and the positive coordinate axes is _____

Ans. 5

Sol. $(y - 2)^2 = x - 1$, $x - 2y + 4 = 0$



$x - 2(y - 2) = 0 \Rightarrow y - 2 = \frac{x}{2}$

$\Rightarrow \frac{x^2}{4} = x - 1 \Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow (x - 2)^2 = 0 \Rightarrow x = 2$ then, $y = 3$

$$\text{Area} = \int_0^2 \left((y-2)^2 + 1\right)dy + \int_2^3 \left((y-2)^2 + 1 - (2y-4)\right)dy$$

$$= \left[\frac{(y-2)^3}{3} + y\right]_0^2 + \left[\frac{(y-2)^3}{3} + 5y - y^2\right]_2^3 = 2 + \frac{8}{3} + \frac{1}{3} + 15 - 9 - 10 + 4 = 5$$

30. The variance σ^2 of the date

x_i	0	1	5	6	10	12	17
f_i	3	2	3	2	6	3	3

is _____

Ans. 29

Sol. Shown in the table:

x_i	f_i	f_ix_i	$x_i - \bar{x}$	$f_i(x_i - \bar{x})^2$
0	3	0	–8	192
1	2	2	–7	98
5	3	15	–3	27
6	2	12	–2	8
10	6	60	2	24
12	3	36	4	48
17	3	51	9	243
		176		640

$$\bar{x} = \frac{176}{22} = 8$$

Variance = $\frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i} = \frac{640}{22} = 29$

PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

31. If 50 Vernier divisions are equal to 49 main scale divisions of a traveling microscope and one smallest reading of main scale is 0.5 mm , the Vernier constant of traveling microscope is

(1) 0.1 mm (2) 0.1 cm

(3) 0.01 cm (4) 0.01 mm

Ans. (4)

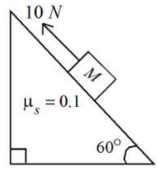
Sol. 50 VSD = 49 MSD

$$1\text{VSD} = \frac{49}{50}\text{MSD}$$

Vernier constant = 1 MSD – 1 VSD

$$= 1\,\text{MSD} - \frac{49}{50}\text{MSD} = \frac{1}{50} \times \frac{1}{2} = 0.01 \text{ mm}$$

*32. A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of 60° by a force of 10 N parallel to the inclined surface as shown in figure. When the block is pushed up by 10 m along inclined surface, the work done against frictional force is :



[g = 10 m/s^2]

(1) $5\sqrt{3}$ J (2) 5J

(3) 5×10^3 J (4) 10 J

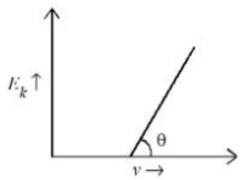
Ans. (2)

Sol. W = F.s

= (μmg cos 60°).S

$$= 0.1 \times 1 \times 10 \times \frac{1}{2} \times 10 = 5 \text{ s}$$

33. For the photoelectric effect, the maximum kinetic energy (E_k) of the photoelectrons is plotted against the frequency (ν) of the incident photons as shown in figure. The slope of the graph gives



(1) Ratio of Planck's constant to electric charge

(2) Work function of the metal

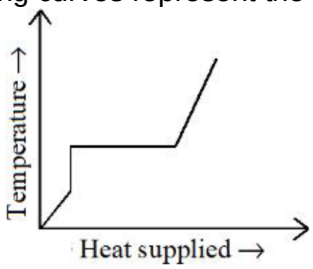
(3) Charge of electron

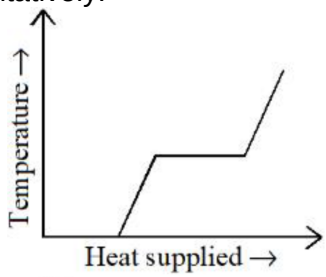
(4) Planck's constant

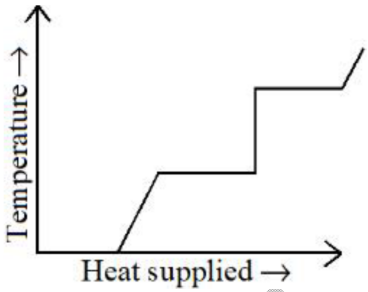
Ans. (4)

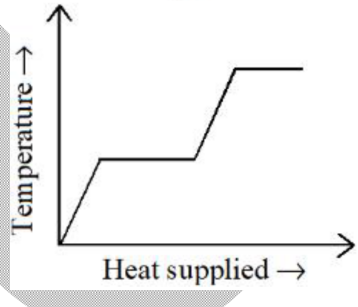
Sol. K.E. = hv – ϕ
Slope = h (plank's constant)

*34. A block of ice at −10°C is slowly heated and converted to steam at 100°C . Which of the following curves represent the phenomenon qualitatively:

(1)


(2)


(3)


(4)


Ans. (4)

Sol. Temperature first increases to zero then remains constant till whole ice melts then it is increases till 100°C then remains constant till whole water converts to steam.

35. In a nuclear fission reaction of an isotope of mass M, three similar daughter nuclei of same mass are formed. The speed of a daughter nuclei in terms of mass defect ΔM will be :

(1) $\sqrt{\frac{2c\Delta M}{M}}$

(2) $\frac{\Delta Mc^2}{3}$

(3) $c\sqrt{\frac{2\Delta M}{M}}$

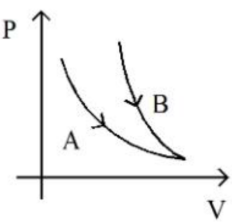
(4) $c\sqrt{\frac{3\Delta M}{M}}$

Ans. (3)

Sol. $\Delta mc^2 = 3\times\frac{1}{2}\frac{mv^2}{3}$

$v = c\sqrt{\frac{2\Delta m}{m}}$

*36. Choose the correct statement for processes A&B shown in figure.



(1) $PV^{\gamma} = k$ for process B and $PV = k$ for process A .

(2) $PV = k$ for process B and A .

(3) $\frac{P^{\gamma-1}}{T^{\gamma}} = k$ for process B and $T = k$ for process A .

(4) $\frac{T^{\gamma}}{P^{\gamma-1}} = k$ for process A and $PV = k$ for process B .

Ans. (1)

Sol. PV^{γ} = constant (adiabatic)
PV = constant (isothermal)
Stope of adiabatic is more

37. An electron revolving in n^{th} Bohr orbit has magnetic moment μ_n. If $\mu_n \alpha n^x$, the value of x is
(1) 2 (2) 1
(3) 3 (4) 0

Ans. (2)

Sol. $\mu_n = \left(\frac{qV}{2\pi r}\right)\left(\pi r^2\right)$

$H_n \propto V_r$

$H_n \propto \frac{1}{n} n^2$

$H_n \propto n$

$n = 1$

38. An alternating voltage $V(t) = 220 \sin 100\pi t$ volt is applied to a purely resistive load of 50Ω. The time taken for the current to rise from half of the peak value to the peak value is:
(1) 5 ms (2) 3.3 ms
(3) 7.2 ms (4) 2.2 ms

Ans. (2)

Sol. $t = \frac{T}{4} - \frac{T}{12} = \frac{3T - T}{12} = \frac{T}{6}$

*39. A block of mass m is placed on a surface having vertical crossection given by $y = x^2/4$. If coefficient of friction is 0.5, the maximum height above the ground at which block can be placed without slipping is:
(1) 1/4 m (2) 1/2 m
(3) 1/6 m (4) 1/3 m

Ans. (1)

Sol. for just slipping

$\tan\theta = \mu$

$\frac{dy}{dx} = \mu$

$\frac{x}{2} = \frac{1}{2}$

$x = 1$

$y = \frac{1}{4} m$

*40. If the total energy transferred to a surface in time t is 6.48×10^5J , then the magnitude of the total momentum delivered to this surface for complete absorption will be:

(1) 2.46×10^{-3} kgm/s (2) 2.16×10^{-3} kgm/s

(3) 1.58×10^{-3} kgm/s (4) 4.32×10^{-3} kgm/s

Ans. (2)

Sol. $\frac{hc}{\lambda} = 6.648 \times 10^5$

$\frac{h}{c} = \frac{6.48 \times 10^5}{3 \times 10^8}$ kg-m/s

$= 2.16 \times 10^{-3}$ kg-m/s

41. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A . The intensity of emergent light is:

(1) $I_0/4$ (2) I_0

(3) $I_0/2$ (4) $I_0/8$

Ans. (1)

Sol. $I = \left(\frac{I_0}{2}\right)\cos^2 45° = \frac{I_0}{4}$

*42. Escape velocity of a body from earth is 11.2 km/s . If the radius of a planet be one-third the radius of earth and mass be one-sixth that of earth, the escape velocity from the planet is :

(1) 11.2 km/s (2) 8.4 km/s

(3) 4.2 km/s (4) 7.9 km/s

Ans. (4)

Sol. $(V_e)_E = \sqrt{\frac{2GM}{R}}$

$(V_e) = \sqrt{\frac{2g\left(\frac{M}{6}\right)}{\left(\frac{R}{3}\right)}} = \sqrt{\frac{GM}{R}}$

$= \frac{(V_e)_E}{\sqrt{2}} = 7.9$ km/s

43. A particle of charge '−q' and mass 'm' moves in a circle of radius 'r' around an infinitely long line charge of linear charge density '+λ '. Then time period will be given as : (Consider k as Coulomb's constant)

(1) $T^2 = \frac{4\pi^2 m}{2k\lambda q} r^3$ (2) $T = 2\pi r\sqrt{\frac{m}{2k\lambda q}}$

(3) $T = \frac{1}{2\pi r}\sqrt{\frac{m}{2k\lambda q}}$ (4) $T = \frac{1}{2\pi}\sqrt{\frac{2k\lambda q}{m}}$

Ans. (2)

Sol. $\frac{mv^2}{r} = qE$ …(i)

$E = \left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)$ …(ii)

$v = \sqrt{\frac{9\lambda}{22\varepsilon_0 m}} = \sqrt{\frac{2kq\lambda}{m}}$

$T = \frac{2\pi r}{V} = 2\pi r\sqrt{\frac{m}{2kq\lambda}}$

*44. If mass is written as $m = kc^P G^{-1/2} h^{1/2}$ then the value of P will be : (Constants have their usual meaning with ka dimensionless constant)

(1) 1/2 (2) 1/3

(3) 2 (4) −1/3

Ans. (1)

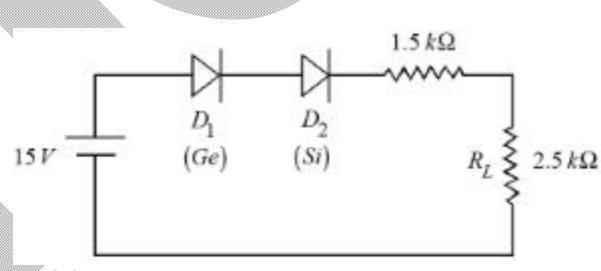
Sol. $M = \left(LT^{-1}\right)^P \left(M^{-1}L^3T^{-2}\right)^{-1/2} \left(ML^2T^{-1}\right)^{1/2}$

$M^1L^0T^0 = L^{P-3/2+1}T^{-P+1-1/2}M^{1/2+1/2}$

$0 = P - 3/2 + 1$

$P = 1/2$

45. In the given circuit, the voltage across load resistance (RL) is:



(1) 8.75 V (2) 9.00 V

(3) 8.50 V (4) 14.00 V

Ans. (1)

Sol. Potential drop of 0.3 in Ge

Potential drop of 0.7 in Si

Potential across 1.5 kΩ and 2.5 kΩ is 15 –1 = 14 V

Voltage across load resistance = $\frac{14}{4} \times 2.5 = 8.75\,V$

*46. If three moles of monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with two moles of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$, the value of adiabatic exponent γ for the mixture is

(1) 1.75 (2) 1.40

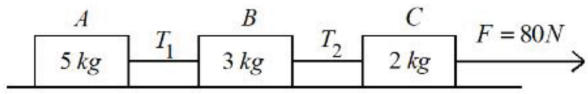
(3) 1.52 (4) 1.35

Ans. (3)

Sol. $\frac{n_1+n_2}{\gamma_{mix}-1}=\frac{n_1}{\gamma_1-1}+\frac{n_2}{\gamma_2-1}$

γ_{mix} = 1.52

*47. Three blocks A, B and C are pulled on a horizontal smooth surface by a force of 80N as shown in figure



The tensions T_1 and T_2 in the string are respectively:

(1) 40N, 64N (2) 60N, 80N

(3) 88N, 96N (4) 80N, 100N

Ans. (1)

Sol. $\frac{80}{10}=a_{net}$

8 m/s = a_{net}

$80 - T_2 = 2 \times a_{net}$

$80 - 2 \times 8 = T_2$

$T_1 = 5 \times a_{net}$

$T_1 = 5 \times 8 = 40$ N

48. When a potential difference V is applied across a wire of resistance R, it dissipates energy at a rate W. If the wire is cut into two halves and these halves are connected mutually parallel across the same supply, the energy dissipation rate will become:

(1) 1/4 W (2) 1/2 W

(3) 2 W (4) 4 W

Ans. (4)

Sol. $W=\left(\frac{V^2}{R}\right)=1$ W

$R=\frac{\rho\ell}{A}$, when cut into half

$R'=\frac{\rho\frac{\ell}{2}}{A}=\frac{R}{2}$

$W'=\frac{V^2}{R/4}=\frac{4V^2}{R}=4$ W

49. Match the List I with List II.

List I		List II	
A.	Gauss's law of magnetostatics	I.	$\oint\vec{E}\cdot\vec{da}=\frac{1}{\varepsilon_0}\int\rho dV$
B.	Faraday's law of electro magnetic induction	II.	$\oint\vec{B}\cdot\vec{da}=0$
C.	Ampere's law	III.	$\oint\vec{E}\cdot\vec{d\ell}=-\frac{d}{dt}\int\vec{B}\cdot\vec{da}$
D.	Gauss's law of electrostatics	IV.	$\oint\vec{B}\cdot\vec{d\ell}=\mu_0 I$

Choose the correct answer from the options given below:

(1) A-I, B-III, C-IV, D-II (2) A-III, B-IV, C-I, D-II

(3) A-IV, B-II, C-III, D-I (4) A-II, B-III, C-IV, D-I

Ans. (4)

Sol. Conceptual

*50. Projectiles A and B are thrown at angles of 45° and 60° with vertical respectively from top of a 400m high tower. If their ranges and times of flight are same, the ratio of their speeds of projection $v_A : v_B$ is :

[Take g = 10 ms^{-2}]

(1) 1 : $\sqrt{3}$ (2) $\sqrt{2}$: 1

(3) 1 : 2 (4) 1 : $\sqrt{2}$

Ans. Bonus (wong question)

Sol. $R_1 = v_A \sin 45° \times T$

$R_2 = v_B \sin 60° \times T$

$R_1 = R_2$ and $T_1 = T_2$

Using $R_1 = R_2$

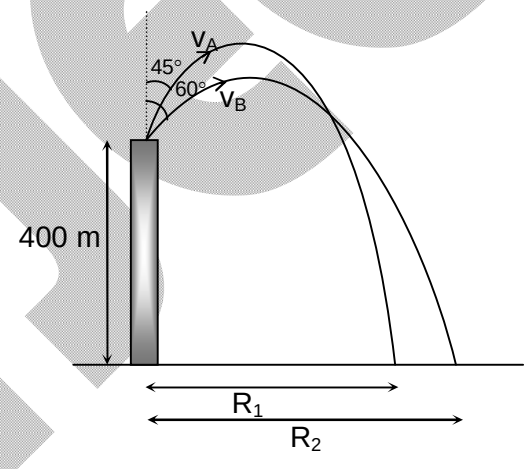
$$\frac{v_A}{\sqrt{2}} \times T = \frac{v_B \sqrt{2}}{2} \times T$$

$$\frac{v_A}{v_B} = \sqrt{\frac{3}{2}}$$

Using $T_1 = T_2$

$v_A \cos 45° = v_B \cos 60°$

$$\frac{v_A}{v_B} = \frac{1}{\sqrt{2}}$$



SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

51. A power transmission line feeds input power at 2.3 kV to a step down transformer with its primary winding having 3000 turns. The output power is delivered at 230 V by the transformer. The current in the primary of the transformer is 5A and its efficiency is 90% . The winding of transformer is made of copper. The output current of transformer is _____A.

Ans. 40

Sol. $$\eta = \frac{O.P}{I.P} \times 100$$

$P = I \times V$

Output power V_S = 230

Input current $I_P = 5A$
Input power $V_P = 2300$
Output current I_S

$$\eta = \left(\frac{I_S \times V_s}{I_P \times V_P}\right) \times 100$$

$$0.9 = \left(\frac{I_S \times 230}{5 \times 2300}\right) \times 100$$

$I_S = 45$ A

*52. A big drop is formed by coalescing 1000 small identical drops of water. If E_1 be the total surface energy of 1000 small drops of water and E_2 be the surface energy of single big drop of water, then $E_1 : E_2$ is $x : 1$ where x = _____.

Ans. 10

Sol. Surface energy = surface tension $\times$ surface area

$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

$$r = \left(\frac{R}{10}\right)$$

$$\frac{E_1}{E_2} = \frac{(S.A)_1}{(S.A)_2}$$

$$\frac{x}{1} = \frac{1000 \times 4\pi\left(\frac{R}{10}\right)^2}{4\pi \times R^2}$$

$x = 10$

*53. Two discs of moment of inertia $I_1 = 4$ kgm^2 and $I_2 = 2$ kgm^2, about their central axes & normal to their planes, rotating with angular speeds 10 rad/s & 4 rad/s respectively are brought into contact face to face with their axes of rotation coincident. The loss in kinetic energy of the system in the process is _____J.

Ans. 24

Sol. $I_1 = 4$ kg m^2, $I_2 = 2$ kgm^2, $\omega_1 = 10$rad/s and $\omega_2 = 4$ rad/s

$$\omega' = \left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right) = \frac{4 \times 10 + 2 \times 4}{6} = 8 \text{ rad/s}$$

$$K.E_I = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 = 216$$

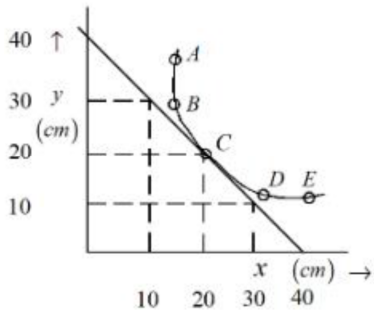
$$K.E_f = \frac{1}{2}(I_1 + I_2)\omega'^2 = \frac{1}{2} \times 6 \times (8)^2 = 192$$

$$\left|K.E_f - K.E_I\right| = \text{loss}$$

$$\left|192 - 216\right| = \text{loss}$$

$\Rightarrow$ loss = 24

54. In an experiment to measure the focal length (f) of a convex lens, the magnitude of object distance (x) and the image distance (y) are measured with reference to the focal point of the lens. The y-x plot is shown in figure. The focal length of the lens is _____ cm .



Ans. 20

Sol. Newton's formula
$xy = -f^2$
x = object distance from 1st four
y = object distance from 2nd four
$(-20) \times (20) = -f^2$
$f = 20$

*55. A vector has magnitude same as that of $\vec{A} = 3\hat{i} + 4\hat{j}$ and is parallel to $\vec{B} = 4\hat{i} + 3\hat{j}$. The x and y components of this vector in first quadrant are x and 3 respectively where x = _____

Ans. 4

Sol. The vector = $|\vec{A}| \times \hat{B}$

$$= 5 \times \left(\frac{4\hat{i} + 3\hat{j}}{5} \right) = 4\hat{i} + 3\hat{j}$$

56. The current of 5 A flows in a square loop of sides 1 m is placed in air. The magnetic field at the centre of the loop is $X\sqrt{2} \times 10^{-7}$ T. The value of X is_____

Ans. 40

Sol. $B_0 = \frac{\mu_0 i}{4\pi R}(\sin\theta_1 + \sin\theta_2) \times 4$

$$= \frac{\mu_0 i}{4\pi \times \left(\frac{a}{2}\right)} 2\sin 45° \times 4 = 40\sqrt{2} \times 10^{-7} \text{ T}$$

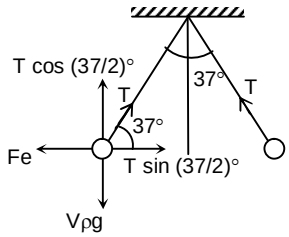
So, x = 40

57. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 37° with each other. When suspended in a liquid of density 0.7 g/cm^3, the angle remains same. If density of material of the sphere is 1.4 g/cm^3, the dielectric constant of the liquid is _____ $\left(\tan 37° = \frac{3}{4} \right)$

Ans. 2

Sol. $T\sin\left(\frac{37°}{2}\right) = Fe$ …(i)

$T\cos\left(\frac{37°}{2}\right) = v\rho g$ …(ii)



From (i) and (ii)

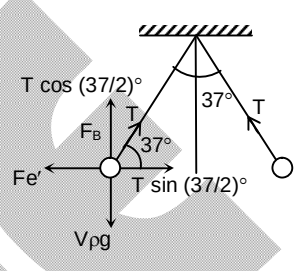
$\tan\left(\frac{37°}{2}\right) = \frac{Fe}{v\rho g}$

When imersed in liquid of density 1.4 g/cm^2

$T\sin\left(\frac{37°}{2}\right) = Fe'$ …(i)

$T\cos\left(\frac{37°}{2}\right) = v\rho g - F_B$

$T\cos\left(\frac{37°}{2}\right) = v\rho g - v\sigma g$ …(ii)



From (i) and (ii)

$\tan\left(\frac{37°}{2}\right) = \frac{Fe}{v\rho g - v\sigma g}$

From (a) and (b)

$\frac{Fe}{v\rho g} = \frac{Fe'}{v\rho g - v\sigma g}$

$\frac{Fe}{Fe'}$ = density of constant of liquid = $\frac{\rho}{\rho - \sigma} = \frac{1.4}{0.7} = 2$

*58. A simple pendulum is placed at a place where its distance from the earth's surface is equal to the radius of the earth. If the length of the string is 4m , then the time period of small oscillations will be_____ s. [take g = $\pi^2 ms^{-2}$]

Ans. 8

Sol. $g_{eff} = \frac{g}{\left(1+\frac{h}{R_e}\right)^2} = \frac{g}{(1+1)^2} = \frac{6}{4}$; $T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} = 2\pi\sqrt{\left(\frac{\ell}{g/4}\right)} = 2\pi\sqrt{\frac{4\times4}{(\pi)^2}} = 2 \times 4 = 8$

*59. A point source is emitting sound waves of intensity 16×10^{-8} Wm^{-2} at the origin. The difference in intensity (magnitude only) at two points located at a distances of 2 m and 4 m from the origin respectively will be _____$\times 10^{-8}$ Wm^{-2}.

Ans. 3

Sol. $I \propto \frac{1}{r^2}$

$\Delta I = 3\times\frac{I}{16} = 3\times\frac{16\times10^{-8}}{16-8} = 3\times10^{-8}$ N/m^2

60. Two resistance of 100 Ω and 200 Ω are connected in series with a battery of 4V and negligible internal resistance. A voltmeter is used to measure voltage across 100Ω resistance, which gives reading as 1 V . The resistance of voltmeter must be _____Ω .

Ans. 200

Sol. $$i = \frac{4}{200 + \left(\frac{100r}{100 + r}\right)}$$

$$V_{100\Omega} = i \times \left(\frac{100r}{100 + r}\right) = 1$$

$$\frac{4}{200 + \left(\frac{100r}{100 + r}\right)} \times \frac{100r}{100 + r} = 1$$

$$\Rightarrow r = 200\ \Omega$$

FIITJEE

PART – C (CHEMISTRY)

SECTION - A

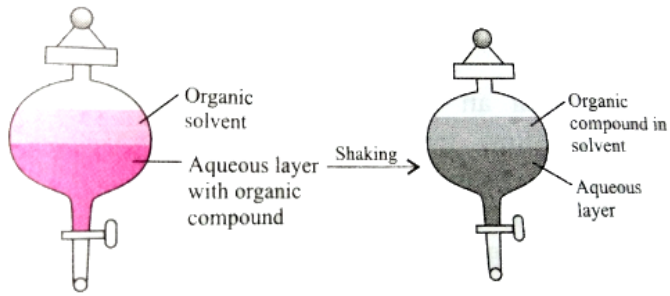
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

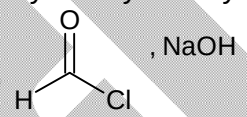
61. Which among the following purification methods is based on the principle of "Solubility" in two different solvents?
(1) Column Chromatography
(2) Sublimation
(3) Distillation
(4) Differential Extraction

Ans. (**4**)

Sol. Column chromatography is the method of separation based on solubility of different components of mixture in a given solvent
Separating funnel is used for separation by use of solubility in two different solvents

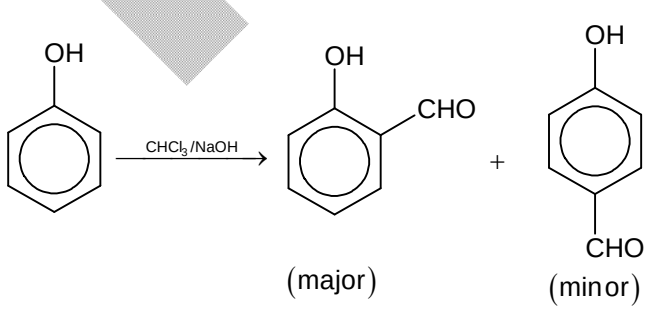


After some time organic compound reach to solvent layer (ether) and gets separated

62. Salicylaldehyde is synthesized from phenol, when reacted with
(1)  , NaOH
(2) CO_2, NaOH
(3) CCl_4, NaOH
(4) $CHCl_3$, NaOH

Ans. (**4**)

Sol.



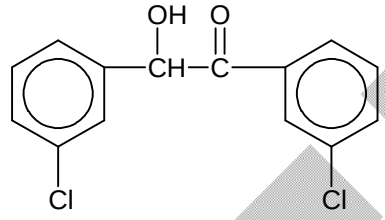
(major) (minor)

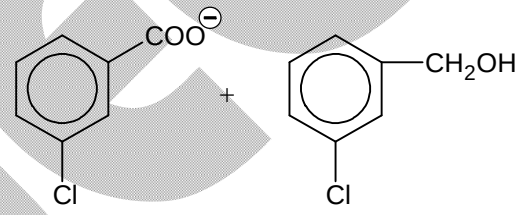
63. Given below are two statements:
Statement I: High concentration of strong nucleophilic reagent with secondary alkyl halides which do not have bulky substituents will follow S_N2 mechanism.
Statement II: A secondary alkyl halide when treated with a large excess of ethanol follows S_N1 mechanism.
In the light of the above statements, choose the most appropriate from the options given below:
(1) Statement I is true but Statement II is false
(2) Statement I is false but Statement II is true
(3) Both Statement I and Statement II are false
(4) Both Statement I and Statement II are true

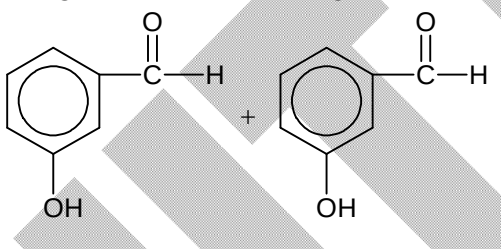
Ans. (**4)**

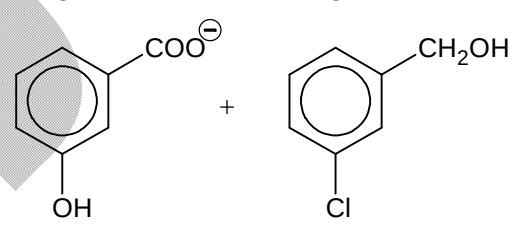
Sol. More substituted alkyl halides with bulky substituents. Undergo elimination reactions, heat is the deriving force of reaction.

64. m-chlorobenzaldehyde on treatment with 50% KOH solution yields

(1) 

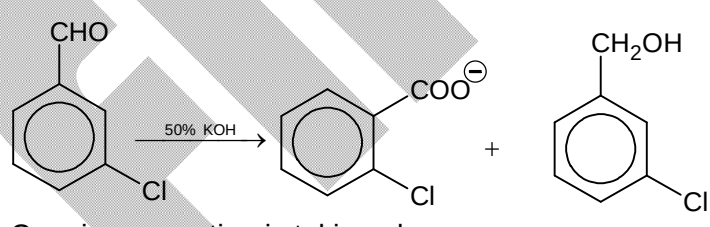
(2) 

(3) 

(4) 

Ans. (**2)**

Sol.



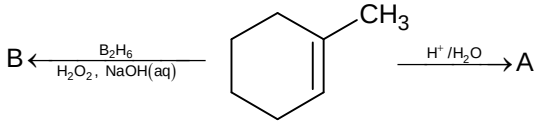
Cannizaro reaction is taking place.

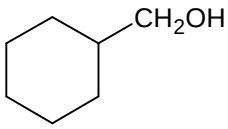
65. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.
Assertion A: H_2Te is more acidic than H_2S.
Reason R : Bond dissociation enthalpy of H_2Te is lower than H_2S.
In the light of the above statements, choose the most appropriate from the options given below:
(1) Both A and R are true but R is NOT the correct explanation of A.
(2) Both A and R are true and R is the correct explanation of A.
(3) A is false but R is true
(4) A is true but R is false

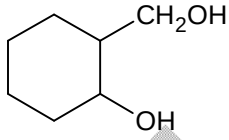
Ans. (**2**)

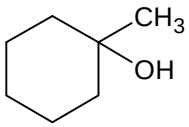
Sol. Acidic strength order $H_2Te > H_2S$

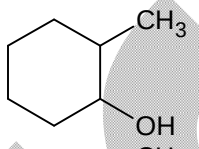
*66. Products A and B formed in the following set of reactions are

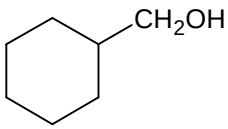


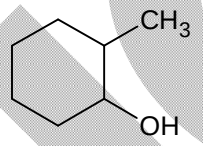
(1) A = 

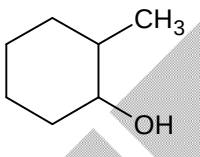
B = 

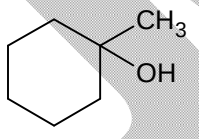
(2) A = 

B = 

(3) A = 

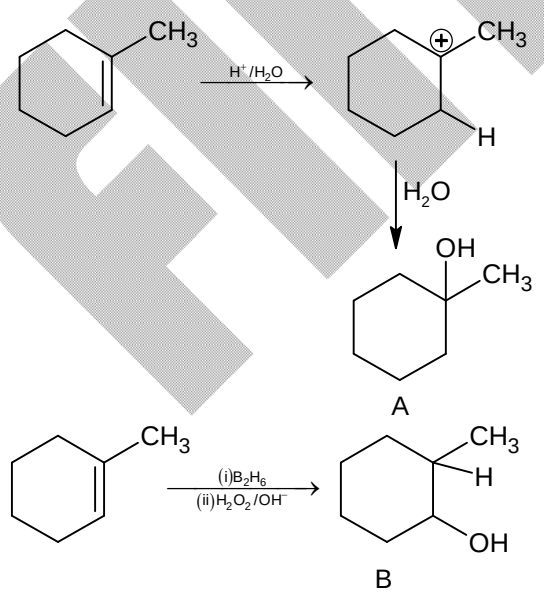
B = 

(4) A = 

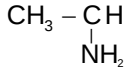
B = 

Ans. (**2**)

Sol.



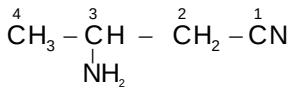
*67. IUPAC name of following compound is :



(1) 2-Aminopentanenitrile
(2) 2-Aminobutanenitrile
(3) 3-Aminobutanenitrile
(4) 3-Aminopropanenitrile

Ans. (**3)**

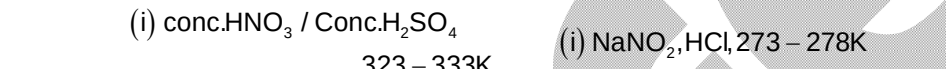
Sol.

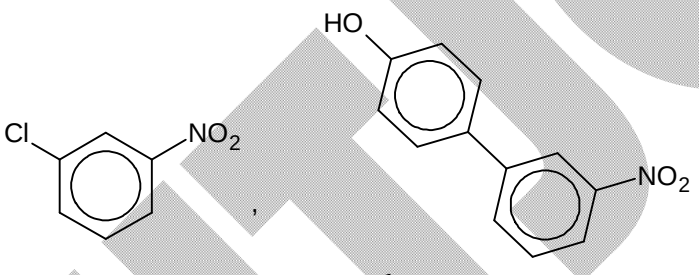


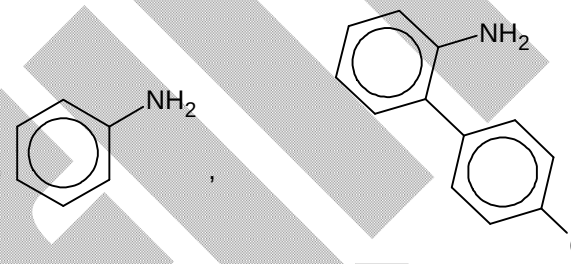
3-aminobutanenitrile

$\rightarrow$ Nitrile functional group has higher priority over amines.

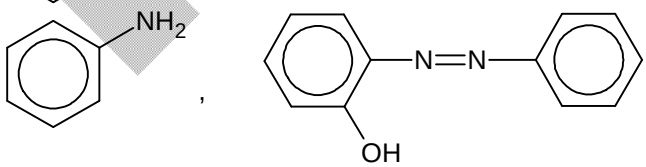
68. The product A and B formed in the following reaction scheme are respectively



(1) 

(2) 

(3) 

(4) 

Ans. (3)

*69. The molecule/ion with square pyramidal shape is

(1) $\left[NI(CN)_4\right]^{2-}$
(2) PCl_5
(3) BrF_5
(4) PF_5

Ans. (3)

Sol.

$\left[Ni(CN)_4\right]^{2-}$

$\rightarrow Ni^{2+} \rightarrow 3d^8 4s^0$

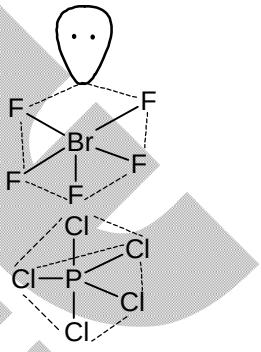
$\rightarrow$ Hyberidisation $\rightarrow dsp^2$

$\rightarrow$ square planar shape

BrF_5

$\rightarrow sp^3d^2$

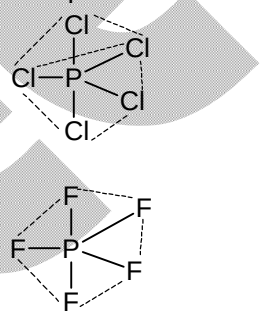
$\rightarrow$ square pyramidal shape



PCl_5

$\rightarrow sp^3d$

$\rightarrow$ Triangular bipyramidal



PF_5

$\rightarrow sp^3d$

$\rightarrow$ Triangular bipyramidal

70. The orange colour of $K_2Cr_2O_7$ and purple colour of $KMnO_4$ is due to
(1) Charge transfer transition in both
(2) $d \rightarrow d$ transitions in $KMnO_4$ and charge transfer transitions in $K_2Cr_2O_7$
(3) $d \rightarrow d$ transitions in $K_2Cr_2O_7$ and charge transfer transitions in $KMnO_4$
(4) $d \rightarrow d$ transition in both

Ans. (1)

Sol. Orange colour of $K_2Cr_2O_7$ and purple colour of $KMnO_4$ is due to charge transfer.

71. Alkaline oxidative fusion of MnO_2 gives 'A' which on electrolytic oxidation in alkaline solution produces B. A and B respectively are
(1) Mn_2O_7 and MnO_4^-
(2) MnO_4^{2-} and MnO_4^-
(3) Mn_2O_3 and MnO_4^{2-}
(4) MnO_4^{2-} and Mn_2O_7

Ans. (2)

Sol.

$$2MnO_2 + 4KOH + O_2 \longrightarrow \overset{(A)}{K_2MnO_4} + 2H_2O$$

(alkaline oxidation fusion)

$$MnO_4^{2-} \longrightarrow MnO_4^-$$

*72. If a substance 'A' dissolves in solution of a mixture of 'B' and 'C' with their respective number of moles as n_A, n_B and n_C. Mole fraction of C in the solution is

(1) $\frac{n_C}{n_A \times n_B \times n_C}$ (2) $\frac{n_C}{n_A + n_B + n_C}$

(3) $\frac{n_C}{n_A - n_B - n_C}$ (4) $\frac{n_B}{n_A + n_B}$

Ans. (**3)**

Sol. $x_c = \frac{n_C}{n_A + n_B + n_C}$

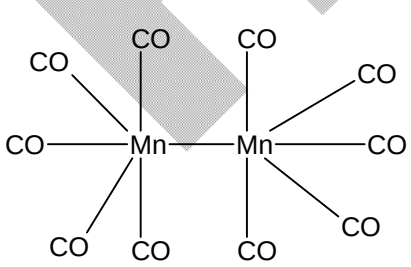
*73. Given below are two statements:
Statement I: Along the period, the chemical reactivity of the elements gradually increases from group I to group 18.
Statement II: The nature of oxides formed by group 1 elements is basic while that of group 17 elements is acidic.
In the light of the above statements, choose the most appropriate from the options given below:
(1) Both Statement I and Statement II are true
(2) Statement I is true but Statement II is False
(3) Statement I is false but Statement II is true
(4) Both Statement I and Statement II are False

Ans. (**3)**

Sol. Metals are very reactive elements. From left to right in period table non metallic character increases reaction decreases.
$\rightarrow$ oxides of s-block are ionic $\rightarrow$ basic
$\rightarrow$ oxides of p-block are covalent $\rightarrow$ acidic

74. The coordination geometry around the manganese in decarbonyldimanganese (0) is
(1) Octahedral (2) Trigonal bipyramidal
(3) Square pyramidal (4) Square planar

Ans. (**1)**
Sol.



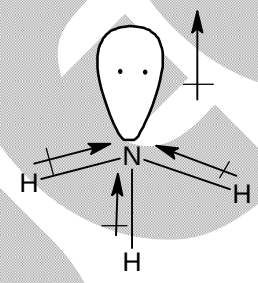
Hybridization = d^2sp^3 , Geometry = Octahedral

*75. Given below are two statements:
Statement I: Since Fluorine is more electronegative than nitrogen, the net dipole moment of NF_3 is greater than NH_3.
Statement II: In NH_3, the orbital dipole due to lone pair and the dipole moment of NH bonds are in opposite direction, but in NF_3 the orbital dipole due to lone pair and dipole moments of N-F bonds are in the same direction.
In the light of the above statements, choose the most appropriate from the options given below:
(1) Statement I is true but Statement II is False
(2) Both Statement I and Statement II are False
(3) Both Statements I and Statement II are true
(4) Statement I is false but Statement II is true

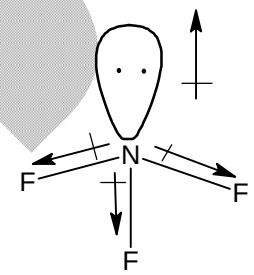
Ans. (**2)**

Sol.

Dipole moment due to lone pair and N-H bonds in same direction
$\mu NH_3 > \mu NF_3$



Dipole moment due to lone pair and N-F bonds are in opposite direction



*76. The correct stability order of carbocations is
(1) $(CH_3)_3C^+ > CH_3-\overset{+}{C}H_2 > (CH_3)_2\overset{+}{C}H > \overset{+}{C}H_3$
(2) $\overset{+}{C}H_3 > (CH_3)_2\overset{+}{C}H > CH_3-\overset{+}{C}H_2 > (CH_3)_3\overset{+}{C}$
(3) $(CH_3)_3\overset{+}{C} > (CH_3)_2\overset{+}{C}H > CH_3-\overset{+}{C}H_2 > \overset{+}{C}H_3$
(4) $\overset{+}{C}H_3 > CH_3-\overset{+}{C}H_2 > CH_3-\underset{CH_3}{\overset{+}{C}H} > (CH_3)_3\overset{+}{C}$

Ans. (**3)**

Sol. $CH_3-\underset{CH_3}{\overset{\oplus}{C}}-CH_3 > CH_3-\underset{H}{\overset{\oplus}{C}}-CH_3 > CH_2-\underset{H}{\overset{\oplus}{C}}-H > \overset{\oplus}{C}H_3$

Stability α number of α-Hydrogen α Hyper conjugating structures

77. The solution from the following with highest depression in freezing point/lowest freezing point is
(1) 180 g of acetic acid dissolved in water
(2) 180 g of acetic acid dissolved in benzene
(3) 180 g of benzoic acid dissolved in benzene
(4) 180 g of glucose dissolved in water

Ans. (**1**)
Sol.

$\Rightarrow$ $CH_3-\overset{\overset{O}{\|}}{C}-OH \rightleftharpoons CH_3-\overset{\overset{O}{\|}}{C}-O^- + H_3O^+$ so, i = 2
$\Rightarrow$ glucose contain alcoholic groups which do not ionizes, so i = 0
$\Rightarrow$ High the i $\rightarrow$ Higher will be depression in Freezing point

78. A and B formed in the following reactions are
$CrO_2Cl_2 + 4NaOH \rightarrow A + 2NaCl + 2H_2O$
$A + 2HCl + 2H_2O_2 \rightarrow B + 3H_2O$
(1) $A = Na_2CrO_4, B = CrO_5$
(2) $A = Na_2Cr_2O_4, B = CrO_4$
(3) $A = Na_2Cr_2O_7, B = CrO_3$
(4) $A = Na_2Cr_2O_7, B = CrO_5$

Ans. (**1**)

Sol. $CrO_2Cl_2 + 4NaOH \rightarrow Na_2CrO_4 + 2NaCl + 2H_2O$
$Na_2CrO_4 + 2H_2O_2 \longrightarrow CrO_5 + 3H_2O + 2HCl$

79. Choose the correct statements about the hydrides of group 15 elements.
(A) The stability of the hydrides decreases in the order $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$
(2) The reducing ability of the hydride increases in the order $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$
(3) Among the hydrides, NH_3 is strong reducing agent while BiH_3 is mild reducing agent
(4) The basicity of the hydrides increases in the order $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$
Choose the most appropriate from the options given below:
(1) B and C only
(2) C and D only
(3) A and B only
(4) A and D only

Ans. (**3**)

Sol. $\text{Reducing power} \propto \frac{1}{\text{stability}}$

80. Reduction potential of ions are given below:

ClO_4^-	IO_4^-	BrO_4^-
$E^0 = 1.19\,V$	$E^0 = 1.65\,V$	$E^0 = 1.74\ V$

The correct order of their oxidizing power is:
(1) $ClO_4^- > IO_4^- > BrO_4^-$
(2) $BrO_4^- > IO_4^- > ClO_4^-$
(3) $BrO_4^- > ClO_4^- > IO_4^-$
(4) $IO_4^- > BrO_4^- > ClO_4^-$

Ans. (**2)**

Sol.

ClO_4^-	IO_4^-	BrO_4^-
$E^0 = 1.19\,V$	$E^0 = 1.65\,V$	$E^0 = 1.74\ V$

Higher the value of $E^0_{R.P}$. Higher the tendency to get reduced. Higher the oxidizing power.

$BrO_4^- > IO_4^- > ClO_4^-$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

81. Number of complexes which show optical isomerism among the following is _____ .

$cis-\left[Cr(ox)_2Cl_2\right]^{3-}$, $\left[Co(en)_3\right]^{3+}$, $cis-\left[Pt(en)_2Cl_2\right]^{2+}$, $cis-\left[Co(en)_2Cl_2\right]^{+}$

$trans-\left[Pt(en)_2Cl_2\right]^{2+}$, $trans-\left[Cr(ox)_2Cl_2\right]^{3-}$

Ans. 4

Sol. The following compound will show optical isomerism.

$cis\left[Co(ox)_2Cl_2\right]^{3-}$, $\left[Co(en)_3\right]^{3-}$, $cis\left[Pt(en)_2Cl_2\right]^{2-}$, $cis\left[Co(en)_2Cl_2\right]^{-}$

82. NO_2 required for a reaction is produced by decomposition of N_2O_5 in CCl_4 as by equation

$2N_2O_{5(g)} \rightarrow 4NO_{2(g)} + O_{2(g)}$

The initial concentration of N_2O_5 is 3 mol L^{-1} and it is 2.75 mol L^{-1} after 30 minutes.

The rate of formation of NO_2 is $x \times 10^{-3}$ mol L^{-1} min^{-1}, value of x is _____. (nearest integer)

Ans. 17

Sol. $-\frac{d[N_2O_5]}{dt} = -\frac{(2.75-3)}{30-0} = \frac{0.25}{30} M/min$

$\frac{1}{2}\frac{-d[N_2O_5]}{dt} = \frac{1}{4} \times \frac{d[NO_2]}{dt}$

$\frac{d[NO_2]}{dt} = \frac{0.25}{30} \times 2 = 1.67 \times 10^{-2} M/min = 16.7 \times 10^{-3}$

*83. Two reaction are given below

$2Fe_{(s)} + \frac{3}{2}O_{2(g)} \rightarrow Fe_2O_{3(s)}$. $\Delta H^0 = -822 kJ/mol$

$C_{(s)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{(g)}$, $\Delta H^0 = -110\ kJ/mol$

The enthalpy change for following reaction $3C_{(s)} + Fe_2O_{3(s)} \rightarrow 2Fe_{(s)} + 3CO_{(g)}$ is _____ kJ/mol.

Ans. 492

Sol. The enthalpy change for following reaction

$$3C_{(s)} + Fe_2O_{3(s)} \longrightarrow 2Fe_{(s)} + 3CO_{(g)}$$

$$\Delta H = 3(-110\ kJ/mol) + (822\ kJ/mol) = 492\ kJ/mol$$

84. The total number of correct statements, regarding the nucleic acid is _____

A. RNA is regarded as the reserve of genetic information
B. DNA molecule self-duplicates during cell division
C. DNA synthesizes proteins in the cell
D. The message for the synthesis of particular proteins is present in DNA
E. Identical DNA strands are transferred to daughter cells.

Ans. 3

Sol. DNA is the chemical basis of heredity and may be regarded as the reserve of genetic information. DNA is exclusively responsible for maintaining the identity of different species of organisms over millions of years. A DNA molecule is capable of self duplication during cell division and identical DNA strands are transferred to daughter cells. Another important function of nucleic acids is the protein synthesis in the cell. Actually, the proteins are synthesised by various RNA molecules in the cell but the message for the synthesis of a particular protein is present in DNA.
Hint: Statement B, D and E are correct.

*85. The pH of an aqueous solution containing 1 M benzoic acid (pK_a = 4.20) and 1 M sodium benzoate is 4.5. The volume of benzoic acid solution in 300 mL of this buffer solution is _____ mL.
(Given : log 2 = 0.3)

Ans. 100

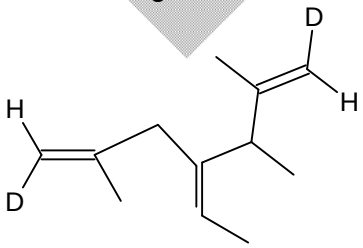
Sol. $$pH = pK_a + \log\frac{[\text{benzoate}]}{[\text{benzoic acid}]}$$

$$4.5 = 4.2 + \log\left(\frac{(300-V)/300}{V/300}\right)$$

$$\frac{300-V}{V} = 10^{0.3} = 2$$

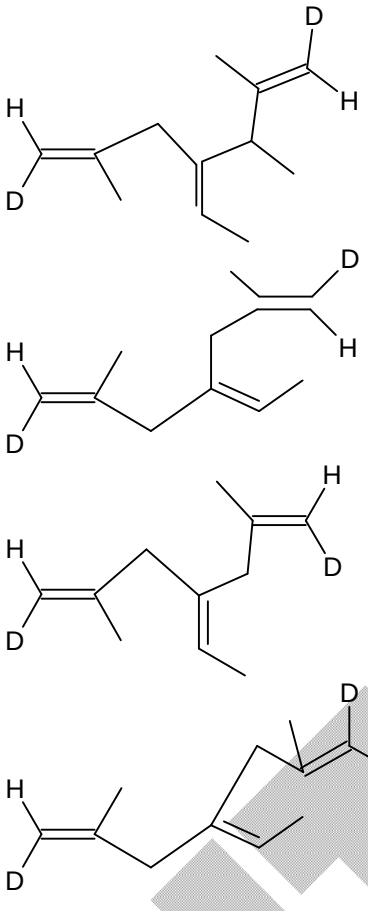
$$V = 100\ ml$$

*86. Number of geometrical isomers possible for the given structure is/are _____ .



Ans. 4

Sol.



*87. Total number of species from the following which can undergo disproportionation reaction is _____.

H_2O_2, ClO_3^-, P_4, Cl_2, Ag, Cu^{+1}, F_2, NO_2, K^+

Ans. 6

Sol. Total 6 species can undergo disproportionation

H_2O_2, ClO_3^-, P_4, Cl_2, Cu^{+1}, NO_2

*88. Number of metal ions characterized by flame test among the following is _____ .

Sr^{2+}, Ba^{2+}, Ca^{2+}, Cu^{2+}, Zn^{2+}, Co^{2+}, Fe^{2+}

Ans. 3

Sol. Only metal give flame test

$Ba^{+2} = \text{yellow} - \text{green}$

$Sr^{+2} = \text{Red}$

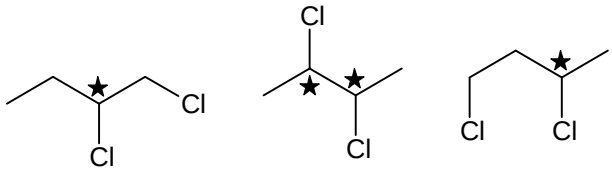
$Ca^{+2} = \text{orange}$

*89. 2-chlorobutane $+Cl_2 \rightarrow C_4H_8Cl_2(\text{isomers})$

Total number of optically active isomers shown by $C_4H_8Cl_2$ obtained in the above reactions is _____

Ans. 6

Sol.



*90. Number of spectral lines obtained in He^+ spectra, when an electron makes transition from fifth excited state to first excited state will be

Ans. 10

Sol. Number of spectral lines $= \frac{\Delta n(\Delta n+1)}{2} = \frac{4(4+1)}{2} = 10$ spectral lines